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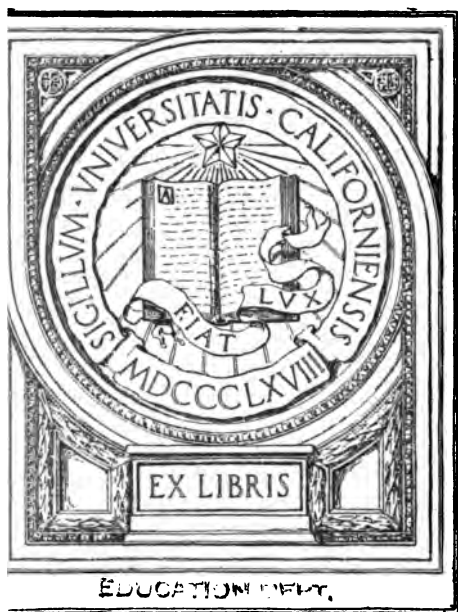


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THE SOUTHWORTH-STONE ARITHMETIC

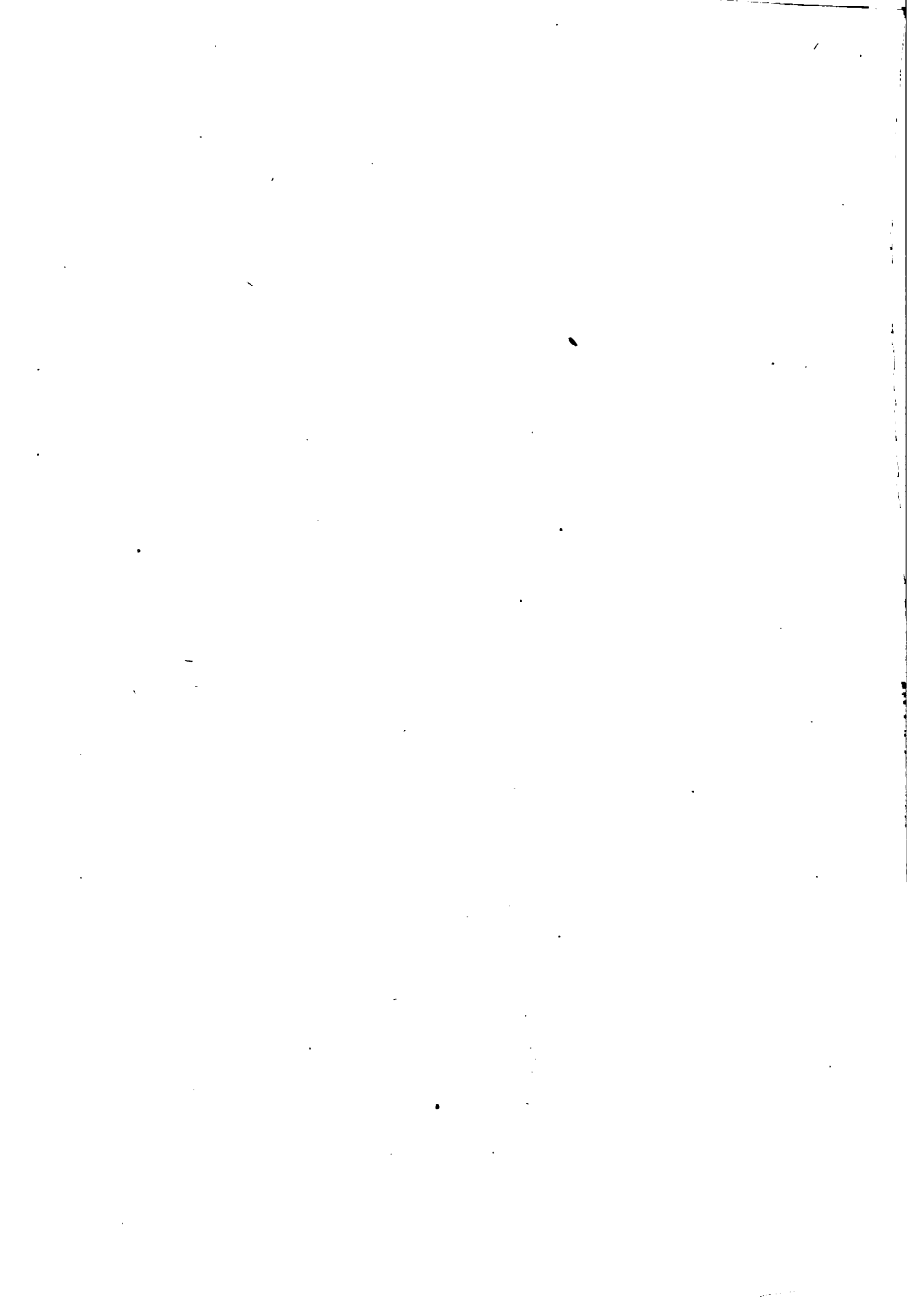
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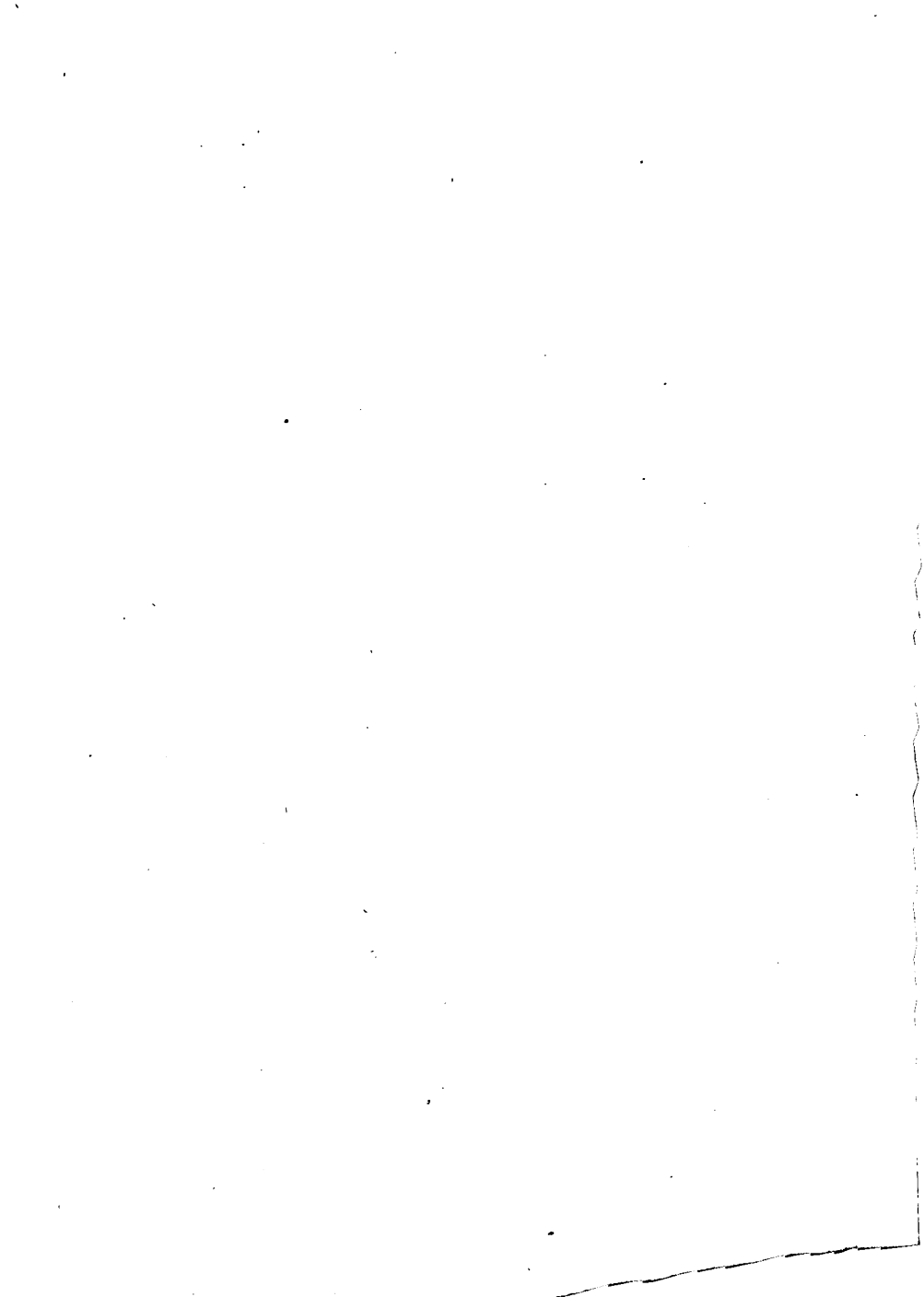
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THE

SOUTHWORTH-STONE ARITHMETIC

A RATIONAL METHOD

BOOK III. FOR ADVANCED GRADES

GORDON A. SOUTHWORTH

SUPERINTENDENT OF SCHOOLS, SOMERVILLE, MASS.

AND

JOHN C. STONE, A.M.

ASSOCIATE PROFESSOR OF MATHEMATICS, STATE NORMAL COLLEGE,
YPSILANTI, MICHIGAN

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BENJ. H. SANBORN & CO.

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TO ALL
ADVERTISERS

THE
SOUTHWORTH-STONE ARITHMETIC.

BOOK I. PRIMARY.

BOOK II. INTERMEDIATE.

BOOK III. ADVANCED.

WITH OR WITHOUT ANSWERS.

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PREFACE

"The Southworth-Stone Arithmetic" is a graded series of three Books each separated into two Parts. The series is designed to cover the work of all the elementary grades in which a text-book is commonly used, beginning with the third-year grade and ending with the last year below the high school.

The books have been prepared not by theorists to exploit their peculiar notions, but by teachers of long and successful experience. They follow the order of subjects and the lines of development established by the highest educational authorities.

No attempt has been made to follow the so-called "spiral plan," now decadent; each grade, however, thoroughly reviews and carries forward the work of the preceding grades, new topics being introduced in order to stimulate the interest of the student and to develop his power.

In the presentation of subjects the inductive method has been employed throughout in a way that calls for study and effort and secures that mathematical training that never comes by mechanical figuring and imitation. This logical development of subjects differentiates the series from mere books of problems.

To secure skill and proficiency in the more important subjects, abundant exercises for drill and practice have been provided. A profusion of oral and written problems is given in about equal proportion. The number to be used must depend upon the need of the student. It will be found that fewer problems carefully solved and logically analyzed will be more valuable than many mechanically performed.

Many subjects heretofore treated in arithmetics have been omitted as non-essential or beyond the legitimate work of the ele-

mentary schools. Enough has been given, however, to meet the demands of business and to furnish the requisite mental discipline.

The methods employed in all the books of the series have been tested in manuscript in the model or training classes in the State Normal College at Ypsilanti, Michigan. The authors acknowledge their indebtedness to Miss Abigail Roe and Miss Mary Steagall and other teachers in that institution for valuable suggestions growing out of such tests. Especial thanks are due to President L. H. Jones of the College, for his counsel as the work has progressed and for his aid in making the books worthy of adoption and use.

Part I of this **Third Book** of "The Southworth-Stone Arithmetic" Series presents, with more prominent reference to principles, a brief review of the fundamental processes, common fractions, ratio, and decimals. It gives a complete presentation of mensuration and its applications to the six quadrilaterals, to triangles, to circles, to rectangular prisms, and to the cylinder. Tables of weights and measures in full are given for reference. Problems, both oral and written, abound.

Part II takes up the subject of percentage analytically and in a way to show that it is only a restatement of principles and processes already familiar, under new names. Its application to business problems, insurance, commission, stocks, bonds, taxes, etc., is fully illustrated. Interest, — simple, exact, compound, — partial payments, bank discount, exchange, proportion, square root and its applications, mensuration of pyramids, cones, spheres, and similar triangles, the metric system, longitude and time, are followed by a large number of practical problems that afford a complete review of all subjects. Definitions of all technical terms used in the book are alphabetically arranged for easy reference and review. (See Index.)

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NOTE.—In this table of contents italics are used to indicate work in which the emphasis is placed on review, practice, drill, and test of power.

Parentheses are used to indicate coördinated or correlated subjects, or to show the purpose of the work covered by the main title.

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THE SOUTHWORTH-STONE ARITHMETIC

THIRD BOOK

PART I

NUMBERS: USE, NAMES, AND NOTATION

1. What need of numbers has a merchant? A carpenter? A farmer? A tailor? A surveyor? A capitalist?
2. What does thirteen mean? Fourteen?
3. Explain the meaning of all the numbers from 13 to 19.
4. What does the syllable *-teen* mean?
5. What does twenty mean? Thirty? Forty?
6. What does the syllable *-ty* mean?
7. What do we call 10 tens? 10 hundreds? 1000 thousands?
8. Mention some numbers that are named by a single word.
9. Show how other numbers are named. Give some examples.
10. How many different figures are used in expressing numbers by the system we use?
11. How is it that all numbers can be expressed by only ten figures?

Our system of writing numbers is called the **Arabic system**, for the system was first introduced into Europe by the Arabs, and they were supposed to be the discoverers. Modern historical research shows that the Hindus were the real discoverers, and hence the system is sometimes called the Hindu system.

1. In 5, 50, 500, 5000, how does the 5 change in value?
2. What value does the zero have? Why is it used?
3. The value of a figure depends upon what two things?
4. What does the figure in the first *order* at the right represent?
In the second order? In the third?

5. In 707,070 name the units in each order.

6. Compare the value of each 7 with that of the other 7's.

7. 100 ones = 1 _____. 10 _____ = 1 million.
 _____ hundreds = 10 thousands. 10 _____ = 1 hundred thousand.
 _____ hundreds = 50 tens. 10 _____ = 1 ten thousand.

8. How many units of any order does it take to make one unit of the next order at the left?

9. In our money system how many cents make one dime? How many dimes make one dollar?

Since in our system of writing numbers, and in our money system, ten units of any order make one of the next higher, we call these **decimal systems**.

Remember that *decem* is Latin for "ten."

In a decimal system ten units of any order make one of the next higher order.

10. In 347,900,903,531 what is the order of each 9? Compare their values.
11. How do the three 3's compare in value?
12. What is the use of the zeros?
13. Why is this number grouped into *periods* of threes?
14. Name each period, beginning with the lowest.

Read without using the word "and":—

1. 4,705	6,137,008	42,200,020	10,063,005
2. 27,003	3,000,975	34,003,007	16,100,005
3. 195,006	600,001	93,040,075	26,013,200
4. 70,590	17,080,005	349,000,672	85,003,017
5. 100,054	2,003,500	34,206,000,127	93,090,006

The next three periods after millions are *billions*, *trillions*, *quadrillions*.

6. Mention something counted in millions. Can you think of any use for billions, trillions, or larger numbers?

Read the following:—

7. The population of New York State in 1900 was 7,268,894. The school enrollment of the state was 1,242,416.

8. The total population of the United States in 1900 was 84,233,069.

9. In the twelve months ending with March, 1903, the exports of the United States were \$1,414,786,954, against \$1,001,596,683 of imports.

10. In 1900 the United States produced 2,105,102,516 bushels of corn valued at \$751,220,034.

11. Write the largest possible number, using these six figures only: 0, 0, 2, 9, 3, 7. Tell why you put each figure in the order that you did.

Write in figures, putting a comma after each period before filling another:—

12. 3 billion, 108 thousand, 11. 15. Ten billion, two million, sixty.

13. 828 million, 7 thousand, 9. 16. One less than a billion.

14. 200 million, 76. 17. The sum of 18000, 200000, 520.

1. In the number 6 ft., what is the *unit* of measure?

2. Name the unit in each of the following :—

6 bu.; 12 in.; 60 ft.; $\frac{3}{8}$ lb.; $\frac{9}{16}$ in.

Any quantity with which another quantity of the same kind is *compared* or *measured* is considered a **unit**.

3. What are numbers called whose units are whole things?

4. Numbers whose units are parts of whole things are what?

5. How many **fractional units** are made by cutting a thing into three equal parts? Into four? How do these units compare in size? Then which is larger, $\frac{3}{8}$ or $\frac{1}{4}$? Why?

6. Give the largest possible fractional unit. Explain your answer.

7. Give a very small fractional unit. How many of these make 1?

Write the fraction denoting the following :—

	NUMBER OF UNITS	THE FRACTIONAL UNIT
8.	3	one fourth.
9.	5	one eighth.
10.	2	one fifth.
11.	7	one ninth.

12. Illustrate the following numbers by drawings :—

$\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{3}$.

13. Read the numbers in the order of their size, the largest first.

14. How many of each would make 1?

15. Upon what does their size depend?

16. How many fractional units in $\frac{3}{8}$, $\frac{7}{10}$, $\frac{11}{12}$, $\frac{9}{15}$, $\frac{14}{15}$?

17. How many of each size make 1? Which of the fractions are nearest 1?

1. The two terms of a fraction are — and —.
2. Which term shows *how many* units the fraction contains?
3. Which term shows *into how many parts* the integral unit has been divided?
4. From which term do you get the *name* of the fractional unit?
5. Give the numerator and denominator and the use of each:—
 $\frac{2}{3}$; $\frac{4}{5}$; $\frac{3}{12}$ yr.; $\$ \frac{5}{8}$; $\frac{4}{7}$ wk.; $\frac{7}{18}$ lb.; $\frac{5}{9}$ gal.
6. Name the fractional unit in each. How many of each make one integral unit?
7. How many kinds of units in $\$ 5\frac{3}{10}$? In $3\frac{1}{2}$ ft.?
8. An integer with a fraction added is called a — number.

DECIMAL FRACTIONS

1. Compare 1 with 0.1; 0.1 with 0.01.
2. In our decimal system of writing numbers how may we give to any figure a certain value and then a tenth of that value?
3. What effect upon the value of a figure has moving it one place to the *right*? To the *left*?
4. What fractional units must fractions have before they can be expressed decimally?
5. How do you know the denominator, or size of the unit, in decimal fractions?
6. For what is the decimal point used?
7. Name the size of the units in each order, beginning at the first order at the right of ones, and reading to the right.

- I. Read the following. II. Give numerator and denominator.
III. Give the value of each figure separately.

- | | | |
|-----------|-----------|-------------|
| 1. 3.6. | 5. 4.053. | 9. 0.1478. |
| 2. 0.54. | 6. 0.765. | 10. 16.47. |
| 3. 2.05. | 7. 0.249. | 11. 18.475. |
| 4. 0.003. | 8. 0.319. | 12. 9.0364. |

13. Which are mixed decimals ?
14. Where is *and* used in reading decimals ?
15. If you should express the above decimals as common fractions, how would the number of decimal places compare with the number of zeros in the denominator ?

ABSTRACT AND CONCRETE NUMBERS

1. 3 ft.; 6 in.; 17; 19; $\frac{5}{8}$ yd.; $\frac{3}{4}$ da.
Which of these numbers are **concrete**, that is, associated with something? Which are **abstract**, that is, which are used alone without reference to any particular thing?
2. Classify: \$275; 362 lb.; 875; 1000; 600 acres; 90 men.
3. What is the unit in each of these numbers ?
4. Select those having *like* units, that is, those having units of the same *kind* and *size*
5. Tell what kind of number, and the unit of each:—
4 ft.; \$2; $\frac{3}{8}$ yd.; \$0.10; 18; 0.36; 16 ft.; $\frac{1}{8}$ yd.
6. Give the integral and the fractional unit in the following:—
 $4\frac{1}{8}$ dozen; $21\frac{1}{8}$; $9\frac{1}{2}$ quarts; 2.1 seconds.
7. Which of the following have the same integral unit:—
 $\frac{1}{4}$ lb.; $\frac{1}{2}$ oz.; $\frac{1}{4}$ ton; 2000 lb.; 1 ton; $\frac{1}{2}$ cwt.
8. Change the unit without changing the value:—
36 in.; 6 ft.; $1\frac{1}{2}$ ft.; 120 sec.; $\frac{1}{8}$ hr.; $\frac{1}{2}$ wk.

1. What is meant by a decimal system ?
2. Why may 1.23 represent dollars, dimes, and cents and not yards, feet, and inches ?
3. How many dimes represented in \$12.625 ?
4. How are dimes usually read ? What does the 5 represent ? How is it usually read ?
5. Read and explain the use of the zeros : \$7.77 ; \$7.07 ; \$7.7 ; \$7.70. Does \$7.7 differ in value from \$7.70 ?

Read the following, (1) as dollars, cents, and mills ; (2) as dollars and cents ; (3) as dollars and thousandths : —

- | | | |
|----------------------------|----------------|--------------------------------|
| 6. \$38.19. | 10. \$0.625. | 14. \$64 $\frac{16}{100}$. |
| 7. \$5.19 $\frac{1}{2}$. | 11. \$309.083. | 15. \$290 $\frac{112}{1000}$. |
| 8. \$24.07 $\frac{1}{2}$. | 12. \$400.040. | 16. \$80,025.95. |
| 9. \$36.05 $\frac{1}{2}$. | 13. \$64.375. | 17. \$8005.025. |
18. Write the preceding numbers from dictation
 19. Express as dollars and cents and mills : —
\$2 $\frac{1}{4}$; \$30 $\frac{1}{10}$; \$ $\frac{7}{8}$; \$24 $\frac{1}{8}$; \$5 $\frac{3}{8}$; \$6 $\frac{1}{8}$.
 20. Name four silver coins. What is a nickel ?
 21. What other metal is coined ? Into coins of what value ?
 22. What is an eagle ? A double eagle ? A quarter eagle ?
 23. What is a mint ? What is bullion ?
 24. Are mills ever coined ? Of what use are they ?
 25. What is counterfeit money ?
 26. What gives value to paper money ?
 27. Are United States coins made of pure silver or of pure gold ? Why is an alloy used ?
 28. Find what is meant by 18-carat gold.

1. Give the sum of 7 and 8. If you had not learned this sum, how could you find it by counting?

2. Find and explain a quick way of adding the following:—

6	1	6	9	3	17
4	9	4	6	6	83
7	6	2	1	7	36
3	4	7	4	4	64
2	3	8	5	5	32
8	7	3	2	2	68
<u>5</u>	<u>2</u>	<u>2</u>	<u>5</u>	<u>8</u>	<u>95</u>

Explain what change you make before adding:—

3. 3 wk. and 14 da.

4. 3 yd., 7 ft., and 24 in.

5. In Exercise 4, why not change the 3 yd. and 7 ft. to inches?

Principle. *Before unlike units can be combined into one sum their units must be made alike.*

6. 2 gal. + 6 pt. = — pt.; 48 oz. + 2 lb. = — lb.

7. 7 yr. + 96 mo. = — yr.; $\frac{7}{12} + \frac{1}{4} = \frac{?}{12}$.

8. 9 ft. + 108 in. = — ft.; 6 yd. + 36 in. = — in.

9. $\frac{2}{3} + \frac{1}{6} = \frac{?}{6}$; $\frac{3}{4} + \frac{1}{8} = \frac{?}{8}$; $\frac{1}{4} + \frac{2}{3} = \frac{?}{12}$.

Practice until you can give the sum of these numbers instantly:—

10. 46, 34.

19. 127, 123.

28. 3000, 1798, 2000.

11. 19, 71.

20. 900, 140.

29. 1300, 2000, 175.

12. 53, 47.

21. 560, 240.

30. 4080, 1507, 6000.

13. 86, 32.

22. 767, 232.

31. 85, 300, 9000.

14. 38, 69.

23. 808, 191.

32. $\frac{2}{15}$, $\frac{7}{15}$, $\frac{3}{15}$, $\frac{1}{15}$.

15. 47, 46.

24. 346, 509.

33. 0.06, 0.18, 0.24.

16. 65, 25.

25. 888, 212.

34. 25%, 8%, 30%.

17. 32, 99.

26. 694, 106.

35. 0.41, 0.19, 0.40.

18. 72, 88.

27. 333, 766.

36. \$4.75, \$3.25, \$7.87

Give directions for five steps in adding:—

I. Arranging the numbers.	378
II. Beginning to add.	492
III. Setting down the sum.	864
IV. "Carrying."	793
V. Checking.	956
	<u>3483</u>

Without copying, first add vertically; then horizontally:—

	1.	2.	3.	4.	5.
6.	\$3.47	\$14.69	\$193.67	\$4769.83	\$6483.47
7.	8.62	48.96	846.84	4392.16	8432.97
8.	9.46	37.81	932.71	8437.66	6432.98
9.	6.58	47.94	683.77	6989.84	8469.32
10.	7.39	82.66	765.75	4329.41	9396.48
11.	<u>9.88</u>	<u>68.43</u>	<u>392.50</u>	<u>6832.47</u>	<u>9375.58</u>

12. Find the sum of the five sums of the columns.

13. Find the sum of the six sums of the lines.

14. Why should these two sums be equal?

Add upward and check by adding downward:—

15.	16.	17.	18.	19.
\$475.21	\$648.93	\$719.63	\$963.94	\$679.83
649.85	973.26	834.56	738.42	759.94
837.64	387.92	784.97	697.18	678.90
246.89	814.78	469.38	346.32	543.21
937.48	687.34	847.86	923.76	783.94
<u>742.37</u>	<u>968.47</u>	<u>952.78</u>	<u>768.93</u>	<u>989.76</u>

A

679,458 a

340,276 b

950,673 c

268,479 d

728,735 e

629,876 f

724,894 g

548,975 h

829,386 i

445,876 j

377,872 k

763,874 l

689,983 m

670,498 n

988,875 o

687,568 p

994,693 q

849,376 r

649,478 s

384,923 t

569,247 u

347,964 v

976,394 w

298,973 x

917,966 y

279,976 z

20. Make frequent use of Col. A for practice in rapid adding.

1. 8 from 17 leaves what? If you had forgotten, how would you have found out that $17 - 8 = 9$?

2. Make a problem in subtraction, using concrete numbers.

3. Which is the *subtrahend*, and which the *minuend*?

4. The three terms used in subtraction are —, —, and —.

5. The largest term is —.

6. How do you find the third term when you have the *difference* and the *subtrahend*?

7. How is the third term found from the minuend and difference?

8. Which is the larger number, 3 ft. or 24 in.? Which is the larger quantity? How can one be subtracted from the other?

9. A boulder weighs 7000 lbs., a stone block $\frac{1}{2}$ ton. Find the difference in weight and explain the process.

Where you can, give the difference, first *like the minuend*, then *like the subtrahend*, in the following:—

10. 4 lbs. — 32 oz. = —. 12. 10 hr. — 240 min. = —.

11. 60 mo. — 2 yr. = —. 13. 2 T. — 2 lb. = —.

Give in one minute or less the difference between each number and the one below it; between each number and the one at the right of it.

a	11	9	13	7	10	13	12	7	15	9	18	14
	2	5	7	3	9	8	6	5	8	4	8	5
b	9	9	10	16	11	6	14	11	15	11	8	16
	3	6	3	8	6	3	9	7	9	8	4	9
c	15	12	7	10	15	17	14	12	11	8	13	10
	7	3	4	5	6	9	8	7	5	5	4	7
d	12	10	17	12	11	13	16	13	18	12	8	12
	8	4	8	9	3	5	7	9	9	5	3	4

1. From 97 count backward rapidly by 6's; 8's; 9's; 12's.
2. From 200 count backward rapidly by 15's; by 12's; by 22's.
3. Give the difference between 100 and each of the following:—

a.	11 88	44 74	52 70	36 13	60 67	37 48	31 87
b.	35 61	82 14	91 33	22 65	42 53	15 59	69 47
c.	83 30	23 57	89 43	95 29	68 32	84 26	79 16
d.	55 81	73 72	21 94	56 99	17 93	41 66	58 51
e.	92 19	63 45	96 18	86 46	76 49	71 62	27 85
f.	64 77	24 28	34 39	78 38	80 25	97 54	98 40

4. Give the difference between each number and the one at its right.
5. Give the difference between each number and the one below it.
6. From 1000 take 120, 175, 225, 350, 760, 807, 901.
7. Take each number in the table from 173. From 182.

What change from a \$ 5.00 bill in payment for :—

- | | | |
|----------------------|---------------------|--------------------|
| 8. Oysters, \$ 0.75; | 9. Gloves, \$ 1.25; | 10. Pens, \$ 0.35; |
| Crackers, .038; | Scarf, 0.75; | Ink, 0.15; |
| Cheese, 0.62? | Pin, 2.50? | Paper, 0.88? |

Subtract at sight :—

11.	12.	13.	14.	15.	16.
700	3000	60503	25000	35111	36459
<u>325</u>	<u>800</u>	<u>40402</u>	<u>37892</u>	<u>46221</u>	<u>47560</u>

Find what remains after receiving and paying as shown below :—

RECEIVED	PAID	RECEIVED	PAID	RECEIVED	PAID
17. \$ 1.16	\$ 0.93	18. \$ 45.00	\$ 28.00	19. \$ 2.25	\$ 1.75
0.24	0.17	95.00	19.00	3.75	2.30
0.60	0.25	70.00	23.00	1.25	1.25

From 683 take 457.

PROCESS
 Minuend, 683
 Subtrahend, 457
 Remainder, $\underline{226}$

1. If you try to subtract one order at a time, what is your first difficulty ?

2. If you had 83 sticks in bundles of 10 each, with 3 sticks over, how would you subtract 7 sticks ?

3. How many bundles would remain ? How many sticks over would remain ?

4. You would then take 5 tens from what ?

5. What terms may be added to check the work ?

6. Give directions for each separate step in the process.

Written Exercise for Drill

Without copying, find quickly the sum of the four differences between : —

1. *e* and *f*. 3. *g* and *h*. 5. *i* and *j*. 7. *k* and *l*. 9. *m* and *n*.
 2. *f* and *g*. 4. *h* and *i*. 6. *j* and *k*. 8. *l* and *m*. 10. *n* and *e*.

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>e</i> .	\$ 3764.82	\$ 4769.31	\$ 5000.37	\$ 9000.15
<i>f</i> .	927.35	3468.97	689.82	794.38
<i>g</i> .	860.83	385.68	1348.75	1866.75
<i>h</i> .	1527.96	2487.32	946.	2889.43
<i>i</i> .	3784.98	694.39	37.89	648.95
<i>j</i> .	2876.45	1748.64	9586.34	1864.37
<i>k</i> .	825.35	4839.87	829.85	624.94
<i>l</i> .	96.47	658.34	1472.98	1739.41
<i>m</i> .	849.33	1987.62	468.32	866.
<i>n</i> .	3276.41	594.83	5500.31	49.75

11. Calling *A* and *B* the two sides of an account, find the balance.

12. Do the same with *C* and *D*.

13. With *B* and *C*.

What will balance : —

14. *A* and *C*.

15. *D* and *A*.

16. *B* and *D*.

1. Four parts of 75 are 18, 9, 13 and 22. Find the fifth part.
2. 37 gallons are in a tank. While 23 gallons run out, 17 run in. How many gallons remain?
3. An engine goes forward 25 rd., back 38 rd., forward 60 rd. How far is it from the starting point?
4. How much farther is it around a 17-foot square than around a 13-ft. square? How did you get your result?
5. By annexing to 57 the figure 6, how much is added?
6. Bought a pony and phaeton for \$500. Sold the pony for \$175, losing \$50. What did the phaeton cost?
7. Having \$400 in the bank, a person draws \$25, deposits \$150, draws \$75 and \$50. How much remains?
8. One horse is worth \$50 more than a second and \$150 more than a third. If the highest priced one is worth \$200, what are they all worth?
9. If you get 5 eggs one day and 6 the next, how many dozen will you get at this rate in a week?
10. How far around a rectangle 15 feet long and 10 feet wide?
11. Around a rectangle 18 feet long and 12 feet wide?
12. My book contains 170 pages. I have read 82 pages. How many more have I to read?
13. I had \$100 in a bank. At different times I drew out \$12, \$18, and \$20. How much remained?
14. A farmer had 75 sheep. He sold 45 and bought 17. How many did he then have?
15. A trader had 29 horses and bought 57. He then sold 69. How many had he left?
16. In a farm of 160 acres, 23 was woodland, 47 pasture, and the remainder grain. How much in grain?

1. How much remained in bank to Mr. Rich's credit Saturday night, if he put in and took out the following:—

Deposits: \$ 26.95, \$ 793.82, \$ 427.96, \$ 839.64, \$ 500, \$ 387.28;

Withdrawals: \$ 18.56, \$ 689.37, \$ 419.28, \$ 649.39, \$ 600, \$ 125.82?

2. I have on hand at the opening of business, cash to the amount of \$846.95. I pay out \$84.92, \$64.87, and have on hand at night \$837.69. What have I received?

3. I received during the day \$249.85, and I paid out \$521.75. I had on hand at night \$37.62. What had I on hand at the opening of business in the morning?

4. Thomas Bond begins business January 1, with cash \$478.37 and merchandise \$1875.28. At the close of the year he has \$1487.63 worth of merchandise and \$738.29 in cash. How much has he gained or lost during the year?

5. The sum of two numbers is 346,301. The smaller is 89,795. What is the larger?

6. What number must be subtracted from one million to leave the difference between 347,698 and 486,931?

7. The distance from A to B is 628 feet, from A to C 1426 feet, and from B to D 1648 feet, all in a straight line. How far is it from C to D? Draw a line and mark off the distance.

Find the excess of exports from this country when the exports and imports from 1898 to 1902 were as follows:—

YEAR	EXPORTS	IMPORTS
8. 1898	\$1,255,546,266	\$634,964,448
9. 1899	1,275,467,971	798,967,410
10. 1900	1,477,946,113	820,140,714
11. 1901	1,405,375,860	880,419,910
12. 1902	1,360,696,355	696,270,009

13. Find the increase or decrease of imports and of exports from year to year.

1. A **debtor** is one who **owes** another, or is in debt to another. A **debit** is something owed.

2. A **creditor** is one to whom another owes a debt. A **credit** is an amount owed to one's account.

3. An account with "Cash" is, as it were, an account with one's pocket book or cash box. Cash is *debtor*, that is, owes me, for all that is put in, and cash is *credited* with all that is taken out.

<i>Dr.</i>		CASH		<i>Cr.</i>			
1908				1908			
Apr. 1	On hand	\$ 100	00	May 3	By Mdse. bought	\$ 450	00
5	To Rent received	50	00	4	By Piano bought	350	00
7	To Mdse. sold	25	00	8	By Clothing bought	25	00
10	To Land	725	00	11	By Balance	75	00
		900	00			900	00
May 22	On hand	75	00				

4. Cash is charged with having received four amounts, which it owes me, that is, for which it is my debtor. How much on hand at the beginning?

5. What is the total amount Cash has received, that is, it owes me or is debtor for how much?

6. When I take out \$350 with which to purchase a piano, Cash has paid me back how much of what it owes me?

7. What other amounts has Cash paid me, that is, for what other amounts should Cash be credited?

8. How much more has Cash received than paid out?

9. How much more might I have spent so as to balance the footings?

10. For what is Cash debtor at the beginning of the next account?

1. Balance the Cash account of Charles Watson. He has on hand \$4.21. He receives at various times \$6.24, \$7.36, \$8.49, \$7.34, \$6.75. He pays out \$8.75, \$9.81, \$8.39.

2. Monday morning a merchant begins business with \$247.84 on hand. He receives \$24.75, \$86.91, \$84.28, \$97.25, \$164.29. He pays out \$18.99, \$37.49, \$64.91, \$83.15. Find the balance on hand.

Find the balance of each of the following accounts:—

3.		4.		5.	
Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
\$987.65	\$629.55	\$4768.82	\$468.34	\$649.81	\$82.46
1839.76	83.74	947.61	984.59	8439.87	981.32
6482.91	968.71	847.77	1483.22	648.38	641.25
478.85	28.46	3998.64		91.76	239.86
<u>698.47</u>	<u>318.93</u>	<u>8372.91</u>	<u> </u>	<u> </u>	<u>728.41</u>

6.		7.		8.	
Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
\$246.94	\$839.75	\$698.32	\$649.83	\$356.78	\$135.72
839.76	646.81	376.59	478.88	938.12	873.54
842.94	794.32	843.26	694.31	45.23	137.92
327.68	546.78	695.98	883.24	938.83	7639.85
<u>946.32</u>	<u>937.89</u>	<u>831.96</u>	<u>695.64</u>	<u>876.23</u>	<u>736.29</u>

9.		10.		11.	
Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
\$94.68	\$986.84	\$346.85	\$249.65	\$192.48	\$129.76
37.95	69.39	976.87	998.54	765.35	947.34
469.38	74.29	695.79	648.36	362.93	274.36
24.38	83.62	949.83	799.35	15.36	1286.54
6.49	45.39	697.87	869.40	746.29	364.92
<u>17.32</u>	<u>169.38</u>	<u>749.78</u>	<u>309.79</u>	<u>126.43</u>	<u>3647.10</u>

1. In combining unequal numbers, as $9 + 8 + 7 + 4 = 28$, what is the process called?

2. $7 + 7 + 7 + 7$ or $4 \times 7 = 28$. What is each of these two processes called?

3. Could the unequal numbers, **addends**, in Exercise 1, have been combined by a shorter process, as in Exercise 2?

4. If you do not know the product of 5×8 from memory, how may you find it? Which number is to be multiplied? Which is the multiplier?

5. What does the multiplier show? Then can it ever be a concrete number, as 5 men, 5 feet, or 5 strokes?

6. Compare addition and multiplication.

7. What are the three terms in multiplication?

Since the multiplier and multiplicand *make* the product, they are called **factors** (makers) of the product.

Name quickly the factors, less than 14, that produce:—

8. 65, 72, 77, 78.

11. 96, 99, 104, 108.

9. 81, 84, 88, 91.

12. 110, 117, 121, 130.

10. 48, 52, 54, 63.

13. 132, 143, 156, 169.

One of the two equal factors that make a number is called the **square root** of the number. $\sqrt{\quad}$ means "the square root of," thus $\sqrt{25} = 5$; $\sqrt{36} = 6$; $\sqrt{49} = 7$.

14. $\sqrt{64} = \text{—}$; $\sqrt{81} = \text{—}$; $\sqrt{121} = \text{—}$; $\sqrt{144} = \text{—}$.

15. $\sqrt{9 \times 25} = 3 \times 5$ or 15; $\sqrt{16 \times 25} = \text{—}$; $\sqrt{25 \times 36} = \text{—}$; $\sqrt{25 \times 49} = \text{—}$.

16. $\sqrt{49 \times 36} = \text{—}$; $\sqrt{36 \times 81} = \text{—}$; $\sqrt{9 \times 144} = \text{—}$; $\sqrt{36 \times 121} = \text{—}$.

17. $\sqrt{4 \times 16 \times 25} = 2 \times 4 \times 5$ or 40; $\sqrt{4 \times 25 \times 36} = \text{—}$; $\sqrt{9 \times 16 \times 49} = \text{—}$.

1. — \times \$8 = \$32; $4 \times$ — = 8 sq. ft.; — \times \$7 = \$56.
2. Give the factors, saying which is the multiplier and which the multiplicand: \$42; 63 ft.; 72 yd.; 12 sq. ft.
3. Make two examples; the multiplicand concrete in one and abstract in the other.
4. What kind of a number was the product in each case?
5. Can the multiplier be concrete? Why?
6. Show by objects that 3×4 things of a kind are 12 of that same kind.
7. Give the factors of \$21; 35 miles; 18 cases; 49 men.
8. Compare 4×5 bu. and 5×4 bu.

Give each product quickly, stating which factor is multiplicand:—

9.	10.	11.	12.	13.	14.	15.
\$800	600	700 rd.	900	800 da.	400	6000
<u>9</u>	<u>12</u> yd.	<u>8</u>	<u>9</u> mi.	<u>12</u>	<u>16</u> men	<u>13</u>

PRINCIPLES. I. *Only one factor can be concrete, both may be abstract.*

II. *The product and the concrete factor are like numbers.*

III. *The order in which factors are used will not affect the product.*

Give rapidly the products:—

16. $4 \times \$8$; $4\frac{1}{2} \times \$8$; $3 \times 8\%$.
17. $13 \times \$3$; $5 \times \$\frac{3}{4}$; 2×37 .
18. $2 \times 37\frac{1}{2}$; $5 \times 12\frac{1}{2}$; $4 \times \$1\frac{1}{2}$.
19. 3×19 ; 11×12 .
20. 13×5 ; 12×9 .
21. 16×4 ; 17×5 .
22. Multiply the following by 8; by 9; by 12:—
7 bales, 70 bales, 80 rods, 30 miles, 90 feet.
23. Multiply by 9 and add 9:—
7, 8, 4, 6, 12, 5, 11, 13, 3, 15, 19.
24. How could you have got the same result in a shorter way?

1. Give two factors making: —
62; 72; 48 rods; 91 feet; 78; 56 days; 81 men.
2. Give two factors of 132 sec.; 125 in.; 144¢; 108 hr.
3. Take 4×7 from 9×7 . How can you do this without finding the products of 4×7 and 9×7 ?
4. From 9×8 take 6×8 in the same way.
5. Take 15×9 from 22×9 ; 16×7 from 21×7 .
6. Add 18×13 to 2×13 . To do this did you need to know the product of 18×13 ?
7. Add 17×25 to 13×25 in the same way.
8. 3×6 in. + 7×6 in. = — in.
9. Add at sight 37×25 to 3×25 .
10. From 63×8 take 13×8 .
11. 3^2 means 3×3 or 9; 5^2 means 5×5 or 25. Find 4^2 ; 6^2 ; 7^2 ; 8^2 ; 9^2 ; 10^2 ; 11^2 ; 12^2 ; 20^2 ; 30^2 .
12. Find 40^2 ; 50^2 ; 60^2 ; 70^2 ; 90^2 .
13. Compare 5 and 50. Tell how you would multiply a number by 10.
14. Compare 5 and 500. Tell how you would multiply a number by 100.
15. How would you multiply by 1000, 10,000, etc.?
16. Multiply the following by 10, 100 and 1000: —
39, 28, 147, 261, 375, 140.

Principle. *Every zero annexed to an integer multiplies it by 10.*

17. Compare 20×5 and $2 \times 10 \times 5$; 50×7 with $5 \times 10 \times 7$.

A	1. Under A what are added to get the result?	B	C
489		489	489
<u>7</u>	2. Explain the position of each product and its real value.	<u>7</u>	<u>70</u>
63		3423	34230
<u>56</u>			
28	3. Show how the same result was got under B without setting down the <i>partial products</i> .		
3423			

4. Compare the work under B and C, and tell how to multiply by 7 tens. By any number of 10's, 100's, etc.

Find quickly the product:—

- | | | |
|----------------------|-----------------------------|------------------------------|
| 5. 6×4763 . | 9. $80 \times 84,965$. | 13. $80 \times 72,397$. |
| 6. 8×4931 . | 10. $70 \times 12,039$. | 14. $70 \times 95,364$. |
| 7. 9×6989 . | 11. $300 \times 842,794$. | 15. $34,000 \times 91,000$. |
| 8. 7×5987 . | 12. $50 \times 1,203,900$. | 16. $28,000 \times 75,000$. |

PROCESS

Multiplying by any Integer

- | | |
|------------------------------|---|
| 578 | 1. In the work at the left read the multiplier. |
| <u>346</u> | |
| $3,468 = 6 \times 578$. | |
| $23,120 = 40 \times 578$. | |
| $173,400 = 300 \times 578$. | |
| $199,989 = 346 \times 578$. | 2. What three partial products are used? |
| | 3. Explain how each is obtained. |
| | 4. Would the result have been changed if we had multiplied by 300 first? By 40 first? |

5. In ordinary work what is omitted from each partial product?
6. Where is the lowest digit of each partial product written if the zero is omitted?

Give directions for six steps in multiplying:—

- | | |
|-------------------------------|--------------------------------|
| 1. Arranging the factors. | 4. Arranging partial products. |
| 2. Beginning to multiply. | 5. Finding the entire product. |
| 3. Setting down and carrying. | 6. Checking the work. |

1. What is the cost of 2378 bbl. flour at \$7?

$\begin{array}{r} \$7 \text{ multiplied by} \\ 2378 \text{ must equal} \\ \hline \$16646 \end{array}$	$\begin{array}{r} \text{for} \\ 2378 \\ \text{multiplied by} \\ 7 \\ \hline \text{equals} \\ 16646 \end{array}$
---	---

2. Which is the true multiplicand? To shorten the work, why must we use abstract numbers as shown at the right?

Find the product of:—

- | | | |
|-------------------|----------------------|-----------------------|
| 3. 536 and 846. | 8. 64 oz. and 976. | 13. 84 qt. and 96. |
| 4. 3976 and 597. | 9. \$627 and 931. | 14. 346 and 19¢. |
| 5. 37¢ and 432. | 10. 36 doz. and 453. | 15. 8477 and 86. |
| 6. 427 and 83 lb. | 11. 864 and \$45. | 16. 98 oz. and 43. |
| 7. 329 and 347. | 12. 387 and \$8. | 17. 304 posts and 97. |

Find the cost of:—

- | | |
|--|-------------------------------------|
| 18. 1579 bbl. sugar @ \$9.87. | 30. 7859 bu. potatoes @ \$0.79. |
| 19. 6496 A. of land @ \$34.89. | 31. 42,000 lb. coffee @ \$0.23. |
| 20. 12,865 T. of coal @ \$4.87. | 32. 6425 pairs shoes @ \$2.25. |
| 21. 16,492 lb. cotton @ \$0.09. | 33. 176,000 bricks @ \$12.84 per M. |
| 22. 24,975 lb. wool @ \$0.35. | 34. 45,761 ft. granite @ \$0.75. |
| 23. 6425 cords wood @ \$6.98. | 35. 43,628 sq. ft. land @ \$0.29. |
| 24. 47,000 ft. lumber @ \$42.25 per M. (per 1000 ft.). | 36. 379 T. ice @ \$4.67. |
| 25. 5600 bu. wheat @ \$0.89. | 37. 47,000 shingles @ \$3 per M. |
| 26. 2745 bbl. cement @ \$1.25. | 38. 87 gal. alcohol @ \$2.65. |
| 27. 47,892 gal. oil @ \$0.08. | 39. 5375 bu. corn @ \$0.87. |
| 28. 135 bbl. pork @ \$16.87. | 40. 3784 lb. butter @ \$0.22. |
| 29. 2892 kgs. nails @ \$3.09. | 41. 967 doz. eggs @ \$0.18. |
42. How many rods in 54,000 miles?
43. How many square inches in 937 square feet?
44. How many seconds in 18 days?

22 MULTIPLICATION; ANALYSIS; STATEMENT OF PROBLEMS

Oral and Written

Give at sight:—

1. $17 \times \$9 + 3 \times \$9 = x$.

4. $125 \times \$9.85 - 25 \times \$9.85 = x$.

2. $87 \times \$7.50 + 13 \times \$7.50 = x$.

5. $937 \times \$125 + 63 \times \$125 = x$.

3. $64 \times \$48 + 36 \times \$48 = x$.

6. $157 \times 316 \text{ ft.} - 57 \times 316 \text{ ft.} = x$.

Use the same method in finding:—

7. $649 \times \$12.84$ less $145 \times \$12.84$.

8. $317 \times \$19.83 + 524 \times \$19.83 + 160 \times \$19.83$.

9. $973 \times 846 + 973 \times 352 - 973 \times 198$.

Analysis and Statement of Problems

Much work may be saved in solving problems if a statement is made showing all that is to be done to get the result before any figuring is done.

1. Which is easier to do; to reason about a problem so as to show how it may be solved, or to figure out the result after being told how to solve it?
2. From which do you learn more?

3. A statement of all that is to be done to get the result written as equal to something denoted (for convenience) by the letter x , is called an equation. Make a problem in which the equation $4 \times 16\frac{1}{2} = x$ will indicate the work to be done.

4. \$2.50 was the expressage on 19 tables at \$12.74, and 28 chairs at \$2.58; x was the total cost.

STATEMENT. $\$2.50 + 19 \times \$12.74 + 28 \times \$2.58 = x$ (or the total cost).

Make an equation showing all that is to be done to find the value of x ; then find it:—

5. 130 men at \$2 a day, 47 at \$1, and 8 at \$3.50 receive x dollars in one day.

6. A nursery contains 1000 trees; 75 are dead; the rest are to be sold at \$2 each. They will bring \$ x .

Finding an Unknown Factor

1. The product of two numbers is 48. One is 6. Find the other.
2. How many 9's in 54? What number is contained 9 times in 63? In 108?
3. 12 is the multiplicand; how many times is it taken to make the product 84?
4. What multiplicand, repeated 12 times, makes the product 108?
5. When one factor and the product are known, how is the other factor found?
6. Illustrate by using $x \times \$6 = \42 ; and $5 \times \$x = \45 .
7. Why is the process called division?
8. Show by the examples in Ex. 6 that: —
 - (a) The *product* becomes the *dividend* (something to be divided).
 - (b) The *known factor* becomes the *divisor*.
 - (c) The *unknown factor*, when found, becomes the *quotient*.
 - (d) The quotient shows either, *how many times* the divisor can be taken out of the dividend, or *the size* of each of the equal parts into which the dividend is separated.

Give the quotients: —

- | | |
|------------------------------|---------------------------------|
| 9. $15 \times x = 30$ days. | 12. $12 \times x = 144$ miles. |
| 10. $x \times \$9 = \108 . | 13. $x \times 9$ ft. = 135 ft. |
| 11. $6 \times x = 120$. | 14. $x \times 11$ yd. = 131 yd. |

15. Give the quotients in the following: —

$$\begin{array}{r} 6 \overline{)120}; 96 \div 24 = x. \quad \frac{360}{18} = x; 72 : 4 = x. \\ x \qquad \qquad \qquad 18 \end{array}$$

16. Describe the *four ways of indicating division* shown in the preceding line.

17. Find any two factors: — 96 bbl.; 91 days; 168 hr.; 182¢.

1. How many \$10 bills make \$300? If one factor of 60 yd. is 10 yd., what is the other?

2. In multiplying two factors to make a product, which factor is always abstract? Which may be concrete?

3. In division, which term corresponds to the product? Which to the factors?

4. Can both dividend and divisor be concrete?

5. Show by the first example on the page that if the dividend is concrete and the divisor like the dividend, the quotient is abstract.

6. If 8 hats cost \$40, what will 1 hat cost?

7. A rod 90 inches long was cut into 10 equal parts. How long was each part?

8. Show by the above example that if the dividend is concrete, and the divisor abstract, the quotient shows the *size* of the equal parts into which the dividend has been divided.

Principles. I. *A divisor that is like the dividend is one of its equal parts and the quotient shows the number of these parts.*

II. *An abstract divisor of a concrete dividend shows into how many equal parts the dividend is to be divided and the quotient shows the size of one of these parts.*

9. When 20 books cost \$40, what part of it will one cost?

10. One factor is 15. The product is 90 yards. What is the other factor?

11. The divisor is 12, the dividend is 108 bushels; what is the quotient?

12. The divisor is 9 ft., the quotient is 11; find the dividend.

13. The dividend is 2100 mi., the divisor 21 mi.; find the quotient.

14. At 17¢ a yard, how many yards can be bought for \$1.53?

1. Division is the reverse of —; since $8 \times 9 = 72$, $72 \div 8 = 9$ and $72 \div 9 = 8$.

2. If you had not known that $8 \times 9 = 72$, how might you have found the number of 9's in 72?

3. Find by subtraction the number of 12's in 60; of 24's in 96.

4. How many 12's in 1740?

5. Are there 200 12's in 1740?

6. Are there 100? Subtract them; what remains?

$$\begin{array}{r} \text{A} \\ 12 \overline{)1740} \\ \underline{1200} = 100 \text{ 12's} \\ 540 \end{array}$$

7. How many 12's in 540? Are there 50?
Are there 40?

$$\begin{array}{r} 480 = 40 \text{ 12's} \\ \underline{60} \end{array}$$

8. Subtract 40 12's; what remains?

$$\begin{array}{r} 60 = 5 \text{ 12's} \\ \text{Total} \quad \underline{\quad} = 145 \text{ 12's} \end{array}$$

9. $60 =$ how many 12's? Subtract them; what remains?

10. How many 12's in all have been taken from 1740 by the three subtractions?

11. In practice what part of the work might be omitted? Where may 1. be written to show by its position that it stands for 100? Where may 4 be written to show it stands for 40?

In practice the work is written as in the margin.

$$\begin{array}{r} \text{B} \\ 145 \end{array}$$

12. *Short Division.* Perform aloud the work of C.
How does short division differ from long division?

$$\begin{array}{r} 12 \overline{)1740} \\ \underline{12} \end{array}$$

13. If 3465 is divided by 58, in what place should the first quotient figure be written?

$$\begin{array}{r} \text{C} \\ 12 \overline{)1740} \\ \underline{145} \end{array}$$

14. $3465 \div 19$. How many figures in the quotient?

$$\begin{array}{r} 54 \\ \underline{48} \\ 60 \\ \underline{60} \end{array}$$

15. When should short division be used?

16. How may division be tested or checked?

17. How may the process of multiplication be proved correct?

1. How many 15's in 4650 ?
2. $27 \times \text{---} = 40,527$.
3. Dividend = 9672; divisor = 372; quotient = --- .
4. 96 and 75 are factors of what dividend ?
5. Product = 33,810; one factor = 245; x = the other.
6. One factor of \$475,000 is \$250; what is the other ?
7. 84 equal numbers make 6300 yards. Find one of them.
8. 360 miles = multiplicand; 25,520 miles = product; x = multiplier.
9. Divisor = \$250; dividend = \$16,750; x = quotient.
10. The divisor = 197; the quotient = \$461; the dividend = x .
11. I received \$1394 for 17 horses. What was the average price ?
12. Divide \$39,624 into 48 equal parts.
13. Find $\frac{1}{8}$ of 3627.
15. Divide 3962 by 49.
14. Find $\frac{1}{47}$ of 39,627.
16. In 3962 ounces, how many pounds ?
17. Divide \$82.36 into 32 equal parts.

Find the quotients:—

- | | | |
|-------------------------|--------------------------|-------------------------|
| 18. $759,470 \div 78$. | 25. $89,175 \div 39$. | 32. $183,974 \div 94$. |
| 19. $624,798 \div 48$. | 26. $284,603 \div 98$. | 33. $265,371 \div 88$. |
| 20. $182,347 \div 57$. | 27. $99,134 \div 49$. | 34. $104,288 \div 78$. |
| 21. $96,343 \div 97$. | 28. $108,264 \div 57$. | 35. $139,267 \div 72$. |
| 22. $192,462 \div 67$. | 29. $346,271 \div 86$. | 36. $204,306 \div 68$. |
| 23. $236,475 \div 77$. | 30. $937,441 \div 163$. | 37. $307,961 \div 96$. |
| 24. $187,931 \div 68$. | 31. $784,267 \div 269$. | 38. $198,001 \div 67$. |
39. Divide each number in Column A, page 9, by the divisor formed by the last 3 figures in the dividend used. Thus: $679,458 \div 458$.

1. Compare 5 with $1\frac{0}{2}$. Compare 5×84 with $1\frac{0}{2} \times 84$.

2. What does $1\frac{0}{2} \times 84$ mean? $\frac{10 \times 84}{2} = \text{what?}$

3. Describe a short method of multiplying by 5.

4. Compare 25 with $1\frac{0}{4}$. Find 25×84 by a short method.

Multiply each of the following by 5 and by 25:—

5. 24. 7. 108. 9. 35. 11. 208. 13. 492. 15. 49.

6. 96. 8. 17. 10. 27. 12. 368. 14. 95. 16. 63.

17. By what must $12\frac{1}{2}$ be multiplied to make 100?

18. Compare $12\frac{1}{2}$ with $1\frac{0}{8}$. Multiply 96 by $12\frac{1}{2}$ by a short method.

19. By what must you multiply $16\frac{3}{8}$ to get 100? $\frac{1}{8}$ of 100 = ?

20. Compare $16\frac{3}{8}$ with $1\frac{0}{8}$. Multiply 42 by $16\frac{3}{8}$.

21. $\frac{1}{8}$ of 100 is what? $33\frac{1}{8} = 1\frac{0}{8}$. Multiply 45 by $33\frac{1}{8}$.

Multiply the following by $12\frac{1}{2}$, $16\frac{3}{8}$, and $33\frac{1}{8}$:—

22. 48. 26. 45. 30. 114. 34. 295.

23. 96. 27. 28. 31. 207. 35. 349.

24. 72. 28. 64. 32. 345. 36. 642.

25. 108. 29. 93. 33. 216. 37. 784.

38. $\frac{1}{8}$ of 1000 = what? Give a short method of multiplying by 125.

39. Multiply by 125: 168; 256; 976; 856; 1040; 375.

40. $5 \times 6 + 8 \times 6 = 13 \times 6$. Why?

41. $13 \times 6 + 13 \times 4 = \text{what?}$

42. Find $17 \times 18 + 23 \times 18 + 60 \times 18$ by a short method.

Find the products:—

43. 25×64 . 46. $16 \times 12\frac{1}{2}$. 49. 125×912 . 52. $16\frac{3}{8} \times 1026$

44. 5×82 . 47. $16\frac{3}{8} \times 144$. 50. $12\frac{1}{2} \times 128$. 53. $33\frac{1}{8} \times 126$.

45. 14×25 . 48. 25×256 . 51. 25×488 . 54. $84 \times 33\frac{1}{8}$.

1. Compare the quotients of $24 \div 2$ and $24 \div 6$.
2. Compare $380 \div 10$ and $380 \div 5$.
3. If 38 is the quotient of some number divided by 10, 2×38 is the quotient of the same number divided by what?
4. Explain: $320 \div 5 = 2 \times 320 \div 10 = 64$.
5. How do you divide a number by 10?
6. Give a short method of dividing by 5.
7. Compare $375 \div 25$ with $4 \times 375 \div 100$.
8. Explain: $925 \div 25 = 4 \times 925 \div 100 = 3700 \div 100 = 37$.

In the same way divide the following by 25:—

- | | | |
|-----------|-------------|-------------|
| 9. 1325. | 12. 1675. | 15. 9675. |
| 10. 8975. | 13. 4325. | 16. 10,275. |
| 11. 1175. | 14. 17,875. | 17. 9625. |
18. $96 \div 16\frac{2}{3} = ? \times 96 \div 100$.
 19. Make a rule for dividing by $16\frac{2}{3}$.
 20. Make a rule for dividing by $33\frac{1}{3}$.

Use these rules in finding quotients in the following:—

- | | | |
|---------------------------------|-------------------------------------|---------------------------------|
| 21. $2500 \div 12\frac{1}{2}$. | 25. $\$15 \div \$0.12\frac{1}{2}$. | 29. $1675 \div 25$. |
| 22. $6400 \div 33\frac{1}{3}$. | 26. $\$24 \div \0.25 . | 30. $1925 \div 25$. |
| 23. $1300 \div 33\frac{1}{3}$. | 27. $\$36 \div \$0.16\frac{2}{3}$. | 31. $8250 \div 50$. |
| 24. $1850 \div 16\frac{2}{3}$. | 28. $\$42 \div \0.50 . | 32. $1750 \div 16\frac{2}{3}$. |
33. How many will \$3 buy at \$0.25 each?
 34. How many yards will \$5 buy at $16\frac{2}{3}$ cents a yard?
 35. I paid \$8 for tea at $33\frac{1}{3}$ cents a pound. How many pounds did I buy?
 36. My milk bill was \$8.75 at 25 cents per gallon. How many gallons did I buy?

() as in $(3+4) \times 5 = 35$, or $\overline{\quad}$ as in $\overline{3+4} \times 5 = 35$, shows that the numbers inclosed or beneath are to be treated as one number.

1. $(9+6) \times (8-5) = x$; $\overline{3+7} \times 10 - 5 = x$.

2. $3 \times 5^2 + 7 = x$; $4 \times 10^2 + 50 = x$; $27 + 3 - \sqrt{4} = x$.

3. $5^2 - 5 \times 2 = x$; $\overline{18-3} + \sqrt{25} = x$; $(10-4) + \overline{6-3} = x$.

Observe the following and tell which process is performed first when \times or $+$ is on one side of a number and $+$ or $-$ on the other:—

4. $3+4 \times 5 = 3+20 = 23$. 6. $6 \times 12 - 8 + 2 = 72 - 4 = 68$.

5. $4+16 \div 8 = 4+2 = 6$. 7. $5 \times 15 + 24 \div 6 = 75 + 4 = 79$.

8. Compare in value $13 - 5 \times 2 + 8$ and $(13-5) \times (2+8)$.

9. $36 \div 4 - 45 \div 9 = x$. 11. $\frac{1}{8}$ of $(7+14) - \frac{1}{4}$ of $(37-9) = x$.

10. $2+5 \times 6 + 8 \div 2 = x$. 12. $\frac{1}{5}$ of $(41-16) + \frac{1}{6}$ of $(19-7) = x$.

13. What expressions are here marked to be treated as one number?

$8 \times 16 \div 8 + 8 \times (16-9) = \sqrt{3 \times 9 \times 3} \times 108 \div 12 - 10 + 1$.

14. What is the value of $\sqrt{64} + 4 + 8 \times 2$?

15. $(\sqrt{100} \times 4 - 5) + 5 + 3 = x$. 18. $(29 + 46 - 1) \times 18 = x$.

16. $(3^2 + 6) \times 5 + 3 + 5 = x$. 19. $(39 + 10) \times 2 + 6 = x$.

17. $4^2 - 4 \times 3 + 10 - 2 \times 3 = x$. 20. $(6^2 + 4 - 10) \times 3 + 9 = x$.

21. What number is to be divided by 11?

$(6 \times 5 + 4 \times 9) \div 11 = [(6+5) \times (10-4)] \div 11$.

22. Show why it was better to use brackets [] than curves ().

23. $x = (12^2 + 24) \times \sqrt{54-5}$.

24. $(4800 \div 100) \div 0.01$ of $600 = x$.

25. $x = [6 \times \overline{8-4} \times (14-4) + 60] \div 100$.

26. $\left[\left(\frac{17+39}{7} \right)^2 + \sqrt{64} \right]^2 = x$. 27. $(12 \times 4 + 6 \times 12) + \sqrt{100} = x$.

28. $x = [(7+3) \times 2 - \frac{1}{8} \text{ of } 39]^2$.

29. $[(\sqrt{25})^2 - \sqrt{6^2} \times \frac{1}{8}] + \frac{1}{8} \text{ of } 72 = x$.

$x = 12 + 25$ means that x stands for some number which is equal to the sum of $12 + 25$, hence $x = 37$.

Find the value of x in the following:—

1. $x = 12 + 25$.
6. $42 - x = 19$.
11. $19 + 11 + x = 50$.
2. $38 + 12 = x$.
7. $x + 17 = 32$.
12. $72 + x + 14 = 96$.
3. $44 - 19 = x$.
8. $28 + x = 50$.
13. $40 + 20 - x = 50$.
4. $x = 100 - 72$.
9. $\frac{7}{8} - x = \frac{3}{8}$.
14. $\$2.75 + x = \4.50 .
5. $x - 24 = 48$.
10. 1 lb. $- x = 12$ oz.
15. $x - \$7.30 = \2.34 .
16. I am x years old; in 8 years my age will be 36 years.

($x + 8$ years = 36 years.)

17. After taking \$14, \$16, and \$12 out of a sum of money \$3.75 remained. There were \$ x at first. $x - \$14 - \$16 - \$12 = \3.75 .

18. A prize cup contains 23 oz. of gold, 10 oz. of silver, and x oz. of alloy. The cup weighs 42 oz. (Make an equation.)

19. 25 gallons run into a tank, and 46 run out. When the faucets were closed, 80 gallons remained. There were x gallons in the tank when the faucets were opened. (Equations.)

20. Make a problem about the weather in March to suit this equation: 31 da. = 12 da. + x da. + 10 da.

3 x means $3 \times x$. $3x = 15$ means that 3 times some number equals 15.

Find the value of x in the following and explain your method:—

1. $4 \times x = 60$.
6. $8x = 400$.
11. $\frac{x}{4} = 8$.
16. $4x + 5 = 21$.
2. $17 \times 5 = x$.
7. $7x = 91$.
12. $\frac{32}{x} = 4$.
17. $17x - 4 = 30$.
3. $\frac{1}{8}$ of $x = 16$.
8. $14x = 700$.
13. $\frac{x}{12} = 15$.
18. $18x \times 10 = 180$.
4. $80 \div x = 4$.
9. $25x = 625$.
14. $\frac{96}{x} = 4$.
19. $42x \div 21 = 70$.
5. $144 \div x = 16$.
10. $4\frac{1}{2}x = 45$.
15. $\frac{x}{24} = 7$.
20. $\frac{1}{2}$ of $16x = 120$.

1. If 16 cords of wood cost \$120, 24 cords will cost what? In solving such a problem, which of these suggestions seem most important?

I. *What is to be found out?* (Cost of 24 cords.)

II. *What facts will help to find this?* (16 cords cost \$120.)

III. *Comparison of what is given to what is wanted.* (24 cords will cost $1\frac{1}{2}$ times as much as 16 cords.)

IV. *Process, briefly set down.* ($1\frac{1}{2} \times \$120 =$ cost of 24 cords.)

V. *Work performed.* ($1\frac{1}{2} \times \$120 = \$120 + \$60 = \$180.$)

VI. *Does the result seem reasonable?*

2. Bought 12 lb. tea at 75¢, and 20 lb. coffee at 40¢. How much butter at 30¢ would cost the same?

STATEMENT.
$$\frac{12 \times \$0.75 + 20 \times \$0.40}{\$0.30} = x.$$

3. Exchanged a 60-acre farm worth \$2400 for 200 acres of woodland valued at \$13.75 an acre. Find the gain.

STATEMENT. $200 \times \$13.75 - \$2400 = x.$

4. Gave 3000 sq. ft. of 20¢ land for a span of horses and \$75. What were the horses valued at?

STATEMENT. $3000 \times \$0.20 - \$75 = x.$

5. A man purchased 130 bbl. of flour at \$4.50 per barrel, and a number of barrels at \$4. He paid \$665. How many barrels of the cheaper flour did he buy?

STATEMENT.
$$\frac{\$665 - 130 \times \$4.50}{\$4} = x.$$

6. Sixty-four men are employed 25 days in digging a sewer. The contract price was \$1200. Nothing was gained or lost. What were the men paid each per day?

7. If 14 lb. cost \$2.94, what will 10 lb. cost?

8. If 17 tons of coal cost \$134, what will 51 tons cost?

Applying the suggestions on the preceding page, state the following and explain orally:—

1. I bought a field of 10 acres for \$1000. I sold 7 acres of it at \$125 an acre, and the remainder at \$85 an acre. How much did I gain?

2. I sold 50 acres of land for \$5000. This was a loss of \$15 per acre. What did the land cost me?

3. A boat goes 10 miles an hour downstream and 6 miles an hour upstream. How long does it take to go 30 miles and return?

4. If 15 men can do a piece of work in 90 days, how long will it take 6 men?

5. If 14 bbl. of apples are worth \$35, what are 21 bbl. worth?

6. A train runs 280 miles in 11 hours. Seven 3-minute stops are made, and a hot axle makes a detention of 39 minutes. The rate per hour was x miles.

7. Six men buy 640 acres at \$125, and sell for \$95,000. Each man gains x . $\frac{1}{6}$ of $(\$95,000 - 640 \times \$125) =$ each man's gain. In the statement what represents the cost of the land? The proceeds of the sale? The whole gain?

8. Bought 39 bbl. of flour at \$4.75; sold 15 bbl. at \$5, and the remainder at \$5.25. Required, my gain.

9. Three 1-pound packages will go by mail each for 1¢ an ounce plus registration; by express, for 25¢ each. Which way is cheaper?

10. A peck, 2 bushels, and 5 quarts are to be divided equally among 7 persons. Any two receive x quarts.

11. I spent \$4485 for cattle and horses, buying the same number of each. If I paid \$75 apiece for the horses, and \$40 each for the cows, how many of each did I buy?

SUGGESTION. What did 1 horse and 1 cow cost? Then how many times can I buy a horse and a cow with \$4485?

1. Explain the process in each of the following:—

$$3 \times 12 = x; \frac{1}{4} \times 12 = x; 3\frac{1}{4} \times 12 = x.$$

2. Give the results of the following:—

$$3\frac{1}{4} \times 8; 8\frac{1}{2} \times 10; 5\frac{1}{3} \times 15.$$

3. What does $\frac{2}{3}$ of 9 mean? Find $\frac{2}{3}$ of 9; $\frac{3}{4}$ of 16; $\frac{5}{6}$ of 10.

NOTE. The sign of multiplication (\times) may be used instead of the word "of" but it must always be read "of" and not "times" when the multiplier is a fraction. Thus,

$$\frac{2}{3} \times 10 \text{ means } \frac{2}{3} \text{ of } 10.$$

4. Give results:— $\frac{3}{5} \times 20$; $\frac{4}{7} \times 28$; $\frac{2}{5} \times 45$; $\frac{7}{8} \times 56$.

5. Give results:— $\frac{8}{9} \times 72$; $\frac{5}{7} \times 63$; $\frac{2}{3} \times 36$; $\frac{4}{5} \times 45$.

6. 0.05 of 20 means $\frac{5}{100}$ of 20 or $5 \times \frac{1}{10}$ of 20.

7. $\frac{1}{10}$ or 0.1 of 20 = —; $\frac{1}{10}$ or 0.1 of 50 = —; 0.1 of 45 = —.

8. Give results:— 0.6 of 20; 0.8 of 60; 0.9×70 ; 0.08×400 .

9. $25\% = \frac{25}{100}$ or —; $6\% =$ —; 6% of 200 = —.

10. 8% of 400; 15% of 400; 16% of 500; 17% of 1000.

11. Compare $3\frac{1}{2}$ with 10. Show a quick way of multiplying by $3\frac{1}{2}$.

12. Compare $33\frac{1}{3}$ with 100 and show a quick way of multiplying by $33\frac{1}{3}$.

13. Give results:—

$$3\frac{1}{3} \times 15; 3\frac{1}{3} \times 18; 33\frac{1}{3} \times 24; 33\frac{1}{3} \times 36.$$

14. Supply omissions in the multiplication at the right. $8\frac{7}{8} \times 576$.

15. Give directions for each step in multiplying by a mixed number.

$$\begin{array}{r} 576 \\ \underline{8\frac{7}{8}} \\ 72 = \frac{1}{8} \text{ of } 576 \\ 504 = \frac{7}{8} \text{ of } 576 \\ \underline{4608} = \text{—} \times \text{—} \\ 5112 = 8\frac{7}{8} \times \text{—} \end{array}$$

Written Work

- | | | |
|------------------------------------|--------------------------------------|---|
| 16. $9\frac{3}{4} \times 280$. | 19. $780 \times 19\frac{2}{10}$. | 22. $13\frac{5}{8} \times 280$. |
| 17. $18\frac{5}{8} \times 942$. | 20. $603\frac{3}{100} \times 2000$. | 23. $14\frac{5}{8} \text{ ft.} \times 784$. |
| 18. $110\frac{7}{12} \times 144$. | 21. $18\frac{7}{8} \times 1728$. | 24. $9\frac{1}{12} \text{ lb.} \times 1080$. |

1. If the Empire State express runs 115 miles in 108 minutes, what is the rate per hour?

2. If 71 cu. ft. of water weigh 2 tons, how much will 2414 cu. ft. of water weigh?

3. I paid \$59.22 for potatoes, and sold them for \$70.56. What was the gain per bushel, if I paid 94¢?

4. My July gas bill was \$5.28 for 4400 feet of gas. In August the gas company raised the price $\frac{1}{4}$. How much should my August bill have been, if 1500 feet of gas were consumed?

5. At the end of August the reading of the meter was 5600. At the end of September the reading was 7400. What is my gas bill for September at \$1.25 per 1000 cubic feet?

6. The mailing clerk in the office of the *Herald* receives \$10.95 for mailing 32,850 papers weekly. How much is that per hundred papers?

7. The Transvaal gold output in August, 1899, was 459,700 oz., and its value was \$194,000. The August output in 1902 was valued at only \$3,200,000. How many ounces were mined?

8. Find the amount of a bill for the following:—

28 $\frac{3}{4}$ yd. silk @ \$1.80.

24 yd. percale @ 0.12 $\frac{1}{2}$.

2 $\frac{7}{8}$ yd. velvet @ 2.25.

$\frac{3}{4}$ yd. silk @ 2.90.

9. A merchant purchased 950 barrels of flour at \$6.90. He sold 325 of these at \$7.20, 460 at \$8.10, and the remainder at \$9. What was his total gain?

10. A fruit dealer bought 500 oranges at the rate of 2 for 3¢, and 400 more at the rate of 4 for 5¢. He sold the whole lot at the rate 3 for 5¢. What was his gain?

1. If 5 lb. of cheese cost 80 cents, 10 lb. will cost what?
2. Why is it needless to find the cost of 1 lb. in Ex. 1?
3. At the rate in Ex. 1, what will $2\frac{1}{2}$ lb. cost?
4. When 21 lb. of steak cost \$3.21, what will 7 lb. cost?
5. Compare the time required by 6 men to do a piece of work with the time required by 2 men. By 12 men.
6. Compare the rent for 5 mo. with the rent for a year.
7. If a house rents for \$300 per year, what is the rent for 5 months?
8. 5 bushels of oats cost \$1.70. At this rate, what will 15 bushels cost? 35 bushels?
9. 9 for \$1 makes 6 cost what? 12 will cost what?
10. \$36 is what part of \$108? Of \$72? Of \$144?
11. What 42 men can do in a week will take 7 men how long? How long will it take 28 men?
12. Supplies that will maintain a regiment of 1000 for a week will maintain 100 how long? 600 men how long?
13. If 6 men can do a piece of work in 10 days, how long will it take 4 men to do it?
14. If 8 men can build a wall in 3 weeks, how many men will be required to build it in 1 week?
15. If 60 horses eat 450 bushels of oats in a month, how long will it last 30? How long will it last 90?
16. If 12 dozen eggs are worth \$1.80, what are 8 dozen worth at the same rate? What are 60 dozen worth?
17. If oranges sell at 3 for 10 cents, what is that a dozen?
18. 4 ounces for 25 cents is how much per pound?

1. Compare the cost of 36 bu. with the cost of 12 bu. The ratio of 36 to 12 is —.

2. If 12 bu. cost \$7.50, what will 36 bu. cost?

3. Compare the cost of 85 yd. with the cost of 17 yd.

4. If 17 yd. of cloth cost 18.70, what will 85 yd. cost?

5. Compare the cost of 114 gal. with the cost of 19 gal.

6. If 19 gal. of alcohol cost \$72, find the cost of 114 gal.

7. Compare the time required for 91 men to do a piece of work with the time required for 13 men.

8. If 13 men can pave a street in 42 days, how long will it take 91 men to do it?

9. What will 364 bbl. of apples cost when 52 bbl. cost \$175?

10. What is the relation (ratio) of all of anything to 20% of it? When 20% of a crop of beans is 325 bushels, what is the whole crop?

11. What is the relation (ratio) of all of anything to $16\frac{2}{3}\%$ of it? If $16\frac{2}{3}\%$ of a certain number is 342, what is the number?

12. A barrel of flour fills 8 bags and costs \$4.50. What is the gain on 3 bbl. sold at \$0.62 $\frac{1}{2}$ per bag?

13. If Hans can haul as much sand in 15 days as Knut can haul in 20 days, which should receive the higher wages?

14. If Knut receives \$60 per 20 days, what should Hans receive per day?

15. What is the ratio of all of anything to 50% of it? If 50% of a certain distance is 168 miles, what is the whole distance? What is 25% of the distance?

16. Compare 9000 lb. with a ton. What should I pay for 9000 lb. of hay at \$12.50 per ton?

Change

1. 3 da. 6 hr. to hours.
2. 18 hr. 43 min. to minutes.
3. 8 yd. 17 in. to inches.
4. 18 sq. ft. 107 sq. in. to square inches.
5. 8 cu. yd. 16 cu. ft. to cubic feet.
6. 5 gal. 3 pt. to pints.
7. 3 mi. 240 rd. to rods.
8. 5 bu. 8 pk. to quarts.
9. 2 wk. 5 da. 11 hr. to hours.
10. 9 A. 120 sq. rd. to square rods.
11. 9 lb. 3 oz. to ounces.
12. 13 T. 1600 lb. to pounds.
13. 9 bu. 8 qt. to quarts.
14. 11 yd. 24 in. to inches.
15. 9 hr. 48 min. to minutes.
16. 8 cu. ft. 1563 cu. in. to cubic inches.
17. 356 qt. to gallons.
18. 150 oz. to pounds.
19. 17,562 cu. ft. to cubic yards.
20. 34,628 sec. to hours.
21. 348 wk. to years.
22. 1963 qt. to bushels.
23. 3468 cu. ft. to cubic yards.
24. 3462 gi. to gallons.
25. Change 120,000 min. to days.
26. Thirteen tons of oatmeal will make how many one-pound packages?
27. How many pint bottles can be filled from 728 gallons of extract?
28. A team of strong horses haul 5 tons of granite. How many cubic feet in the load if each cubic foot weighed 165 pounds?
29. Bought 3 acres of land and cut it into house lots, each containing 3267 square feet. How many lots were there?
30. How many rails 60 feet long will be required to lay 20 miles of double track railroad?
31. How many 3-ounce packages can be made from a quarter of a ton of pepper?
32. How many pint bottles of mineral water can be filled from a tank holding 300 gallons of mineral water?

1. Compare 24 and 12, thus: $24 = 2 \times 12$; $12 = \frac{1}{2}$ of 24.

What is the difference between two ways of comparing 24 and 12 ?

In this way compare the following numbers :—

- | | | |
|---------------|----------------|---------------|
| 2. 12 and 36. | 5. 20 and 80. | 8. 49 and 7. |
| 3. 15 and 60. | 6. 25 and 125. | 9. 63 and 9. |
| 4. 25 and 75. | 7. 30 and 90. | 10. 42 and 6. |

11. Compare 3 in. and 2 ft.; 2 yd. and 6 in.

12. What is the relation of 3 to 9 ? Of 4 to 16 ? Of 5 to 15 ?

The relation of one number to another is their ratio. Ratio is expressed as the quotient of the first number divided by the second. Thus the ratio of 6 to 2 = 3; of 2 to 6 = $\frac{1}{3}$.

13. Read these five ways of expressing ratio :—

The ratio of 12 to 24 is $\frac{1}{2}$; $12 : 24 = \frac{1}{2}$; $12 \div 24 = \frac{1}{2}$; $\frac{12}{24} = \frac{1}{2}$; 12 is to 24 as 1 is to 2.

14. What is the ratio of 12 to 60 ? Of $12\frac{1}{2}$ to 100 ? Of 15 to 45 ?

What is the ratio of :—

- | | | |
|---|---|------------------|
| 15. $12\frac{1}{2}$ to 25 ? | 18. $8\frac{1}{3}$ to $16\frac{2}{3}$? | 21. 20 to 30 ? |
| 16. 16 to 48 ? | 19. $6\frac{1}{4}$ to 25 ? | 22. 60 to 90 ? |
| 17. $16\frac{2}{3}$ to 50 ? | 20. $37\frac{1}{2}$ to 75 ? | 23. 108 to 144 ? |
| 24. What is the ratio of $12\frac{1}{2}$ to $18\frac{3}{4}$? | | |

SUGGESTION. $12\frac{1}{2}$ is 2 $6\frac{1}{4}$'s, and $18\frac{3}{4}$ is 3 $6\frac{1}{4}$'s, hence the ratio is 2 to 3, or $\frac{2}{3}$.

What is the ratio of :—

- | | | |
|--|---|---|
| 25. $12\frac{1}{2}$ to $31\frac{1}{4}$? | 28. $6\frac{1}{4}$ to 25 ? | 31. $16\frac{2}{3}$ to $41\frac{2}{3}$? |
| 26. $12\frac{1}{2}$ to 75 ? | 29. $8\frac{1}{3}$ to $16\frac{2}{3}$? | 32. $33\frac{1}{3}$ to $266\frac{2}{3}$? |
| 27. $37\frac{1}{2}$ to $87\frac{1}{2}$? | 30. $16\frac{2}{3}$ to 100 ? | 33. $66\frac{2}{3}$ to 300 ? |

1. If 15 oranges cost 40 cents, 3 will cost what part of 40 cents?
2. If 3 cost 7 cents, what will 15 cost?
3. If $6\frac{1}{4}$ lb. of honey cost 95 cents, what will 25 lb. cost?
4. Give two equal factors of 3600, that is, $\sqrt{3600} = \text{what?}$
5. Divide 21,000 by 3000. Explain your method.
6. Find the value of $\frac{1}{4}$ of 2400 $-\frac{5}{8}$ of 1500.
7. Find the sum of $3600 \div 120$, $\frac{1}{80}$ of 2400, and $\frac{1}{800}$ of 75,000.
8. $\sqrt{121}$; $\sqrt{81}$; $\sqrt{144}$; $\sqrt{400}$; $\sqrt{1600}$; $\sqrt{2500}$.
9. $3 \times 5 \times 7 \div 5 \times 6 = \text{what?}$ What common factor in both dividend and divisor may be dropped?
10. Divide $4 \times 6 \times 8$ by 6; by 8; by 4; by 24.
11. How many 7's from 910 leave 700?
12. Compare the work done by 9 men with the work done by 15 men. Compare the time required by them to do a piece of work.
13. How long will it take 15 men to do what 9 men can do in 10 days?
14. At the rate of 3 for 9 cents, what will a dozen cost? What will 24 cost?
15. 2 ounces for 40 cents is how much per pound?
16. 40 sq. rd. of land for \$1000 is how much per acre?
17. If 6 inches of pipe cost 35¢, what will 1 yard cost?
18. If 18 pounds of sugar is worth \$1.00, what will a bag containing 63 lb. be worth?
19. Divide 360,000 by 4000; 90,000 by 4500.
20. Divide 3600 by 100; by 1000. Divide 280,000 by 100; by 1000.

1. Bought 44 barrels of flour at \$ $5\frac{1}{4}$ and sold it in $\frac{1}{4}$ -bbl. bags at 70¢. Find my total profit.
2. When lessons are \$ 30 per quarter, what is the average cost per week?
3. How many days in a leap year? How many weeks and days are there?
4. Bought an 8-peck barrel of cranberries for \$ 8 and retailed them at 2 qt. for \$ 0.25. What was my profit?
5. A bushel of wheat weighs 60 lb. What is a ton of wheat worth at 86 cents per bushel?
6. At 60 lb. to a bushel, 3 bushels to the barrel, 9 tons of beans will fill how many barrels?
7. Bought a field 600 ft. long and 413 ft. wide. What will it cost me to fence it at \$ 1.25 per rod?
8. A man travels 23 miles per day on every day except Monday, when he goes 6 miles farther, after resting on Sunday. How many weeks will it take him to go 576 miles?
9. Bought 130 tons of coal by long ton (2240 lb.) at \$ 4.00 per ton, and sold it by short ton (2000 lb.) at \$ 6.50 per ton. How much did I make on the lot?
10. I paid \$ 300 for an acre of land and sold it for 25¢ per square foot. How much did I gain or lose?
11. Give the length of a double track railroad laid with 1056 rails 30 ft. long.
12. Ocean steamers sometimes use 300 T. of coal every 24 hours. This is how many pounds per minute?
13. The velocity of light is 186,337 miles per second. Light from the sun reaches the earth in 8.3 minutes. What is the sun's mean distance from us?

1. What is an integer? Name some integers. What is the unit in each?

2. What is a fraction? Compare a fraction and an integer. A fraction is a number made up of one or more — units, and an integer is a — made up of — units.

3. What 5 coins may $\$ \frac{5}{4}$ mean? What 2 coins?

4. A fraction equal to more than a whole integral unit is what kind of fraction? Name some improper fractions.

5. Compare the values of the expressions $\$ \frac{2}{3}$, $\frac{1}{4}$ of $\$ 9$, and $\$ 9 \div 4$.

6. Read the following, first as fractions, then as expressed divisions: $\frac{12}{4}$, $\frac{27}{9}$, $\frac{84}{12}$, $\frac{100}{25}$, $\frac{500}{50}$. Give the value of each in integers.

7. Give the value of these improper fractions: $\frac{21}{5}$, $\frac{48}{8}$, $\frac{77}{7}$, $\frac{15}{4}$.

8. What two kinds of units are mixed in $17\frac{1}{2}$? In $\$ 18\frac{1}{2}$?

9. What are such numbers called?

10. Use objects to show that $\frac{2}{3}$ of 1 thing is equal to $\frac{1}{3}$ of 2 things.

11. What kind of number is the quotient when the divisor is larger than the dividend?

$$2 + 3 = \text{---}; \quad 5 + 7 = \text{---}; \quad 9 + 13 = \text{---}.$$

Give the value of x in the following:—

$$12. \frac{28}{6} = x. \quad 13. \frac{35}{x} = 7. \quad 14. \frac{x}{3} = 16. \quad 15. 11 + 12 = x.$$

16. How many 7ths in 1? In 5? In $5\frac{1}{2}$?

17. How many 12ths in 1? In 8? In $7\frac{1}{2}$? In $8\frac{1}{2}$?

18. In changing a mixed number to an improper fraction:—

(a) What numbers are multiplied?

(b) What numbers are added?

(c) How may your result be proved correct?

19. Change to improper fractions: $2\frac{2}{3}$, $3\frac{1}{2}$, $4\frac{7}{10}$, $5\frac{1}{2}$, $6\frac{4}{15}$, $2\frac{5}{8}$.

20. Change to mixed numbers: $\frac{11}{4}$, $\frac{40}{18}$, $\frac{28}{12}$, $\frac{50}{11}$, $\frac{120}{10}$, $\frac{18}{7}$, $\frac{51}{4}$.

Change to mixed numbers or improper fractions:—

1. $39\frac{3}{4}$. 3. $497\frac{1}{2}$. 5. $511\frac{1}{8}$. 7. $401\frac{1}{10}$. 9. 465.27.
2. $\frac{800}{18}$. 4. $11\frac{2}{5}$. 6. $\frac{5000}{12}$. 8. 968%. 10. $1\frac{800}{9}$.
11. $2\frac{9000}{288}$ gal. = how many gallons?
12. How many eggs in $196\frac{1}{2}$ dozen?
13. Add: $1\frac{728}{144}$, $1\frac{936}{4}$, and $2\frac{276}{18}$. Hint. Change to integers.
14. Which is larger, $1\frac{817}{18}$ or $1\frac{422}{4}$?
15. Add: $\frac{79}{100}$, $1\frac{30}{100}$, 67%, 0.97, and 37%.

Change to other forms:—

16. $342\frac{3}{4}$. 18. $\frac{57682}{21}$. 20. $961\frac{2}{8}$. 22. $846\frac{5}{17}$. 24. $2\frac{934}{28}$.
17. $1\frac{246}{2}$. 19. $48\frac{5}{18}$. 21. $\frac{3482}{218}$. 23. $\frac{8896}{10}$. 25. $462\frac{1}{2}$.

CHANGES IN THE NUMBER OF FRACTIONAL UNITS TO CORRESPOND WITH CHANGES IN THEIR SIZE

Oral

1. 3 weeks; 21 days. Compare the size of the units; the number of units; the value of the two expressions.
2. Why is the larger number of no greater value than the smaller?
3. Give other examples of equal values expressed in units of different size.
4. Compare $\frac{1}{4}$ and $\frac{3}{12}$ as to the size of the units; as to the number of units; as to their value.
5. Show, by drawing diagrams, that in changing 4ths to 12ths you change to a unit $\frac{1}{3}$ as large, but that you have 3 times as many fractional units; that is, that $\frac{1}{4} = \frac{3}{12}$.
6. In the same way compare $\frac{3}{4}$ and $\frac{9}{12}$.
7. If each term of $\frac{1}{8}$ be made $\frac{1}{2}$ as large, how is the value affected? Will the unit be made larger or smaller?

1. How does an increase in the numerator affect the value of a fraction? A decrease? Compare $\frac{8}{15}$ and $\frac{4}{15}$; $\frac{6}{11}$ and $\frac{2}{11}$.

2. How does an increase in the denominator affect the value of a fraction? A decrease? Compare $\frac{3}{5}$ and $\frac{3}{10}$; $\frac{5}{8}$ and $\frac{5}{14}$.

3. If each term of $\frac{2}{3}$ be multiplied by 3, what effect upon the value of the fraction?

4. What is done to the size of the unit when the denominator is doubled? When it is halved?

5. What is done to the fraction when its numerator is doubled? When it is halved?

6. How can you tell whether a fraction is large or small?

7. Is $\frac{1}{1750}$ a very large or a very small fraction?

Principle. Both terms of a fraction may be multiplied or divided by the same number without changing the value of the fraction.

8. Applying this principle, how would you change 3ds to 12ths? 12ths to 3ds?

9. Change to 12ths: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$.

10. Change to 20ths: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{9}{10}$.

11. When you change to larger terms, as in Exercises 10 and 11, are you changing to larger or smaller units?

12. What number will divide both 36 and 42? How can $\frac{36}{42}$ be changed to smaller terms? What will the unit then be?

13. Has the unit been made larger or smaller? How does the change in size correspond to the change in number?

14. Change to larger units: $\frac{21}{85}$, $\frac{34}{72}$, $\frac{27}{98}$, $\frac{36}{108}$, $\frac{78}{95}$.

$$1. \frac{3}{7} = \frac{x}{21}, \frac{x}{49}, \frac{x}{28}, \frac{x}{42}, \frac{x}{35}, \frac{x}{84}, \frac{x}{63}, \frac{x}{91}, \frac{x}{56}.$$

$$2. \frac{x}{60} = \frac{1}{20}, \frac{4}{15}, \frac{7}{12}, \frac{11}{30}, \frac{10}{5}, \frac{1}{5}, \frac{5}{6}, \frac{9}{10}.$$

$$3. \frac{5}{9} = \frac{x}{36}, \frac{x}{45}, \frac{35}{x}, \frac{10}{x}, \frac{x}{54}, \frac{60}{x}, \frac{x}{90}, \frac{80}{x}, \frac{100}{x}.$$

4. Without changing the value, make the denominators of these fractions 156: $\frac{1}{4}, \frac{1}{52}, \frac{1}{13}, \frac{1}{12}, \frac{102}{12}, \frac{1}{48}, \frac{1}{2}, 7\frac{5}{8}$.

5. In changing a fraction to lower terms, how do you affect the size of the units? Their number?

6. Why does multiplying both terms of $\frac{1}{8}$ by 8 leave the value unchanged?

At sight give an equivalent fraction in larger or smaller units:—

$$7. \frac{3}{4} = \frac{?}{56}, \text{ or } \frac{7.5}{?}, \text{ or } \text{---}\%.$$

$$13. \frac{21}{?}, \frac{34}{?}, \frac{27}{?}, \frac{86}{108}.$$

$$8. \frac{7}{8} = \frac{?}{120}, \text{ or } \frac{63}{?}, \text{ or } \text{---}\%.$$

$$14. \frac{73}{93}, \frac{85}{170}, \frac{33}{5}, \frac{125}{200}.$$

$$9. \frac{4}{5} = \frac{?}{45}, \text{ or } \frac{80}{?}, \text{ or } \text{---}\%.$$

$$15. 75\%, 0.50, 80\%, 0.90.$$

$$10. \frac{5}{9} = \frac{?}{45}, \text{ or } \frac{35}{?}.$$

$$16. \frac{30}{105} = x; \frac{75}{500} = x; \frac{75}{500} = \text{---}\%.$$

$$11. \frac{3}{16}, \frac{5}{24}, \frac{12}{96} \text{ to 48ths.}$$

$$17. \frac{7}{8} = \frac{?}{40} = \frac{105}{?} = \frac{?}{220}.$$

$$12. \frac{5}{12}, \frac{48}{120}, \frac{3}{4} \text{ to 60ths.}$$

$$18. \frac{96}{144} = \frac{16}{?} = \frac{12}{?} = \frac{?}{24}.$$

19. Why do we call 72 a **composite** number and 73 a **prime** number? Of what two factors is 72 composed?

20. What divisor is **common** to 24 and 36? To 60 and 90?

21. What is the **greatest common divisor** of 60 and 90? Of 45 and 60?

22. What advantage is there in using the greatest common divisor in changing fractions to smaller terms; that is, to larger units?

23. Make use of the greatest common divisors (G.C.D.) in changing the following to smallest terms: $\frac{25}{125}, \frac{108}{144}, \frac{63}{81}, 80\%, 0.125, \frac{26}{130}$.

1. Any number is a multiple of 2 if the last digit is zero or a multiple of 2, that is, if it ends in 0, 2, 4, 6, or 8.

2. Any number is a multiple of 3 if the sum of the digits is a multiple of 3.

For example, 3 will divide 1278 for it will divide $1 + 2 + 7 + 8$ or 18.

3. Any number is a multiple of 4 if the last two figures are zeros or express a number divisible by 4.

Thus 38,236 is divisible by 4 for 36 is.

4. Any number is a multiple of 5 if it ends in 5 or 0.

5. Any number is a multiple of 6 if the last digit is even and the sum of the digits is divisible by 3.

6. Any number is a multiple of 9 if the sum of the digits is.

Thus 327,654 will contain 9 for $3 + 2 + 7 + 6 + 5 + 4$ or 27 will contain 9.

7. Any number is a multiple of 10 if it ends in a zero.

8-14. Which of these numbers are multiples of 2? Of 3? Of 4? Of 5? Of 6? Of 9? Of 10?

| | | | | |
|------|------|------|------|------|
| 360 | 6984 | 2160 | 3654 | 3741 |
| 1728 | 8397 | 3240 | 1782 | 1746 |
| 3123 | 6624 | 9270 | 4662 | 8460 |

Making use of these tests to find the common factors, change the following to largest units:—

| | | |
|--|-------------------------|-------------------------|
| 15. $6\frac{128}{128} = 3\frac{21}{11} = \frac{7}{16}$. | 22. $\frac{178}{178}$. | 26. $\frac{312}{312}$. |
| 16. $\frac{243}{243}$. | 19. $\frac{108}{108}$. | 23. $\frac{178}{178}$. |
| 17. $\frac{248}{248}$. | 20. $\frac{228}{228}$. | 24. $\frac{278}{278}$. |
| 18. $\frac{218}{218}$. | 21. $\frac{248}{248}$. | 25. $\frac{178}{178}$. |
| | | 29. $\frac{178}{178}$. |

1. What are like numbers? Give examples.
2. What is an integer? Name some integers and the unit of each.
3. What is a fraction? Name some fractions and the unit of each.
4. Show the difference between an integral and a fractional unit.
5. In the following numbers name (a) the integral unit, (b) the size and kind of fractional unit, (c) the number of fractional units:—
 $\frac{5}{8}$ pk.; $\frac{7}{12}$ yr.; $\frac{5}{18}$; $\$ \frac{3}{4}$; $\frac{7}{8}$ in.; $\$ 0.15$; 6% of a day.
6. Which of the following fractions have units of the same size? Of the same kind? Of the same kind and size? Which are *like fractions*?
 $\frac{5}{8}$ yr.; $\frac{1}{8}$ yr.; $\frac{4}{8}$ yr.; $\frac{7}{8}$ da.; $\$ \frac{3}{4}$; $\frac{5}{8}$ yd.; $\frac{3}{8}$ mi.
7. Why not add $\frac{7}{8}$ lb. and $\$ \frac{1}{8}$? $\frac{3}{4}$ wk. and $\frac{1}{2}$ wk.?

Principle. *Before fractions can be added (or their difference found) their units must be of the same kind and size, that is, the fractions must be like fractions.*

8. $\frac{7}{12} + \frac{3}{12} + \frac{1}{12} = \frac{11}{12}$.
9. $\frac{3}{84} + \frac{9}{84} + \frac{1}{84} = \frac{13}{84}$.
10. $0.55 - x = 0.30$.
11. $\frac{7}{88} - \frac{14}{88} = \frac{31}{88}$.
12. $72\% - \frac{2}{100} = 0.48$.
13. $16\frac{2}{3} + 18\frac{2}{3} = x$.
14. 14 da. + 3 wk. = — da. or — wk. What change did you make before adding?
15. Mention some unlike fractions. Why are they unlike?
16. $\frac{4}{8}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{5}{8}$. Which of these fractions have a *common numerator*? A *common denominator*? Which are like fractions? Which are unlike?
17. $\frac{9}{16} + \frac{3}{16} + \frac{5}{16} + \frac{7}{16} + \frac{1}{4} + \frac{3}{8} = x$. (Add the like fractions first.)
18. $\frac{1}{8} + \frac{1}{8} = x$. Can $\frac{1}{8}$ be changed to 6ths? Why will it be better to change $\frac{1}{8}$ to 6ths rather than $\frac{1}{8}$ to 18ths?
19. In Exercise 14, why would it be better to change the 14 days to weeks?
20. $\frac{3}{8} + \frac{1}{4} = x$. Why change $\frac{3}{8}$ to 24ths?

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1. Which term is common to like fractions?
2. Change $\frac{2}{5}$ and $\frac{3}{10}$ to like fractions, or a *common denominator*. Change to the **least common denominator** (L. C. D.).
3. In adding $\frac{3}{4}$ and $\frac{1}{6}$ shall we use 4ths or 60ths as the common unit? Will 4 or 60 be the least common denominator?
4. Why is it easier to use the largest common unit (or the least common denominator) in adding?
5. Give three steps in adding $\frac{1}{3}$ and $\frac{5}{24}$.

Method. *In adding or subtracting unlike fractions.*

- I. *Change to a common unit.*
- II. *Add numerators or find their difference.*
- III. *Simplify the result.*

Perform the operation indicated in the following:—

- | | | |
|--|---|-------------------------------------|
| 6. $\frac{2}{8} + \frac{1}{4}$. | 9. $\frac{3}{7} + \frac{2}{8}$. | 12. $\frac{3}{18} - \frac{7}{24}$. |
| 7. $\frac{9}{16} + \frac{2}{8}$. | 10. $\frac{2}{8} + \frac{3}{4}$. | 13. $24\% + \frac{1}{5}$. |
| 8. $\frac{7}{100} - \frac{3}{4}$. | 11. $\frac{7}{8} - \frac{5}{8}$. | 14. $\frac{3}{8} + \frac{5}{16}$. |
| 15. $\frac{1}{8} + \frac{1}{8} + \frac{1}{12} + \frac{1}{24} + \frac{1}{72}$. | 20. $\frac{2}{7} + \frac{5}{12} + \frac{5}{7} + \frac{1}{8} + \frac{1}{4}$. | |
| 16. $20\% + \frac{1}{5} + 0.20 + \frac{4}{5}$. | 21. $\frac{5}{8} + \frac{1}{8} + \frac{2}{8} + 2\frac{1}{2}$. | |
| 17. $\frac{2}{10} + \frac{1}{4} + \frac{7}{8} + \frac{1}{8} + 25\%$. | 22. $36\% - \frac{1}{4} + 0.14 - \frac{1}{5}$. | |
| 18. $\frac{4}{5} + \frac{7}{12} + \frac{2}{3} + \frac{3}{5}$. | 23. $\frac{2}{8} - \frac{2}{8} + \frac{5}{8} + \frac{7}{8}$. | |
| 19. $13\% + 0.12 + \frac{3}{4} + \frac{3}{8}$. | 24. $\frac{1}{4} + \frac{3}{5} + \frac{1}{2} - \frac{1}{10} + \frac{7}{10}$. | |
| 25. $\frac{1}{4}$ wk. + $\frac{2}{865}$ yr. + $\frac{1}{8}$ da. - $\frac{1}{15}$ mo. + 180 min. = x. | | |

Multiples; Common Multiples; Least Common Multiples.

1. Show that 36, 60, 72 and 120 are **multiples** of 12.
2. Show that 50, 60, 80, and 120 are *multiples* of 10.
3. Show that 60 and 120 are *common multiples* of 10 and 12.
4. Show that 60 is the **least common multiple** (L. C. M.) of 10 and 12.

1. Find the *least common multiple* of 30 and 42, *i.e.* a multiple containing only such factors as are needed to produce each number separately.

WORK

$$30 = 2 \times 3 \times 5.$$

$$42 = 2 \times 3 \times 7.$$

$$42 \times 5 = 210, \text{ the least common multiple (L. C. M.).}$$

EXPLANATION. A multiple of 30 must contain all the prime factors of 30, and a multiple of 42 all the prime factors of 42. Now, a number whose factors are 2, 3, 5, and 7 will contain $2 \times 3 \times 5$, or 30, and also $2 \times 3 \times 7$, or 42. In practical work we say 30×7 , or 42×5 , since 7 is not found in 30, or 5 in 42.

2. Find the L. C. M., *i.e.* the *least number divisible by* 60, 72, and 108.

WORK

$$60 = 2 \times 2 \times 3 \times 5.$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$108 = 2 \times 2 \times 3 \times 3 \times 3.$$

$$108 \times 2 \times 5 = 1080, \text{ the L. C. M.}$$

- (a) What prime factor of 60, needed in the L. C. M., is not found in 108?
- (b) What prime factor of 72 is not found in 108, that is, found more times in 72 than 108?
- (c) What is meant by the least common multiple of several numbers?

3. Find the L. C. M. of 60, 84, and 132.

4. What is the least common multiple of 45, 90, 100, 200?

SUGGESTION. Is a multiple of 90 a multiple of 45?

Is a multiple of 200 a multiple of 100?

Notice that we need simply to find the L. C. M. of 90 and 200.

Find the L. C. M. of:—

5. 15, 21, 45.

6. 16, 18, 27, 72.

7. 16, 25, 80, 100.

1. Add
- $\frac{11}{8}$
- and
- $\frac{7}{8}$
- .

PROCESS

$$\begin{array}{rcl}
 15 & = & 3 \times 5. \\
 18 & = & 3 \times 3 \times 2. \\
 18 \times 5 & = & 90, \text{ the L. C. M.}
 \end{array}
 \qquad
 \begin{array}{rcl}
 \frac{11}{8} & = & \frac{33}{24} \text{ (why?) } \\
 \frac{7}{8} & = & \frac{21}{24} \text{ (why?) } \\
 \hline
 \text{sum} & = & \frac{54}{24} \text{ or } 2\frac{3}{2}
 \end{array}$$

EXPLANATION. Since the common fractional unit could not be readily seen, we found the L. C. M. of the denominators. 90ths, then, is the largest common unit to which both 15ths and 18ths can be changed.

2. $\frac{7}{15} + \frac{4}{21} = x$.

7. $\frac{3}{7} + \frac{5}{12} + \frac{2}{3} = x$.

3. $\frac{3}{8} + \frac{1}{8} = x$.

8. $\frac{7}{10} + \frac{1}{10} + \frac{6}{10} + \frac{1}{10} = x$.

4. $\frac{7}{24} + \frac{1}{11} = x$.

9. $\frac{5}{88} + \frac{1}{88} + \frac{1}{16} = x$.

5. $\frac{2}{48} + \frac{2}{48} = x$.

10. $\frac{3}{60} + \frac{4}{112} = x$. (Reduce each to larger units. Why?)

6. $\frac{8}{10} + \frac{625}{1000} = x$.

11. $\frac{4}{9} + \frac{3}{26} + \frac{2}{13} = x$.

12. $24\frac{1}{4} + 23\frac{1}{4} + 72\frac{3}{4} + 16\frac{5}{8} + 18\frac{1}{2}$. Add the fractions mentally and this result to the sum of the integers.

13. Add
- $32\frac{5}{9}$
- ,
- $17\frac{1}{3}$
- ,
- $24\frac{7}{18}$
- ,
- $36\frac{1}{3}$
- and
- $21\frac{2}{3}$
- .

14. From
- $75\frac{7}{8}$
- take
- $57\frac{1}{4}$
- .

PROCESS

$$\begin{array}{r}
 75\frac{7}{8} = 75\frac{28}{32} = 74\frac{10}{32} \\
 57\frac{1}{4} = 57\frac{8}{32} = 57\frac{8}{32} \\
 \hline
 \text{Difference} = 17\frac{2}{32}
 \end{array}$$

EXPLANATION. Since the fraction in the minuend is smaller than the one in the subtrahend, we take 1 or $\frac{32}{32}$ from 75 and add to the fraction $\frac{28}{32}$. How is this like subtraction of integers when a figure in the subtrahend is larger than the one above in the minuend?

15. $63\frac{1}{2} - 4\frac{1}{2}$.

20. $73\frac{1}{2} - 16\frac{1}{2}$.

16. $94\frac{1}{6} - 18\frac{5}{6}$.

21. $77\frac{1}{2} - 19\frac{1}{2}$.

17. $72\frac{1}{2} - 15\frac{1}{4}$.

22. $84\frac{3}{4} - 15\frac{3}{5}$.

18. $19\frac{3}{8} - 5\frac{1}{4}$.

23. $171\frac{5}{22} - 91\frac{9}{10}$.

19. $14\frac{9}{25} - 6\frac{51}{100}$.

24. $48\frac{1}{5} - 13\frac{4}{7}$.

25. $114\frac{1}{10} - 16\frac{1}{2} + 84\frac{3}{10} - 16\frac{1}{2}$. Can you find some short way of doing this without changing the fractions to a common unit?

I. To multiply a fraction by increasing the number of units.

1. 9×7 units = ——— units.

2. In Exercise 1, does it matter whether the units are integral or fractional?

3. Then it follows that 9×7 eighths = 63 eighths; $9 \times \frac{7}{8} = \frac{63}{8}$.

4. $7 \times \frac{1}{3}$.

7. $12 \times \frac{1}{4}$.

10. $16 \times \frac{2}{5}$.

5. $6 \times \frac{7}{10}$.

8. $13 \times \frac{2}{5}$.

11. $14 \times \frac{2}{5}$.

6. $11 \times \frac{7}{12}$.

9. $15 \times \frac{2}{3}$.

12. $10 \times \frac{1}{4}$.

13. In the preceding exercises did we change the number or the size of the units?

II. To multiply a fraction by increasing the size of the units.

1. Compare $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{2}$ and $\frac{1}{8}$; $\frac{1}{4}$ and $\frac{1}{16}$.

2. Compare $\frac{3}{4}$ and $\frac{3}{16}$; $\frac{5}{8}$ and $\frac{5}{12}$; $\frac{7}{8}$ and $\frac{7}{4}$.

3. How does dividing the denominator affect the size of the units?

4. Compare 3 days and 3 weeks. What is the unit in each? Why is 3 weeks 7 times as large as 3 days?

5. $4 \times \frac{7}{16} = \frac{7}{4}$. What change in the value of the fraction when

we take $\frac{7}{4}$ instead of $\frac{7}{16}$? Why?

Find the product by increasing the size of the units:—

6. $6 \times \frac{7}{12}$.

9. $10 \times \frac{7}{10}$.

12. $15 \times \frac{7}{10}$.

7. $8 \times \frac{1}{3}$.

10. $12 \times \frac{1}{3}$.

13. $25 \times \frac{1}{5}$.

8. $3 \times \frac{1}{2}$.

11. $18 \times \frac{2}{3}$.

14. $36 \times \frac{1}{2}$.

Principle. A fraction is multiplied by any number either by multiplying its numerator or dividing its denominator by the number.

I. *The multiplier a fraction; the multiplicand an integer.*

1. How can you get $9 \times \frac{1}{3}$ by addition?
2. What does the multiplier show?
3. Is it proper to say $\frac{1}{3}$ times \$6? That is, can \$6 be taken $\frac{1}{3}$ of a time as an addend?
4. In finding $\frac{1}{3}$ of \$6 to be \$2, do we multiply or divide?

While such expressions as $\frac{2}{3} \times \$9$ are called multiplication, the \$9 has not really been multiplied, that is, increased.

When preceded by a fraction the sign \times must be read "of." The expression means $\frac{2}{3}$ of \$9, that is, 2 times one of the 3 equal parts of \$9, or $2 \times \$3$.

5. $\frac{3}{8} \times 24$ hr. = $3 \times (\frac{1}{8} \text{ of } 24 \text{ hr.})$ or 3×3 hr. or 9 hr. In practice we say $\frac{3}{8} \times \cancel{24}^3 \text{ hr.} = 9$ hr.

- | | | |
|-------------------------------|---------------------------------|---------------------------------|
| 6. $\frac{5}{7} \times 30$. | 10. $\frac{8}{9} \times 20$. | 14. $\frac{1}{10} \times 105$. |
| 7. $\frac{7}{8} \times 36$. | 11. $\frac{7}{12} \times 50$. | 15. $\frac{1}{10} \times 144$. |
| 8. $\frac{1}{12} \times 84$. | 12. $\frac{5}{8} \times 100$. | 16. $\frac{1}{10} \times 225$. |
| 9. $\frac{2}{15} \times 60$. | 13. $\frac{3}{16} \times 100$. | 17. $\frac{1}{10} \times 840$. |

II. *One factor a mixed number.*

PROCESS

$$\begin{array}{r}
 \$0.95 \\
 74\frac{1}{2} \\
 \hline
 9) \$6.65 = 7 \times \$0.95 \\
 \quad 0.73\frac{1}{2} = \frac{7}{8} \text{ of } 0.95 \\
 \quad 3.80 = 4 \times 0.95 \\
 \quad 66.50 = 70 \times 0.95 \\
 \hline
 \$71.03\frac{1}{2} = 74\frac{1}{2} \times \$0.95
 \end{array}$$

When the final result includes a fraction of $\frac{1}{2}$ cent or more, it is customary to count the fraction as another cent.

18. At \$0.95 a pound what will $74\frac{1}{2}$ pounds of tea cost?
19. $18\frac{3}{8}$ cords of wood at \$8.75?
20. 15 yards at \$0.18 $\frac{1}{2}$?
21. $27\frac{5}{8}$ yards at \$6.25?
22. $7\frac{3}{15}$ months' rent at \$28?
23. $3\frac{1}{4}$ tons at \$8.75?
24. $9\frac{5}{8}$ bushels at \$3.40?
25. 400 dozen at \$0.16 $\frac{3}{4}$?
26. 125 feet at \$0.34 $\frac{1}{2}$?

1. $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{1}{4}$ of a number = 110. Is the unknown number larger or smaller? How do you know?
2. A barrel is $\frac{3}{4}$ full. Draw off $\frac{1}{4}$ of a barrel and $\frac{2}{3}$ of a barrel. What part remains?
3. A stone wall cost \$1 a rod. What costs 3 days' work, or $6\frac{3}{4}$ rd., $5\frac{1}{4}$ rd., and $7\frac{5}{8}$ rd.?
4. A chimney contains 132 courses of brick. $\frac{4}{8}$ are under ground, $\frac{2}{8}$ roofed in; how many courses are exposed?
5. How many cords of wood in $2\frac{3}{4}$ cd. sawed by hand, $1\frac{2}{8}$ cd. by machine, and $\frac{1}{8}$ cd. chopped?
6. Two pumps contribute $\frac{1}{4}$ and $\frac{1}{4}$ toward filling a reservoir, springs contribute $\frac{1}{8}$ and $\frac{1}{8}$, surface water the rest. How much more do the pumps yield than other sources?
7. In an 11-acre marsh lot three men cut $\frac{1}{4}$, $\frac{5}{12}$, and $\frac{2}{3}$ of the whole. What part remains for a fourth man to cut? How many acres?
8. If you invest $\frac{2}{8}$ of what you have in one way and $\frac{3}{8}$ in another, what remains?
9. At \$3 a day what is due a man for working half a day, $\frac{5}{8}$ da., $\frac{3}{4}$ da., and $2\frac{1}{2}$ da.?
10. What remains of a 49-yard piece of cloth after selling $\frac{5}{8}$ of it, $\frac{2}{4}$ of the rest, and 4 yards? What is the remnant worth at $8\frac{5}{8}$ ¢ per yard?
11. Another piece of 47 yards is damaged. One half sold at 7¢. Of the other half $2\frac{3}{4}$ yards were unsalable, but the rest went at 5¢. Give the total receipts.
12. At the rate of \$1 $\frac{1}{2}$ a day, figure a board bill in dollars and cents for 3 months from August 1.
13. If a glacier moves uniformly a hundred feet a year, how far does it go in 181 days?

Finding the Fractional Part of a Fraction

1. $\frac{2}{3}$ of 6 things (apples, collars, fourths) = x things.
2. $\frac{2}{3}$ of 10 twelfths = x twelfths; $\frac{2}{3}$ of $\frac{10}{12} = \frac{1}{12}$ or $\frac{1}{3}$.
3. $\frac{5}{8}$ of $\frac{1}{3}$.
4. $\frac{1}{3}$ of $\frac{1}{3}$.
5. $\frac{5}{8}$ of $\frac{2}{3}$.
6. $\frac{1}{10}$ of $\frac{4}{9}$.
7. $\frac{5}{8}$ of $\frac{3}{8}$.
8. $\frac{1}{12}$ of $\frac{7}{8}$.

To multiply $\frac{2}{3}$ by $\frac{3}{4}$ is to find $\frac{2}{3}$ of $\frac{3}{4}$: that is, to divide $\frac{3}{4}$ into 5 equal parts and find the value of 3 of these parts.

9. How does increasing the denominator affect the size of the fractional unit?

10. If we take a denominator 5 times as large, how is the size of the fraction changed?

11. Then $\frac{1}{5}$ of $\frac{3}{4} = \frac{3}{20}$. If $\frac{1}{5}$ of a fraction = $\frac{3}{20}$, $\frac{3}{5}$ of the same fraction will be how many times $\frac{3}{20}$?

12. Make a rule for finding the product of two fractions, i.e. for finding one or more of the equal parts of a fraction.

13. Find $\frac{10}{12} \times \frac{24}{35}$.

PROCESS

$$\frac{10}{12} \times \frac{24}{35} = \frac{10 \times 24}{12 \times 35} = \frac{240}{420} = \frac{4}{7}$$

or
$$\frac{10}{12} \times \frac{24}{35} = \frac{4}{7}$$

(a) Of what use is cancellation?

(b) On what principle is it based?

(c) Which is easier, to change the product to lowest terms or to cancel first?

14. $\frac{10}{12} \times \frac{24}{35}$.

15. $\frac{1}{17} \times \frac{5}{8}$.

16. $\frac{1}{21} \times \frac{3}{8}$.

23. $8\frac{3}{4} \times 6\frac{3}{4} = \frac{35}{4} \times \frac{27}{4} = x$.

24. $\frac{2}{3} \times 7\frac{1}{2}$.

25. $2\frac{2}{3} \times 7\frac{1}{6}$.

26. $4\frac{1}{2} \times 15\frac{5}{8}$.

27. $6\frac{1}{2} \times 246$.

17. $\frac{3}{8} \times \frac{2}{3}$.

18. $85\% \times \frac{1}{10}$.

19. $96\% \times \frac{2}{3}$.

28. $\frac{1}{4}$ of $\frac{3}{8}$ of $3\frac{1}{2}$.

29. $2\frac{2}{3} \times 4\frac{1}{5} \times 7\frac{1}{18}$.

30. $\frac{3}{4} \times 1\frac{1}{2} \times \frac{1}{3}$.

31. $1\frac{1}{2} \times 5\frac{1}{4} \times 1\frac{2}{5}$.

20. $\frac{3}{12} \times \frac{5}{8}$.

21. $\frac{5}{18} \times \frac{4}{9}$.

22. $\frac{1}{12} \times \frac{2}{12}$.

32. 16% of $5\frac{1}{2}$.

33. 84% of $3\frac{1}{2}$.

34. 72% of $3\frac{7}{8}$.

35. 19% of $1\frac{1}{2}$.

Multiply each fraction in the table by the number at the end of its line or column. Change any fraction in the product to a smaller denomination when possible. Thus: — $4 \times \frac{1}{3}$ yd. = $2\frac{2}{3}$ yd. = $2\frac{2}{3}$ yd. = 2 yd. 8 in.

| | 12 | 4 | 6 | 10 | 15 | |
|---|-------------------|--------------------|--------------------|------------------------|-------------------|----|
| 2 | $\frac{1}{4}$ qt. | $\frac{5}{8}$ yd. | $\frac{1}{15}$ hr. | $\frac{7}{8}$ sq. yd. | $12\frac{1}{2}\%$ | 12 |
| 3 | $\frac{2}{3}$ yd. | $\frac{7}{8}$ gal. | $\frac{1}{11}$ da. | $\frac{8}{15}$ min. | $16\frac{2}{3}\%$ | 11 |
| 4 | $\frac{5}{8}$ pk. | $\frac{3}{16}$ lb. | $\frac{1}{12}$ ft. | $\frac{1}{11}$ sq. ft. | $33\frac{1}{3}\%$ | 10 |
| 5 | $\frac{2}{3}$ ft. | $\frac{5}{12}$ yr. | $\frac{1}{10}$ T. | $\frac{2}{10}$ sec. | $62\frac{1}{2}\%$ | 9 |
| 6 | $\frac{1}{4}$ wk. | $\frac{9}{10}$ in. | $\frac{8}{8}$ cd. | $\frac{7}{10}$ hr. | $37\frac{1}{2}\%$ | 8 |
| | 7 | 8 | 9 | 12 | 7 | |

Written Work. Problems

Make out bills in full for: —

1.

17 doz. — at \$ 1.62 $\frac{1}{2}$.

17 $\frac{1}{2}$ yd. — at 10¢.

2.

15 $\frac{3}{8}$ doz. — at \$ 1.00.

1 $\frac{3}{4}$ yd. — at 62 $\frac{1}{2}$ ¢.

3. Find the cost of 9 $\frac{3}{4}$ tons of coal at \$ 7.41.

4. Twenty pounds of sugar bought at 4 $\frac{7}{8}$ ¢ are sold for \$ 1.25. At this rate what is gained on a barrel of 200 lb.?

5. Oil is bought at \$ 3.50 for a 42-gal. barrel and retailed at 12 $\frac{1}{2}$ ¢. The gain is what part of the cost?

6. Oranges bought at 3 for 5¢ are sold at 4 for 9¢. What is gained on a box of 9 doz., 1 in 12 of which are worthless?

7. I can buy blank books of one dealer at the rate of \$ 1.25 a hundred; of another at \$ 1.60 a gross. How much is one offer better than the other?

8. Find the cost of seven 50-gal. barrels of oil at three for \$ 16.71.

9. Supposing an empty barrel to be worth \$ 1.25, what is the oil worth per gallon?

I. When the divisor is an integer

1. To divide $\frac{3}{4}$ into 5 equal parts, that is, $\frac{3}{4} \div 5 = \frac{1}{5}$ of $\frac{3}{4} = x$.

2. $\frac{1}{7} \div 4 = \frac{x}{17}$; $\frac{1}{7} \div 4 = \frac{15}{x}$.

Observe that a fraction may be divided by a number either by dividing the numerator or by multiplying the denominator by that number.

3. Observing Exercise 2, tell when you use one method and when the other, and why. Which is shorter?

4. $\frac{21}{100} \div 7$.

7. $\frac{4}{5} \div 12$.

10. $\frac{15}{100} \div 6$.

5. $\frac{4}{5} \div 8$.

8. $\frac{6}{15} \div 12$.

11. $48\% \div 16$.

6. $\frac{24}{5} \div 8$.

9. $\frac{4}{15} \div 20$.

12. $\frac{85}{100} \div 17$.

II. When the divisor is a fraction

13. 2 ft. + 3 in. = x. (Since dividend and divisor must represent like units, we have 24 in. + 3 in.)

14. $\frac{3}{4} \div \frac{2}{3}$. Are the units of the same size?

15. $\frac{3}{4} \div \frac{2}{3} = \frac{9}{12} \div \frac{8}{12}$; $9 \div 8 = 1\frac{1}{8}$. 16. $\frac{4}{5} \div \frac{1}{3} = \frac{x}{15} \div \frac{y}{15} = x \div y = z$.

17. What is the first step in dividing days by hours? Feet by inches? 4ths by 5ths? One fraction by another?

18. $\frac{5}{6} \div \frac{2}{3}$.

21. $\frac{7}{10} \div \frac{3}{4}$.

24. $\frac{27}{2} \div \frac{5}{16}$.

19. $1\frac{1}{2} \div \frac{3}{4}$.

22. $\frac{5}{12} \div \frac{7}{8}$.

25. $1\frac{3}{8} \div \frac{3}{4}$.

20. $\frac{8}{9} \div \frac{3}{4}$.

23. $\frac{4}{5} \div \frac{4}{7}$.

26. $\frac{7}{15} \div \frac{2}{3}$.

27. How many 4ths in 1? In 2? In 5?

28. How many 3ds in 1? In 4? In 10?

29. How many 5ths in 1? In 10? In 20?

30. $1 \div \frac{1}{4} = \text{what?}$

33. $1 \div \frac{1}{7} = x$.

36. $1 \div \frac{1}{10} = x$.

31. $1 \div \frac{1}{6} = \text{—?}$

34. $1 \div \frac{1}{8} = x$.

37. $1 \div \frac{1}{10} = x$.

32. $1 \div \frac{1}{8} = \text{—?}$

35. $1 \div \frac{1}{9} = x$.

38. $1 \div \frac{1}{9} = x$.

1. $1 + \frac{2}{3} = \frac{5}{3} + \frac{2}{3} = \frac{7}{3} = 2\frac{1}{3}$. 4. $1 + \frac{3}{4}$. 7. $1 + \frac{1}{12}$.
 2. $1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4} = x$. 5. $1 + \frac{3}{4}$. 8. $1 + \frac{3}{4}$.
 3. $1 + \frac{7}{8} = x$. 6. $1 + \frac{5}{16}$. 9. $1 + \frac{5}{12}$.

Notice that in each of the preceding exercises the quotient was the same as that obtained from dividing the denominator by the numerator. Why was this?

10. Compare $6 \div 2$ and $12 \div 2$.
 11. What effect on the quotient if the dividend is increased?
 12. How does $2 \div \frac{2}{3}$ compare with $1 \div \frac{2}{3}$?
 13. Compare $8 \div 2$ with $4 \div 2$.
 14. Compare $\frac{1}{2} \div \frac{2}{3}$ with $1 \div \frac{2}{3}$.
 15. $\frac{3}{4} \div \frac{2}{3}$.

SUGGESTION. Since $1 \div \frac{2}{3}$ is $\frac{3}{2}$, $\frac{3}{4} \div \frac{2}{3}$ is $\frac{3}{4}$ of $\frac{3}{2}$ or $\frac{9}{8}$.

16. $\frac{7}{8} \div \frac{3}{8} = \frac{7}{8} \times \frac{8}{3} = \frac{7}{3}$.

EXPLANATION. Since $1 \div \frac{3}{8}$ is $\frac{8}{3}$, $\frac{7}{8} \div \frac{3}{8}$ is $\frac{7}{8}$ as great as $1 \div \frac{3}{8}$, or is $\frac{7}{8}$ of $\frac{8}{3}$.

17. The two methods compared:—

FIRST METHOD

$$\frac{7}{8} \div \frac{3}{8} = \frac{35}{40} \div \frac{3}{8}$$

$$35 \div 24 = \frac{35}{24} = 1\frac{11}{24}$$

SECOND METHOD

$$\frac{7}{8} \div \frac{3}{8} = \frac{7}{8} \times \frac{8}{3} = \frac{7}{3} = 1\frac{1}{3}$$

- (a) What advantage has the second method over the first?
 (b) What disadvantage may it possibly have?

NOTE. Cancellation may be used in division of fractions as in multiplication.

Apply the shorter process and explain why it is shorter:—

18. $\frac{5}{7} \div \frac{3}{8}$. 22. $\frac{4}{5} \div \frac{7}{12}$. 26. $\frac{2}{3} \div \frac{5}{12}$.
 19. $\frac{6}{11} \div \frac{4}{9}$. 23. $\frac{4}{5} \div \frac{7}{15}$. 27. $\frac{5}{4} \div \frac{2}{11}$.
 20. $\frac{3}{7} \div \frac{2}{14}$. 24. $\frac{3}{4} \div \frac{5}{8}$. 28. $0.32 \div \frac{1}{25}$.
 21. $\frac{3}{7} \div \frac{1}{12}$. 25. $\frac{3}{4} \div \frac{5}{16}$. 29. $0.042 \div \frac{7}{25}$.

Complex Fractions

1. Divide $12\frac{3}{4}$ by $3\frac{2}{3}$. (Change mixed numbers to improper fractions.)

2. $9\frac{3}{4} \div 8\frac{3}{8}$.

5. $15\frac{5}{8} \div 9\frac{3}{8}$.

8. $82\frac{2}{7} \div 32\frac{2}{7}$.

3. $10\frac{3}{8} \div 7\frac{1}{8}$.

6. $73\frac{5}{8} \div 57\frac{1}{8}$.

9. $1000 \div 66\frac{2}{3}$.

4. $8\frac{5}{8} \div 5\frac{3}{8}$.

7. $72\frac{9}{10} \div 42\frac{3}{10}$.

10. $100 \div 62\frac{1}{2}$.

11. $\frac{2\frac{1}{2}}{7\frac{1}{8}}$ is a way of indicating a division of $2\frac{1}{2}$ by $7\frac{1}{8}$. $\frac{\frac{5}{2}}{\frac{57}{8}} = \text{---}$.

Such expressions are called **complex fractions**. To change them to simple fractions, multiply each term by a common denominator of the fractions. Thus:—

$$\frac{2\frac{1}{2} \times 6}{7\frac{1}{8} \times 6} = \frac{15}{44}$$

12. $\frac{16\frac{3}{4}}{100}$.

13. $\frac{18\frac{3}{4}}{100}$.

14. $\frac{5\frac{1}{4}}{30\frac{1}{4}}$.

15. $\frac{\frac{2}{3}}{\frac{1}{5}}$.

Mixed Numbers Divided

1. Divide $1645\frac{5}{8}$ by 9. Name two steps.

PROCESS

$$\begin{array}{r} 9 \overline{) 1645\frac{5}{8}} \\ \underline{182\frac{5}{8}} \end{array}$$

EXPLANATION. (a) $\frac{1}{8}$ of $1645\frac{5}{8} = 182$, and $7\frac{5}{8}$ remain-
ing.

(b) $\frac{1}{8}$ of $7\frac{5}{8} = \frac{1}{8}$ of $5\frac{5}{8} = \frac{5}{64}$.

2. $3476\frac{5}{8} \div 8$.

4. $9213\frac{7}{8} \div 9$.

6. $2648\frac{3}{4} \div 9$.

3. $7329\frac{3}{8} \div 12$.

5. $876\frac{3}{4} \div 11$.

7. $468\frac{3}{4} \div 7$.

8. Divide $289\frac{3}{4}$ by 26. To what common fractional unit are both dividend and divisor changed?

PROCESS

$$\begin{array}{r} 26 \overline{) 289\frac{3}{4}} \\ \underline{4} \quad \underline{4} \\ 104 \overline{) 1159} \quad (11\frac{15}{104}) \end{array}$$

9. $367\frac{3}{8} \div 24$.

11. $4723\frac{3}{8} \div 105$.

10. $846\frac{3}{8} \div 39$.

12. $6928\frac{1}{8} \div 216$.

13. If 75 boxes weigh $847\frac{7}{8}$ lb., what will one weigh?

$\frac{1}{75}$ of $15 = \frac{15}{75}$

14. $89 \text{ rd.} = 1468\frac{1}{2} \text{ ft.}; 1 \text{ rd.} = x \text{ ft.}$

| <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> | <i>e</i> | <i>f</i> | <i>g</i> | <i>h</i> | <i>i</i> |
|----------|----------------|-----------------|-----------------|------------------|------------------|------------------|--------------------|----------|
| 1. | $\frac{2}{3}$ | $\frac{5}{6}$ | $1\frac{1}{2}$ | $2\frac{2}{3}$ | $6\frac{1}{2}$ | $15\frac{2}{3}$ | $316\frac{1}{2}$ | \$100 |
| 2. | $\frac{3}{4}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | $7\frac{3}{4}$ | $18\frac{1}{2}$ | $28\frac{1}{2}$ | $493\frac{3}{4}$ | 250 |
| 3. | $\frac{5}{6}$ | $\frac{4}{5}$ | $\frac{4}{5}$ | $5\frac{2}{10}$ | $9\frac{1}{2}$ | $21\frac{1}{2}$ | $641\frac{1}{2}$ | 500 |
| 4. | $\frac{7}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ | $6\frac{5}{8}$ | $10\frac{3}{10}$ | $64\frac{3}{8}$ | $827\frac{3}{8}$ | 576 |
| 5. | $\frac{8}{9}$ | $\frac{5}{6}$ | $\frac{8}{9}$ | $9\frac{8}{9}$ | $15\frac{2}{3}$ | $85\frac{2}{3}$ | $936\frac{2}{3}$ | 600 |
| 6. | $\frac{9}{10}$ | $\frac{7}{8}$ | $\frac{9}{10}$ | $8\frac{9}{10}$ | $21\frac{1}{2}$ | $90\frac{2}{10}$ | $1464\frac{1}{2}$ | 640 |
| 7. | $\frac{7}{10}$ | $\frac{9}{10}$ | $\frac{10}{10}$ | $4\frac{1}{10}$ | $9\frac{7}{10}$ | $16\frac{1}{2}$ | $2525\frac{7}{10}$ | 720 |
| 8. | $1\frac{1}{2}$ | $\frac{1}{10}$ | $\frac{6}{10}$ | $9\frac{3}{4}$ | $12\frac{4}{10}$ | $72\frac{4}{10}$ | $4769\frac{3}{10}$ | 800 |
| 9. | $\frac{4}{5}$ | $\frac{3}{5}$ | $\frac{10}{10}$ | $10\frac{4}{10}$ | $18\frac{4}{10}$ | $36\frac{4}{10}$ | $8461\frac{4}{10}$ | 960 |
| 10. | $1\frac{1}{2}$ | $\frac{10}{10}$ | $\frac{10}{10}$ | $3\frac{1}{2}$ | $20\frac{1}{2}$ | $25\frac{1}{2}$ | $7550\frac{7}{10}$ | 1000 |

TO THE TEACHER. Each of the following combinations may be applied to each number in the designated column so as to furnish ten examples which may be assigned consecutively from 1 to 10, making 450 in all. How much to use depends on the degree of accuracy and facility attained or desired.

- | | | | |
|------------------------|-------------------------------|---|---------------------|
| 1. $b + \frac{2}{3}$. | 10. $f + g + h$. | 19. $g - d$. | 28. $f \times g$. |
| 2. $c + \frac{3}{4}$. | 11. $20 - e$. | 20. $g - e - f$. | 29. $67 \times h$. |
| 3. $d + \frac{7}{8}$. | 12. $f - 5\frac{1}{2}$. | 21. $5 \times b$. | 30. $g \times i$. |
| 4. $b + c$. | 13. $b - c$. | 22. $12 \times c$. | 31. $b \div a$. |
| 5. $c + d$. | 14. $d - c$ or $c - d$. | 23. $a \times g$. | 32. $a \div c$. |
| 6. $b + c + d$. | 15. $f - e$. | 24. b of d . | 33. $b \div c$. |
| 7. $e + f$. | 16. $g - f$. | 25. $c \times d$. | 34. $e + d$. |
| 8. $f + g$. | 17. $94\frac{7}{10} - g$. | 26. $b \times c \times d$. | 35. $f \div e$. |
| 9. $e + f + g$. | 18. $h - g$. | 27. $c \times d \times e$. | 36. $g + f$. |
| 37. $h + 9$. | 40. $i + g$. | 43. c of $f + e$. | |
| 38. $h \div 75$. | 41. $b + c - \frac{1}{2} d$. | 44. $g + e \times f$. | |
| 39. $i + e$. | 42. $g - (e + f)$. | 45. $(b \text{ of } e) + (e \text{ of } f)$. | |

1. An heir gets $\frac{4}{5}$ of an estate, then loses $\frac{3}{8}$ of his share. What part of the estate does he keep?

2. I buy at 20% discount. What is the total cost to me of goods sold regularly for \$ 1.42, \$ 3.98, \$ 57, \$ 0.16 $\frac{3}{8}$, and 9 pieces at \$ 0.31 $\frac{1}{4}$?

3. If 8 $\frac{1}{2}$ T. of coal cost \$ 48 $\frac{7}{8}$, what is the cost of 68 T.?

4. Property which cost \$ 5000 is rented for \$ 43 $\frac{1}{3}$ a month; what is the annual income to the owner after paying a tax of \$ 15 on a thousand?

5. Three cheeses, weighing respectively 34 $\frac{1}{2}$, 42 $\frac{3}{4}$, and 47 $\frac{5}{8}$ lb., were sold for \$ 20.60. What was the price per pound?

6. J. F. Sampson bought 72 $\frac{1}{2}$ bu. potatoes at 62 $\frac{1}{2}$ ¢ a bushel, and sold $\frac{3}{8}$ at 64 $\frac{1}{2}$ ¢, the remainder at 75¢. What did he gain?

7. An electric launch was sold for \$ 285, or $\frac{1}{2}\frac{2}{5}$ of the cost. Find $\frac{2}{5}\frac{2}{5}$ of the cost, or the whole cost. Compare $\frac{2}{5}\frac{2}{5}$ with $\frac{1}{2}\frac{2}{5}$.

8. $\frac{2}{3}$ of a ton of hay at \$ 20 pays for 1 $\frac{1}{3}$ T. of coal at how much a ton?

9. 16 $\frac{1}{2}$ ft. of 2-in. pipe at 6 $\frac{1}{2}$ ¢, and 1020 ft. of 1-in. pipe at 4 $\frac{1}{4}$ ¢, are exchanged for 120 lb. of tubing at 11 $\frac{1}{8}$ ¢, and 134 ft. at 9¢. What is the difference in value?

10. Two trains start together in the same direction. How far apart will they be in an hour if one goes a mile in 1 $\frac{3}{10}$ min. and the other in 85 sec.?

11. An express train runs 240 mi. in 5 $\frac{1}{4}$ hr. How far will it run in 3 $\frac{1}{2}$ hr.? (Compare 3 $\frac{1}{2}$ with 5 $\frac{1}{4}$.)

12. A tank holds 168 gal. and is $\frac{3}{8}$ full. $\frac{3}{8}$ of the quantity is drawn off. How many gallons will fill the tank?

13. An automobile started at 10.45 A.M., and at 3.20 P.M. had covered 64 $\frac{1}{4}$ mi. What was the rate per hour?

1. A man had in a bank a certain sum of money. He withdrew $\frac{2}{3}$ of $\frac{3}{4}$ of it, and gave his son $\frac{5}{8}$ of $\frac{1}{2}\frac{3}{4}$ of that. What part of the whole amount did his son receive?

2. A man walked $29\frac{1}{4}$ mi. one day, and $21\frac{5}{8}$ mi. the next day. On the third he walked the difference between the two distances. How far did he walk in the three days?

3. A owned a farm as follows: $33\frac{1}{4}$ A. of wood land; $63\frac{5}{8}$ A. of meadow; $25\frac{1}{2}$ A. of good grass land; and $19\frac{3}{4}\frac{1}{2}$ A. producing fruit. He sold $\frac{3}{4}$ of the whole farm to a speculator. This was x A.

4. If 3 bbl. of flour will supply 30 people $13\frac{1}{4}$ wk., how many barrels will it take to supply them 46 wk.?

5. A man traveled $\frac{3}{4}$ of his journey the first month, and $\frac{2}{3}$ the next month, when he found that he still had 660 mi. to go. How far had he traveled?

6. A boy bought $2\frac{3}{4}$ bu. grain and sold me $\frac{7}{12}$ of it. Another boy bought 3 bu. and sold me an amount equal to what I bought from the first boy. What fractional part of the second boy's grain did I buy?

7. Bought $\frac{3}{8}$ of a barrel of flour of one man, $\frac{1}{4}$ bbl. of another, and $\frac{17}{100}$ of the third. What was the cost at the rate of \$6 a barrel?

8. I purchased 360 A. of land at \$75 per acre, and sold $\frac{5}{8}$ of it at \$85 per acre. After paying for the land, I had left x A. and \$y.

9. If $13\frac{3}{4}$ bu. of rice cost \$11.75, what will $37\frac{1}{4}$ bu. cost?

10. A man bought 100 yd. of carpeting, and sold $37\frac{1}{2}$ yd. to one man and $\frac{1}{3}$ of the remainder to another. How many yards had he left?

11. How much larger is $24\frac{3}{4} + \frac{7}{8}$ than $24\frac{3}{4} \times \frac{7}{8}$?

Compare with 100:—

1. 50, 25, 75, 20, 40, 10, 30, 70. 2. 5, 15, 4, 12, 16, 2, 6, 8.

Repeat rapidly until thoroughly learned the values of the following parts of 100:—

- | | |
|--|--|
| 3. $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}$. | 7. $\frac{1}{10}, \frac{1}{20}, \frac{1}{30}, \frac{2}{10}, \frac{3}{10}, \frac{2}{5}, \frac{1}{100}$ |
| 4. $\frac{2}{3}, \frac{4}{3}, \frac{1}{6}, \frac{5}{6}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}$. | 8. $\frac{3}{10}, \frac{1}{12}, \frac{5}{16}, \frac{6}{16}, \frac{7}{16}, \frac{8}{16}, \frac{1}{4}$. |
| 5. $\frac{7}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{16}, \frac{1}{20}, \frac{1}{25}, \frac{1}{30}$. | 9. $\frac{3}{12}, \frac{1}{16}, \frac{1}{40}, \frac{1}{80}, \frac{1}{160}, \frac{1}{320}$. |
| 6. $\frac{2}{3}, \frac{3}{8}, \frac{5}{8}, \frac{2}{5}, \frac{4}{5}, \frac{5}{6}$. | 10. $\frac{1}{12}, \frac{1}{16}, \frac{1}{24}, \frac{3}{40}, \frac{1}{160}, \frac{1}{12}$. |

11. A nurseryman sells 2500 strawberry plants at \$6 a hundred. They cost him $\frac{2}{3}$ as much to raise, and he gives an agent $\frac{1}{3}$ of the profit. How much does he gain?

Compare one number with the other in each of the following columns. Thus:—

1. (a) The ratio of 8 to 24 is $\frac{1}{3}$; (b) 8 is $\frac{1}{3}$, or $33\frac{1}{3}\%$, of 24.
 2. (a) The ratio of $\frac{5}{8}$ to $\frac{1}{4}$ is 5 to 2, or $2\frac{1}{2}$; (b) $\frac{5}{8}$ is $2\frac{1}{2}$ times, or 250%, of $\frac{1}{4}$.

| I. | II. | III. | IV. |
|-----------------------------------|--------------------------------|--------------------------------|--|
| 1. 8, 24 | 25, $6\frac{1}{4}$ | 90, 18 | 1 da., 1 hr. |
| 2. 60, 12 | $3\frac{1}{3}$, 10 | 1 oz., 1 lb. | 1 wk., 1 da. |
| 3. 24, 60 | $\frac{5}{8}, \frac{1}{4}$ | 2 gal., 3 qt. | 5 min., 25 sec. |
| 4. 48, 72 | 72, 60 | 30, 50 | 144, 148 |
| 5. $16\frac{2}{3}, 66\frac{2}{3}$ | $37\frac{1}{2}, 6\frac{1}{4}$ | $\frac{4}{5}, 0.16$ | $\frac{1}{1000}, \frac{1}{1000}$ |
| 6. $\frac{3}{8}, \frac{3}{4}$ | $\frac{3}{8}, \frac{5}{8}$ | 0.68, 0.51 | $4 \times \frac{1}{100}, 3 \times \frac{1}{100}$ |
| 7. 375, 25 | $\frac{5}{6}, \frac{2}{3}$ | $10\frac{1}{2}, 5\frac{1}{4}$ | 0.93, 0.31 |
| 8. 1, 100 | 10%, 20% | $\frac{1}{100}, \frac{5}{100}$ | 200, $16\frac{2}{3}$ |
| 9. 2.75, 5.50 | \$0.75, \$1.25 | \$1, 100¢ | \$1, \$1.50 |
| 10. 1, 1000 | $37\frac{1}{2}, 87\frac{1}{2}$ | $33\frac{1}{3}, 100$ | $1.83\frac{1}{3}, 1.00$ |

Compare the following and give the ratio in per cent : —

Thus, to compare 5 with 20, say 5 is 25% of 20.

1. 2 with 28, 15 with 40.
2. 16 with 80, 6 with 42.
3. 25 with 150, 24 with 42.
4. 45 with 15, 48 with 36.
5. 18 with 54, 18 with 24.
6. 35 with 105, 3 da. with 1 wk.
7. 4 in. with 1 yd., 2 mo. with 1 yr.
8. 1 oz. with 1 lb., 1 hr. with 1 min.
9. 1 lb. with 1 oz., 100 lb. with 1 T.
10. 2 qt. with 4 gal., 1¢ with \$1.
11. What is the relation of the whole of anything to $\frac{2}{3}$ of it, i.e. the relation of $\frac{2}{3}$ to $\frac{2}{3}$?

12. If 12 is $\frac{2}{3}$ of a number, what is the number?

SUGGESTION. Since all, or 1, is $2\frac{1}{2}$ times $\frac{2}{3}$, the whole number is $\text{---} \times 12$.

13. If 28 is $\frac{2}{3}$ of a number, what is the number? (Compare $\frac{2}{3}$ with $\frac{2}{3}$. Is the number larger or smaller than 28? How many times as large?)

In this way find the whole when a part is given : —

14. 12 is $\frac{1}{2}$ of what?
15. 16 is $\frac{1}{3}$ of what?
16. 24 is $\frac{1}{6}$ of what?
17. 19 is $\frac{1}{5}$ of what?
18. 28 is $\frac{2}{3}$ of what?
19. 36 is $\frac{3}{4}$ of what?
20. 72 is $\frac{4}{5}$ of what?
21. 100 is $\frac{1}{10}$ of what?
22. 450 is $\frac{1}{8}$ of what?
23. 175 is $\frac{3}{4}$ of what?
24. What part of $2\frac{3}{4}$ is $\frac{1}{4}$?
25. $3\frac{2}{3}$ is $\frac{1}{2}$ of what?
26. $12\frac{1}{2}$ is $\frac{5}{6}$ of what?
27. $14\frac{2}{3}$ is $\frac{2}{3}$ of what?
28. What part of $\frac{2}{3}$ is $\frac{5}{12}$? (Change to like units.)
29. $18\frac{3}{4}$ is what part of $62\frac{1}{2}$? ($3 \times 6\frac{1}{2}$ and $10 \times 6\frac{1}{2}$.)
30. A farmer sold 200 bu. of beans. This was $\frac{4}{5}$ of his crop. How many had he?
31. 20, or $\frac{2}{5}$ of a farmer's sheep, are black. How many sheep has he?

1. What does the phrase *per cent* mean? How many per cent in the whole of anything?

2. The sign % takes the place of what denominator?

3. What is the unit in 8%?

4. What is the ratio of all, or 100%, of anything to 50% of it?

5. If 50% of a crop is 400 bu. what is the whole crop?

Find the whole when the specified part is known:—

6. 16 is $\frac{1}{2}$, or 50%, of what? 10. 56 is $\frac{7}{8}$, or $87\frac{1}{2}\%$, of what?

7. 24 is $\frac{3}{4}$, or 75%, of what? 11. 20 is 25% of what?

8. 32 is $\frac{2}{3}$, or $66\frac{2}{3}\%$, of what? 12. 15 is $12\frac{1}{2}\%$ of what?

9. 40 is $\frac{5}{8}$, or $83\frac{1}{8}\%$, of what? 13. 80 is $37\frac{1}{2}\%$ of what?

14. $2\frac{1}{2}$ is 10% of _____. 17. $\sqrt{25}$ is 5% of ____.

15. $3\frac{1}{3}$ is $33\frac{1}{3}\%$ of _____. 18. 36 is 18% of ____.

6. $\frac{1}{8}$ is 20% of _____. 19. 9^2 is 27% of ____.

20. A teacher pays \$6 per week for board and room. This is 40% of her salary. What is her salary for a school year of 40 weeks? (Her salary is how many times 40% of it?)

21. A man's expenses are \$7.50 per week. This is $8\frac{1}{8}\%$ of his income. What is his income? (His whole income is how many times $8\frac{1}{8}\%$ of it?)

22. How much have I if \$1.20 is 20% of my money?

23. Mary Smith's salary is \$750 per year. This is $33\frac{1}{3}\%$ of her father's income. What does Mr. Smith receive annually?

24. After spending \$5, James had 60% of his money remaining. How many per cent did he spend and how much had he at first?

25. I sold 60 of my flock of sheep. If this was 20% of my whole flock, how many had I left?

HINT. How many per cent left? How many times 20% is the per cent left?

Explain exactly how you get each result:—

1. $10 = \frac{1}{10}$ of what number? $10 = \frac{1}{2}$ of what number?
2. Give $\frac{1}{2}$ and $\frac{1}{3}$ their least common denominators.
3. $\frac{1}{2}$ of a hill is dug away. How many times as much remain?
4. $10 + \frac{1}{2} = \text{what?}$ $10 + \frac{1}{3} = \text{what?}$
5. A train goes a mile in $1\frac{1}{2}$ min. How far will it go in an hour?
6. At 1 mi. in 90 sec., how much in an hour?
7. 50 mi. an hour = how much a minute?
8. At 48 mi. an hour, how long does it take to go 1 mi.?
9. A mile in $1\frac{1}{2}$ min. is the same as 60 mi. in —
10. 34% of certain telegraph lines are under ground. What per cent are above ground?
11. A foundry uses 100 T. of Swedish iron to 50 T. from other sources. What part or per cent of each class is used?
12. \$160 was $\frac{1}{5}$, or 16%, of the profits. What were the profits?
13. 23% of a certain stock was glassware, 69% was china. The rest was in brass goods, which were $x\%$ of the whole?
14. The 5,000,000 sq. mi. of the Arctic Ocean are what per cent of the area of the Pacific, which is 16 times as large?
15. I gain 100% on $\frac{1}{2}$ my goods and sell the rest at cost. How much do I gain on \$100 invested?
16. In a 36-column newspaper what part of the whole space would be filled by 20 columns of advertisements? What per cent?
17. I lose half that I have, and 25% of the rest. What I keep is what part of what I lose?
18. The board of a horse is \$20, shoeing \$1.25, harness repairs \$0.25, use of carriage \$3, new whip \$0.50. Each item is what part of the whole? Give per cents when you can.
19. $3^2 = x\%$ of 9^2 .
20. $2^3 = x\%$ of 8^3 .

1. Change to lowest terms $\frac{510}{1400}$; $\frac{707}{1400}$.
2. Compare results: $224.7 \text{ yd.} + 7$; $224\frac{7}{10} \text{ yd.} + 7 \text{ yd.}$
3. $\frac{1}{11}$ of $10 = x$. $\frac{5}{8}$ of $10 = y$.
4. $\frac{2}{11}$ league = what part of $\frac{3}{11}$ league?
5. $\frac{3}{8} \text{ A.} =$ what part of $\frac{1}{8} \text{ A.}$? Of $1\frac{7}{14} \text{ A.}$?
6. $\frac{3}{8}$ is contained in $1\frac{1}{8}$ how many times?
7. Cancel mentally: $\frac{3}{8}$ of $\frac{3}{8} \times \frac{1}{4}$ of $\frac{1}{8} = x$.
8. By getting a discount of $\frac{2}{3}$ I pay only \$3.33. What is the regular price?
9. $1\frac{7}{8} \div 2\frac{7}{8} = x$.
10. What is the least that will pay for 1 article, when the price per dozen is —
 $\$1.05?$ $\$1.10?$ $\$1.15?$ $\$1.25?$ $\$1.30?$ $\$1.35?$ $\$1.40?$ $\$1.50?$
 $\$1.75?$ $\$2.00?$ $\$2.25?$ $\$2.50?$ $\$2.75?$ $\$3.00?$ $\$5.00?$
11. $\frac{3}{4}$ rd. at \$1.25.
12. $5\frac{7}{8}$ lb. at \$1.28.
13. $3\frac{1}{8}$ in $25x$ times.
14. $40 - 16\frac{2}{3} - 7\frac{2}{3} = x$.
15. $412\frac{1}{2} + 87\frac{1}{2} + 62\frac{1}{2} + 37\frac{1}{2} + 112\frac{1}{2} + 12\frac{1}{2}$; $66\frac{2}{3} - 16\frac{2}{3} - 8\frac{1}{3}$.
16. By a Fahrenheit thermometer what is the temperature when the top of the mercury column is $\frac{5}{16}$ of the distance in degrees from zero to the freezing point?
17. After gaining $\frac{1}{10}$, or 10%, I have \$99. What had I at first?
18. Find $2\frac{1}{2} \times 90$. What is $\frac{1}{100}$ of it? 6% of it?
19. What part of 100 is $9\frac{1}{11}$? Give $\frac{1}{11}$ of 100; $\frac{3}{11}$; $\frac{6}{11}$; $\frac{9}{11}$.
20. How much for a dozen at 16 for a quarter? At 4 for 5¢? At 20 for a quarter? At 3 for 10¢? At 3 for 5¢?
21. A newspaper weighing 4 oz. may be mailed for 1¢. What will it cost to send —
 1 lb.? 10 oz.? 5 oz.? $4\frac{1}{8}$ oz.? $\frac{3}{4}$ oz.?

1. Paid at different times $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{7}{12}$ of a debt. The balance was \$1170.61. What was the whole debt?

2. $\frac{1}{3}$ of an estate is divided equally among 14 persons, another $\frac{1}{3}$ among 9 persons. One of each of these shares is to be given to the heir of the remaining third. What part of the whole does he receive?

3. From a life-saving station to the end of the northern beat is $2\frac{1}{10}$ mi. How many full steps will a surfman take in going and returning, if his steps average $2\frac{1}{2}$ ft. each?

4. The Minot's Ledge light revolves twice a minute. It is lighted from sunset to sunrise. How many revolutions does it make between 5.52 P.M. and 5.52 A.M.?

5. Four equal farms, all adjoining, are offered for house lots. Parts of each are sold as follows: $\frac{3}{8}$, $\frac{9}{10}$, $\frac{9}{10}$, $\frac{9}{12}$. Add the fractions and tell what the sum shows.

6. What part of a mile is covered by 22 revolutions of a wheel 18 ft. round?

7. Divide 231 cu. in. into 3 equal integral parts; into 33. How else can it be exactly divided?

8. Just when does $\frac{1}{4}$ of a common year end? $\frac{1}{4}$ of a leap year?

9. A bushel of potatoes is commonly 60 lb. A thousand 56-lb. bushels are what part by weight of 1000 60-lb. bushels?

10. A barrel of 42 gal. will fill how many cans containing $1\frac{1}{8}$ pt.?

11. When \$80 are earned in a month and $\frac{5}{12}$ of it spent, the saving of $2\frac{1}{12}$ yr. at that rate would be how much?

12. A chest contained $7\frac{1}{4}$ lb. of tea at 55¢, $13\frac{1}{4}$ lb. at 35¢, and $9\frac{3}{8}$ lb. at 27¢. The mixture is worth x ¢ a pound, and \$10 would buy y lb., with z ¢ remaining.

Find the cost of the following purchases: —

1. $3\frac{1}{2}$ yd. silk at \$1.37 $\frac{1}{2}$; $8\frac{1}{8}$ doz. buttons at 15¢; $7\frac{1}{2}$ sticks braid at 75¢; 23 yd. ribbon at 16 $\frac{2}{3}$ ¢.

2. 6470 ft. fencing at 9 $\frac{5}{12}$ ¢; 3 lots land, 10,280, 7595, 8122 sq. ft. at 5 $\frac{1}{4}$ ¢; 3400 bricks at \$8.15 per M.

3. 3450 lb. coal at \$8.25 a ton; 324 cu. ft. wood at \$5.75 per cord; 5460 lb. coke at \$6.40 per ton.

4. Go over the computations in the following bill or invoice to find the errors it contains.

CHICAGO, Aug. 1, 1904.

MR. HENRY D. WARREN

Bought of JOHN V. FARWELL & Co.

| | | | | | | |
|---------|---------------------------------|----------------------|----|----|----|----|
| May 18 | 23 yd. Brussels Carpeting | @ \$1.50 | 34 | 50 | | |
| June 22 | 16 $\frac{1}{2}$ yd. Black Silk | @ 1.75 | 18 | 98 | | |
| July 6 | 38 yd. Wamsutta Cotton | @ 0.12 $\frac{1}{2}$ | 4 | 75 | | |
| | | | | | 58 | 23 |
| | Less $\frac{1}{10}$ | | | | 5 | 82 |
| | | | | | 52 | 41 |
| | <i>Cr.</i> | | | | | |
| June 2 | By Cash | \$25.00 | | | | |
| | By Cash | 20.00 | | | 45 | 00 |
| | <i>Received payment,</i> | | | | 7 | 41 |
| | JOHN V. FARWELL & Co., | | | | | |
| | By Smith. | | | | | |

5. Make out bill in proper form. Supply dates and names.

13 tons Franklin coal at \$ 7.25

6 $\frac{1}{2}$ tons Lackawanna at 5.50

1 Cord Hard Wood at 11.00

3 $\frac{1}{2}$ bbl. Cement at 3.25

1. Friday, Jan. 1, 1904, Sam'l Chase had \$32.76 to his credit in a bank. If he deposited \$25 every week day during the month, and \$100 extra every Saturday, what amount could he draw against Feb. 1?

2. In buying 785 music books @ 85¢, a discount of $\frac{1}{4}$ or 20% is allowed on cash payments. The net cost is what?

3. Bill 6 $\frac{1}{2}$ lb. Formosa Oolong Tea @ 60¢
 30 lb. Maracaibo Coffee @ 24 $\frac{1}{2}$ ¢
 2 bbl. "Bridal Veil" Flour @ \$5.25

Discount 2% [for cash].

4. Bill 37 $\frac{1}{2}$ yd. Dwight Cotton @ 11¢
 42 $\frac{1}{2}$ yd. Scotch Gingham @ 23¢
 11 $\frac{3}{4}$ yd. India Silk @ \$1.75

Credit mdse. returned, \$8.75.

5. Invoice 3 $\frac{1}{2}$ gro. No. 314 Eagle Pencils @ \$4.20
 54 gro. No. 404 Gillott's Pens @ 0.37 $\frac{1}{2}$
 $1\frac{1}{2}$ gro. 4to Blank Books @ 3.66

6. Colonel S., a Kansas farmer, harvested 4000 acres of wheat in 1903. He estimates the cost per acre as follows: for plowing, \$1.00; for drilling, 25¢; for seed, 3 pecks to the acre at 60¢ a bushel; for heading and stacking, \$1.25; for threshing, 20 bushels per acre at 6¢; for hauling 4¢ a bushel. The wheat was sold at 60¢ a bushel, and the use of fields for grazing during the winter is worth \$2500. Estimate the profits.

Rule paper for an account that you keep with John Holmes. It will show that he is debtor for all that is sold to — by — and creditor for all that is paid to — by — as below. (See p. 15.)

| JOHN HOLMES | | Dr. | Cr. |
|-------------|---------------------------------------|-----|-------|
| 19— | | | |
| May 1 | To 3 Shares Mill Stock | 312 | 00 |
| | By $\frac{1}{2}$ River Pasture (3 A.) | | 45 00 |

1. Complete Holmes's account, found begun on page 68, from the following data: $3\frac{1}{2}$ days repairing Holmes's fence at \$2.50. Credit him for use of his oxen same time at \$2. Sold him 23 bbl. apples at \$1.40. Bought of him 3 hogs (732 lb.) at $11\frac{1}{2}\phi$, and $1\frac{1}{2}$ tons hay at \$15. Sold him 15 young maples at \$1.06 $\frac{1}{4}$, and 3 hoops containing 35 ft. strap iron at $2\frac{3}{4}\phi$. Holmes paid cash \$100. What does he now owe me?

2. January 1 my gas meter read 67,500; March 31 it read 91,500. At \$1.60 per thousand my quarter's gas bill is \$x.

3. A week's sales of wheat in bushels: 2137, 3476, 972, 3041, 6732, 1849. Valued at $62\frac{3}{4}\phi$. What are the gross proceeds?

4. What did my house cost me as shown by these items:—

Cellar, 18 days @ \$14.75; mason's contract, \$4375.86; carpenter, 137 days @ \$2.15 and 96 at \$3; materials, \$576.84; painting, etc., \$397.68?

5. At an auction sale of land the following prices were obtained: 3648 ft. at 23ϕ ; 2894 at $31\frac{1}{2}\phi$; 7642 at $19\frac{1}{2}\phi$; 8641 at 25ϕ . The auctioneer's commission was 2ϕ on the dollar, and advertising, etc., cost \$37.50. Required the net proceeds.

6. Bought a 100-acre wood lot for \$800. Paid 23 men \$517.50 for 18 days' work at cutting. Sold 175 cords at \$2.37, 215 at \$4.25, and the remainder with the lot for \$800. What did I gain?

7. A farmer wintered 17 horses from December 1 to April 1 at \$12 a month. He paid \$23 a ton for 22 tons of hay, and 42 cents each for 280 bushels of oats. He had \$14 worth of provender left. He made \$x a month.

8. I can buy of one firm 732 tons of coal at \$4.20 and 75 cords of wood at \$8.16. Another firm bids \$4.16 for the coal and \$8.35 for the wood. Shall I buy of the first or of the second, and save what?

9. There are 2741 operatives on a corporation. 12 overseers get \$3.50 a day, 25 second hands get \$2.50, 1305 earn \$1.50, 215 men and 731 women earn \$1.25 each, and the remainder on the average receive 96 cents. What is the weekly pay roll?

1. 290 ft. hemlock boards at $1\frac{3}{8}$ ¢ will cost how much?
2. 3.25 bu. of beans at 2 qt. for $12\frac{1}{2}$ ¢ will sell for what?
3. $\frac{1}{4}$ of a 235-lb. barrel of sugar at $4\frac{7}{12}$ ¢ is worth x dollars.
4. $\frac{1}{16}$ T. of cream of tartar fills how many 4-oz. boxes?
5. $32\frac{3}{8}$ yd. lace billed at \$87 costs how much a yard?
6. When hay is \$13 $\frac{3}{4}$ a ton, what fraction of a ton is worth \$13?
How many pounds?
7. When 115 votes are in favor of a project and 46 are against it, what part of the whole are opposed?
8. Find the gain in 2250 lb. of wool bought at $16\frac{1}{4}$ ¢ and sold at $16\frac{3}{8}$ ¢.
9. A man sold $\frac{2}{3}$ of his interest in a mill for \$30. If his share amounted to $\frac{3}{4}$ of the whole property, what part of the whole did he sell? What was the value of the whole property at this rate?
10. Broadway Park measures $\frac{1}{4}$ of a mile wide and $\frac{3}{8}$ of a mile long. What will a walk 8 feet wide around the park cost at $62\frac{1}{2}$ ¢ per running foot? (Draw a diagram. Do not leave the corners without a walk.)
11. If $\frac{2}{5}$ of a piece of work can be done in $\frac{5}{8}$ of a day, how long will it take to complete the work?
12. At an auction one buyer bids $\frac{7}{12}$ of the cost, another $\frac{4}{5}$. The difference was \$75. What did each bid?
13. If a glass jar contains a hundred thousand fish eggs, how many jars will hold 8,000,000? If $\frac{1}{3}$ are hatched, how many on an average are lost from each jar?
14. The catch of shad for a certain period is valued at \$145,000. What part of this is \$4000, the cost of hatching the eggs and stocking the waters?
15. When oysters yield $1\frac{1}{4}$ gal. to the bushel, a 25-gal. barrel can be filled from x bushels in the shell.

(Review pp. 5 and 6)

1. By a decimal system we mean what ?
2. Compare the values of the 2's in 222.
3. Compare the values of the 3's in 33; in 3.3; in 0.33; in 0.033.
4. Which of the preceding numbers are integers? Decimals? Mixed decimals?
5. The value of a figure depends upon what two things?
7654321.234567.
6. What figure stands for tenths? Hundredths? Thousandths?
7. Compare the position and value of the 3's. Of the 5's.
8. Of what orders are the 6's? The 7's?
9. What is the use of the decimal point?
10. How is the denominator of a decimal fraction determined?
11. Write the following with denominators: 0.24, 0.08, 0.175, 0.036, 0.0025, 0.0001, 0.00017.
12. Are $\frac{8}{10}$, $\frac{8}{100}$, $\frac{8}{1000}$ common or decimal fractions?
13. How would you write them decimally?
14. Compare the number of places each takes up with the number of zeros in the denominator.
15. What part of 140.040 should be read first?
16. Where is "and" used in reading mixed decimals?
17. Which is the easier way of finding the denominator of 0.040, (a) by counting from the decimal point, — "tenths," "hundredths," "thousandths," — or (b) by imagining 1 and *three* zeros annexed?

Read:—

18. 0.307, 0.0307.

21. 0.710, 0.0071.

19. 330.03, 0.303.

22. 64.0019, 6400.019.

20. 3003.075, 3.00375.

23. 6000.006, 0.6006.

Since 84 per cent, or 84%, means 84 hundredths, or 0.84, in what two ways might the following be read?

1. 0.06, $33\frac{1}{3}\%$, $0.16\frac{2}{3}$.
2. 0.14, $0.37\frac{1}{2}$, $41\frac{2}{3}\%$.
3. $0.87\frac{1}{2}$, $37\frac{1}{2}\%$, $0.00\frac{1}{2}$.
4. $0.00\frac{1}{4}$, $\frac{7}{8}\%$, $0.06\frac{1}{4}$.
5. $3\frac{1}{3}$ thousandths is written $0.003\frac{1}{3}$. To what order of units does the $\frac{1}{3}$ belong?
6. $\frac{1}{2}\%$ is written $0.00\frac{1}{2}$. Write $\frac{3}{4}\%$, $1\frac{1}{2}\%$, $2\frac{1}{3}\%$. A number made up of decimal and common fractions is a **complex decimal**.
7. Give other complex decimals.

Writing Decimals

1. If the denominator contains three zeros, how far from the decimal point must the numerator end?
2. Write decimally: $\frac{345}{1000}$, $\frac{12}{100}$, $\frac{817}{1000}$, $\frac{1284}{10000}$.
3. If the numerator contains but one figure and the denominator three, where is the numerator written? What is written in the other two places?
4. Write: $\frac{3}{1000}$, $\frac{1}{10000}$, $\frac{9}{100}$, $\frac{5}{10000}$, $\frac{17}{1000}$, $\frac{12}{10000}$.
5. Write: $1\frac{3}{100}$, $20\frac{20}{10000}$, $16\frac{1}{10000}$, $20\frac{24}{10000}$.
6. Write as common fractions or mixed numbers: 1.003, 20.020, 19.03, 0.0013, 125% , 250% .

Write from dictation or at sight:—

7. 8 thousandths.
8. 17 tenths.
9. Three and a fifth %.
10. 3075 millionths.
11. $\frac{1}{2}$ a hundredth.
12. 804 hundred thousandths.
13. 400 and 4 ten-thousandths.
14. Forty thousand forty millionths.
15. Seven hundred six thousandths.
16. Two million 71 and 404 millionths.

1. Change $1\frac{1}{2}$ to larger units. What principle applies?

2. Annex a zero to 0.8. Write 0.8 and 0.80 as common fractions and compare their values.

3. Compare the numerators, the denominators, and the value of 0.90 and 0.9.

4. What effect on the *number* of units has annexing a zero to a decimal? What effect on their *size*?

5. Omit the zeros at the right of 0.360 and 0.400. How is the numerator affected? The denominator? The value?

Read as printed; then in smallest decimal terms, that is, in largest decimal units:—

6. 0.0400, 6.6450.

8. 8.0500, 0.050.

10. 9.400, 0.100.

7. 7.0900, 0.0600.

9. 10.010, 8.450.

11. 0.50, 0.050.

12. What is the effect upon the value of a number when the decimal point is moved one place to the *left*? One place to the *right*?

13. If a zero is annexed to 8 is the decimal point moved? Which way?

14. To 1.2 annex a zero. Is the decimal moved, that is, have *ones* and *tenths* been changed?

15. Tell how each figure is changed in value when a zero is annexed to 135. To 13.5.

16. Explain any change in value made by annexing a zero to 8; to 120; to 0.3; 0.03.

Read the following as printed; then with one or more zeros annexed:—

17. 0.17; 12.3; 0.07; 0.101; 0.008; 245.6; 36.008.

18. Read and announce the change when the decimal point is moved one place to the left.

19. When the decimal point is moved one place to the right.

74 DECIMALS CHANGED TO COMMON FRACTIONS *Oral or Written*

1. $0.75 = \frac{75}{100} = \frac{3}{4}$. Give directions for the two steps in this process.

Change to common fractions. Give each step.

2. 0.2, 0.4, 0.5.

5. 0.125, 0.480, 0.375.

3. 0.80, 0.25, 0.12.

6. 0.000125, 0.0625.

4. 0.50, 0.75, 0.70.

7. 0.00375, 0.001728.

$$8. 0.18\frac{3}{4} = \frac{18\frac{3}{4}}{100} = \frac{18\frac{3}{4} \times 4}{100 \times 4} = \frac{75}{400} = \frac{3}{16}.$$

9. Multiplying both terms of $\frac{18\frac{3}{4}}{100}$ by 4 has what effect on the value? On the form? Upon what principle does this depend?

10. Explain the whole process of reducing a complex decimal to a common fraction.

Change to common fractions:—

11. $33\frac{1}{2}\%$, $0.16\frac{2}{3}\%$, $66\frac{2}{3}\%$.

14. $0.03\frac{1}{4}\%$, $4\frac{1}{8}\%$, $0.06\frac{1}{4}\%$.

12. $0.83\frac{1}{3}\%$, $0.08\frac{1}{3}\%$, $58\frac{1}{3}\%$.

15. $6\frac{2}{3}\%$, $31\frac{1}{4}\%$.

13. $12\frac{1}{2}\%$, $0.37\frac{1}{2}\%$, $87\frac{1}{2}\%$.

16. $41\frac{2}{3}\%$, $0.62\frac{1}{2}\%$, $91\frac{2}{3}\%$.

Change the following per cents to common fractions:—

17-26. $43\frac{3}{4}\%$, $56\frac{1}{4}\%$, $62\frac{1}{2}\%$, $68\frac{3}{4}\%$, $81\frac{1}{4}\%$, $41\frac{3}{8}\%$,
 $93\frac{3}{4}\%$, $22\frac{2}{3}\%$, $29\frac{1}{8}\%$.

27-36. Subtract each of the preceding from 100% and change the remainder to a common fraction.

37. Change $\frac{7}{8}$ to 100ths or %.

$$\frac{7}{8} = \frac{700}{800} = \frac{700 \div 8}{800 \div 8} = \frac{87\frac{1}{2}}{100} = 0.87\frac{1}{2} = 87\frac{1}{2}\%.$$

PRACTICAL CALCULATION

(See Book II, p. 145.)

$$8 \overline{) 7.000}$$

$$0.875 = 87\frac{1}{2}\%.$$

38. $\frac{7}{80}$.

41. $\frac{27}{80}$.

44. $\frac{15}{16}$.

39. $\frac{13}{16}$.

42. $\frac{3}{80}$.

45. $\frac{5}{16}$.

40. $\frac{13}{16}$.

43. $\frac{7}{80}$.

46. $\frac{3}{8}$.

I. *Without copying, write the sum of each column and of each line.*

- | | 1. | 2. | 3. | 4. | 5. |
|-----|--------|----------|----------|-----------|-----------|
| 6. | 96.475 | + 186.32 | + 0.4875 | + 0.64985 | + 396.47. |
| 7. | 83.8 | + 62.379 | + 2.93 | + 3.207 | + 82.379. |
| 8. | 5.42 | + 4.8 | + 8.479 | + 0.0439 | + 6.4. |
| 9. | 16.783 | + 9.54 | + 6.53 | + 9.642 | + 180.09. |
| 10. | 4.09 | + 72.683 | + 2.946 | + 8.78314 | + 34.769. |

II. *Find the difference, first explaining whether the denominators must be alike:—*

- | | 1. | 2. | 3. | 4. | 5. | 6. | | |
|-----|--|-----------|-----|------|-----------|-----|-------|-----------|
| 7. | 3.64 | − 1.873. | 9. | 41. | − 13.074. | 11. | 6.39 | − 0.497. |
| 8. | 1.9 | − 0.3694. | 10. | 9.87 | − 4.3. | 12. | 2.641 | − 0.0994. |
| 13. | Take seventeen hundred eight ten thousandths from twenty-four and six thousandths. | | | | | | | |
| 14. | From eighty-six tenths take forty-three thousandths. | | | | | | | |

III. *Rewrite as integers and decimals; then add each column and each row:—*

- | | 1. | 2. | 3. | 4. | 5. | 6. |
|-----|--------------------|-------------------|-----------------|-------------------|------------------|--------------------|
| 7. | 16.372 | $8\frac{5}{8}$ | 79.42 | $14\frac{1}{4}$ | $86\frac{3}{4}$ | $21.05\frac{1}{2}$ |
| 8. | $216\frac{1}{2}$ | $34\frac{7}{8}$ | 19.3794 | $12\frac{1}{20}$ | $97\frac{3}{10}$ | 16.25 |
| 9. | $25\frac{25}{100}$ | 0.847 | $96\frac{4}{8}$ | $29\frac{13}{2}$ | $\frac{3}{10}$ | $62\frac{1}{2}\%$ |
| 10. | $\frac{3}{8}$ | $87\frac{1}{2}\%$ | 0.758 | $93\frac{1}{2}\%$ | $\frac{1}{8}$ | $12\frac{1}{2}\%$ |
| 11. | 0.3 | 2.03 | $7\frac{5}{8}$ | $40\frac{5}{8}$ | 116% | 6.4837 |

IV. *Find the difference between:—*

- | | | | | | |
|----|----------------------------|----|--------------------------------------|----|------------------------------|
| 1. | 17.38 and 200. | 4. | $9\frac{1}{8}$ and 7969. | 7. | 1 and $0.83\frac{1}{2}$. |
| 2. | 2.0875 and $\frac{5}{8}$. | 5. | $\frac{35}{10}$ and $2\frac{1}{2}$. | 8. | 0.1 and $0.083\frac{1}{2}$. |
| 3. | $4\frac{1}{8}$ and 7.011. | 6. | $\frac{7}{8}$ and 64%. | 9. | 10 and $8.3\frac{1}{2}$. |

1. Compare 3 and 0.3. Moving the decimal point one place to the left has what effect?

2. Move the point one place to the left in 18.4 and 0.15, and give the result and the effect on the number.

3. Compare 0.1 of 18.4, $\frac{1}{10}$ of 18.4, and $18.4 \div 10$.

4. Find 0.1 of the following: 3.4, 0.2, 0.05, 12.5.

5. Compare 400 and 4.00. Moving the point two places to the left has what effect?

6. Read the quotients after dividing these by 10. By 100.

24.5, 3.65, 42.1, 0.04, 0.875, 625, 0.264, 1.82.

7. Instead of reading the preceding as divided by 10 or 100, we may read them as — or — of the number.

Give at sight:—

8. $\frac{1}{10}$ of 2.46.

10. $\frac{1}{1000}$ of 37.6.

12. 0.0001 of 3500.

9. 0.01 of 32.

11. 0.001×0.9 .

13. 0.001×25 .

14. Having found 0.1 or 0.01 by moving the point, how would you find 0.3 or 0.05, etc.?

15. Compare 0.1 and 0.8; 0.01 and 0.15; 0.01 and 0.08.

16. What is 0.01 of 300? 0.03 of 300?

17. $0.01 \times 32.45 =$ what? How many decimal places in the result?

18. $0.09 \times 0.03 =$ what?

19. 0.06×300 .

$0.01 \times 0.03 = 0.0003$ (why?)

20. 0.15×0.6 .

$0.09 \times 0.03 = 9 \times 0.0003$ (why?)

21. 0.05×0.005 .

22. Compare the number of decimal places in the product with the number in both factors.

The product contains as many decimal places as there are in both its factors.

1. 0.6 of 30 = 6×3 . Why?
2. $33\frac{1}{3}\%$ of 60 = $\frac{1}{3}$ of 60. Why?

Give results and show what process you use:—

3. 0.06 of 200. 5% of 500. 7. 2.5×0.6 . 2.5×0.30 .
4. $\$90 \times 0.9$. 12% of 1000. 8. 80% of \$400. 16% of 40.
5. 0.6 of 0.6. 0.12 of 0.12. 9. $33\frac{1}{3}\%$ of 360. $0.2 \times 0.2 \times 0.2$.
6. 0.08×0.5 . 20% of 60 yr. 10. 1.2×1.2 . 200×0.003 .
11. Had you first to multiply or divide in these examples?
12. A man of 50, spending 30% of his life abroad, is at home x yr.
13. 2% of \$5000 being counterfeit, the rest or \$ x is good money.
14. $23 \times 375 = 8625$. 15. $0.23 \times 37.5 = y$.
- $2.3 \times 375 = x$. $23 \times 3.75 = z$.
- $w \times 3.75 = 0.8625$. $230 \times u = 8625$.
16. How would you write the product of 0.02×0.004 ?

Written

1. 3.46×2.008 .

WORK

| | |
|--------------|--------------|
| 2.008 | 2.008 |
| <u>3.46</u> | <u>3.46</u> |
| .12048 | .12048 |
| .8032 | .8032 |
| <u>6.024</u> | <u>6.024</u> |
| 6.94768 | 6.94768 |

EXPLANATION. The first partial product is 0.06×2.008 or 0.12048, which may be written in either of the ways given. The important thing to remember is how to determine the number of decimals in the product and to put the decimal point where it belongs.

(a) Why are there 5 decimals in the first partial product, 4 in the second, and 3 in the last?

2. $0.8\frac{1}{2} \times 5.076$. 5. $0.037\frac{1}{2} \times 12.50$. 8. $8\% \times \$456$.
3. 8.47×9.432 . 6. $1.8\frac{1}{2} \times 0.360$. 9. $0.24 \times \$9.60$.
4. $0.84 \times \$9.60$. 7. $0.36 \times 3.6 \times 36$. 10. $17\% \times 34.6$.
11. A man's salary is \$1200. If reduced $12\frac{1}{2}\%$, what will it be?

1. $0.48 \div 8$, or $\frac{1}{8}$ of $0.48 =$ — hundredths or 0. —.
2. $0.48 \div 0.06$. Have these a common unit? What is it?
3. What is $\frac{1}{8}$ of 0.81 ? $\frac{1}{8}$ of 0.072 ? $\frac{1}{4}$ of 0.063 ?
4. What is $6.3 \div 7$? $8.1 \div 9$? $1.05 \div 7$?
5. Compare $0.8 \div 0.4$ with $8 \div 4$. $20 \div 4$ with $10 \div 2$.
6. Does multiplying both dividend and divisor affect the quotient?
7. Compare $0.049 \div 0.07$ with $4.9 \div 7$. How many places to the right was the point moved in each? By what did this multiply each?
8. When any number of units (dollars, feet, tenths, hundredths) is divided into any integral number of parts, what is the unit of each part?
9. Then if the divisor is an integer as in $4.9 \div 7$, the quotient will be expressed in the same units as the —. Hence $4.9 \div 7$, or 49 tenths, $\div 7 = 0.7$.

PROCESS

$$\begin{array}{r} 0.09 \overline{)0.144} = 9 \overline{)14.4} \\ \underline{1.6} \end{array}$$

*To divide 0.144 by 0.09.**To divide 63.44 by 25.6.*

PROCESS

$$\begin{array}{r} 2.47 + \\ 25.6 \overline{)63.44} \quad 256 \overline{)634.4} \\ \underline{512} \\ 122.4 \\ \underline{102.4} \\ 20.00 \\ \underline{17.92} \\ 2.08 \end{array}$$

10. In the example at the left how is the divisor changed from 0.09 to 9? For what purpose?
11. How and why must the dividend also be changed? State the principle.
12. After dividing 14 by 9, how many tenths *in all* remain to be divided?
13. Explain the process shown in the second example. When there is a remainder, how do you continue the division?
14. How may you always have an integral divisor?
15. $38.7 \div 4.34$.
16. $3.485 \div 0.95$.
17. $24.6 \div 0.17$.

Principle. *Multiplying or dividing both dividend and divisor by the same number does not affect the quotient.*

Give directions for division of decimals in five steps:—

I. Setting down. II. Pointing. III. Dividing. IV. Placing and pointing the quotient. V. Managing the remainder.

Divide, noticing whether the quotient will be larger or smaller than the dividend:—

- | | |
|--------------------------|--|
| 1. $21.6 \div 0.006$. | 6. $102.01 \div 1.01 + 12.5\%$ of 100. |
| 2. $0.4913 \div 1.7$. | 7. $\$8.281 \div \frac{91}{100} - 6.25\%$ of \$3.00. |
| 3. $2.1952 \div 0.028$. | 8. $4.096 \div 0.0064 + 0.82369 \div 28.7$. |
| 4. $1.521 \div 3.9$. | 9. $67.24 \times 82\% - 67.24\% \div 82$. |
| 5. $0.6345 \div 0.009$. | 10. $400 \div 0.66\frac{2}{3} + 876.16 \div 0.296$. |

Common Fractions Changed to Decimals

Since $3 \div 8$ or $\frac{3}{8}$ of 3 = $\frac{3}{8}$, a fraction may be changed to a decimal by considering it a problem in division of decimals; thus,—

$$\frac{3}{8} = 8 \overline{)3.000} \\ 0.375$$

Change to complex decimals of three places:—

- | | | | |
|----------------------|----------------------|---------------------|---------------------|
| 11. $\frac{7}{12}$. | 15. $\frac{5}{9}$. | 19. $\frac{4}{9}$. | 23. $\frac{8}{9}$. |
| 12. $\frac{9}{16}$. | 16. $\frac{7}{80}$. | 20. $\frac{8}{8}$. | 24. $\frac{4}{8}$. |
| 13. $\frac{4}{15}$. | 17. $\frac{1}{8}$. | 21. $\frac{1}{8}$. | 25. $\frac{1}{8}$. |
| 14. $\frac{3}{4}$. | 18. $\frac{2}{8}$. | 22. $\frac{4}{8}$. | 26. $\frac{8}{8}$. |

Change to incomplete decimals of four places:—

- | | | | |
|---------------------|----------------------|-----------------------|----------------------|
| 27. $\frac{1}{2}$. | 29. $\frac{6}{11}$. | 31. $\frac{7}{5}$. | 33. $\frac{7}{18}$. |
| 28. $\frac{5}{9}$. | 30. $\frac{1}{10}$. | 32. $\frac{1}{100}$. | 34. $\frac{3}{18}$. |

35. A long ton is $\frac{2240}{2000}$ of a short ton. Express the relation as a mixed decimal.

36. 9 is what part of 144? Express the relation as a complex decimal of two places. As a per cent.

37. What per cent of \$6000 is \$4680?

1. 38.025 is to be divided into 195 parts. Will each part be more or less than $\frac{1}{2}$?
2. How many places would there be in the quotient if the division were exact?
3. A camel goes 3.5 miles per hour. How far will it go in 10 hr.?
4. Of what two equal numbers less than 1 is 0.25 the product?
5. Find the square root of 0.0081.
6. 12 is what per cent of 50? Of 25?
7. Find the balance of this account by inspection. Which party is described as Dr. and Cr.?

NEW YORK, Jan. 30, 1904.

MR. J. Q. ADAMS.

In acct. with JOHN REYNOLDS & Co.

| Dr. | | | | Cr. | | | |
|--------|---------------------|-------|----|--------|----------------------|-------|----|
| Jan. 2 | To Mdse. as by bill | \$114 | 81 | Jan. 1 | By bill for services | \$150 | 00 |
| 9 | Freight prepaid | 2 | 13 | 5 | By goods returned | 14 | 81 |
| | Storage of barrels | | 50 | | By allowance for | | |
| 16 | Cash on acct. | 50 | 00 | | damages | 2 | 00 |

8. Multiply the sum of 11.507 and 4.493 by the difference between 10.85 and $10\frac{1}{2}$.

Read rapidly and change to or from decimal forms:—

1. $\frac{9001}{10000}$, $\frac{9}{8000}$, $\frac{35}{500}$, $\frac{1000}{125}$.

6. $\frac{30250}{2500}$, 0.0006, $\frac{333}{8000}$.

2. $0.31\frac{1}{2}$, $0.037\frac{1}{2}$, $3.7\frac{1}{2}$.

7. 20.2020, 202.020, 2020.20.

3. $6\frac{3}{8}\%$, $\frac{1}{8}\%$, $\frac{17}{4}$, $\frac{17}{40}$.

8. 10.12, \$10.012, $\$11\frac{1}{10}$.

4. $\frac{10101}{10}$, $\frac{40800}{2000}$, $\frac{935}{5000}$.

9. $\frac{17}{20}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{3}{7}$.

5. 60.004, 0.048, 6.666+

10. $\frac{7}{2100}$, $\frac{48}{1820}$, $\frac{880}{1860}$.

1. A city contains 40,000 persons, 26% in the first ward, 32% in the second, 21% in the third, and the remainder in the fourth. How many in each?

2. 21% of a gang of 200 workmen receive \$48.30. The wages of the rest are 20% higher. What does a workman of each class receive per day?

3. The binding of 390 books cost \$54.60. What was the entire cost of each book if the binding was 14% of it?

4. One girl lives 370 rd. east from school; another 142.35 rd. in the same direction. Their houses are how many feet apart?

5. When the cost of transporting coal is 2¢ per ton for each mile the freight on 400 tons is \$800. What is the distance?

6. At 1.25 cu. ft. to the bushel, compute the value at $57\frac{1}{2}$ ¢ a bushel of a bin of corn containing 4000 cu. ft.

7. Find the profit on 274 bbl. of flour at \$4.11 $\frac{1}{2}$, and 128 bbl. at \$3.96 $\frac{3}{8}$, if sold at 3¢ per pound. (196 lb. to the barrel.)

8. After melting $\frac{5}{8}$ of a sheet of metal, and later $\frac{1}{8}\frac{1}{2}$, there was $\frac{1}{8}\frac{1}{2}$ of a square foot left. How many square inches in the part first melted?

9. £1 English money = \$4.8665. Find the value of £23,738.

10. \$81,271.08 is to be divided among 7 heirs. 5 of them share equally; the others receive each a double portion. What is the amount of a 2% tax on 1 of the 5 equal shares?

11. A lot of cord wood is $\frac{2}{16}$ beech, 0.21875 birch, 0.1875 maple, $\frac{2}{8}\frac{1}{2}$ ash, 10 cords oak, $\frac{1}{8}$ poplar, and 3 $\frac{1}{8}$ % pine. How many cords in all?

12. When 5.20 francs = \$1, how many dollars will 3302 francs equal?

1. I live in a hired house worth \$6000. For the use of the house for a year I pay $\frac{1}{10}$, or 10%, of its value. What do I pay per year?

2. If I had used the money which the house cost, \$6000, for a year at the same *rate*, 10%, the annual *interest* would have been what? The interest for 6 mo. would have been what?

3. The value of a house used was \$3000; rate of *rent*, 5%. Find a year's rent. A month's rent.

4. Money used, \$3000; rate of *interest*, 5%. Find a year's interest. A month's interest.

5. What is the difference between *rent* and *interest*?

Interest is an allowance to the owner for the use of his money.

The *principal* is the money used.

The *amount* is the sum of the interest and the principal.

The *rate* of interest is the number of hundredths of the principal paid for a year's use of it.

6. The principal is \$200. The rate is 6%. Give the interest for 1 yr.; 2 yr.; 3 yr.; 4 yr.; 5 yr.; the interest for 1 mo.; 2 mo.; 3 mo.; 4 mo.; 5 mo.; 6 mo.; 7 mo.; 8 mo.; 9 mo.; 10 mo.; 11 mo.

7. What is a year's interest of \$300 at 2%? 3%? 4%? 5%?

Find the interest: —

What shall I pay for the use: —

8. Of \$300 at 4% for 2 yr.

11. Of \$1000 for 2 yr. at 10%?

9. Of \$500 at 6% for 3 yr.

12. Of \$600 for $\frac{5}{8}$ yr. at 10%?

10. Of \$800 at 7% for $\frac{1}{2}$ yr.

13. Of \$800 for $\frac{3}{4}$ yr. at 4%?

14. In most business transactions 30 da. make a month. If a month's interest is \$60, what is the interest for 1 da.? For 10 da.? For 20 da.?

15. At 6% the interest of \$300 for 1 yr. is ——. For 1 mo. it is ——. For 1 da. it is ——.

1. Find the interest of \$ 240 for 2 yr. 5 mo. at 5%.

| |
|--|
| A |
| \$ 240 = principal |
| .05 = rate |
| <hr/> \$ 12.00 = int. for 1 yr. |
| 2 $\frac{5}{12}$ = time in years |
| \$ 5.00 = int. for $\frac{5}{12}$ yr |
| 24.00 = int. for 2 yr. |
| <hr/> \$ 29.00 = int. for 2 $\frac{5}{12}$ yr. |

| |
|--|
| B |
| $29 \times \frac{1}{12} \times \frac{5}{100} \times \frac{12}{20} \times \$ 240 = \$ 29$ |

(a) In B what represents a year's interest ?

(b) A month's interest ?

(c) How is the process shortened ?

What is the interest of:—

- | | |
|---|---|
| 2. \$ 840 for 1 yr. 9 mo. at 10% ? | 5. \$ 1000 for 12 $\frac{1}{2}$ yr. at 8% ? |
| 3. \$ 360 for 4 yr. 10 mo. at 5% ? | 6. \$ 400 for 2 yr. 5 mo. at 7% ? |
| 4. \$ 960 for 1 yr. 8 mo. at 4% ? | |
| 7. Find the interest of \$ 500 for 1 yr. 5 mo. 6 da. at 5%. | |

| | |
|---|---|
| A | B |
| \$ 500 | 43 |
| .05 | 129 |
| <hr/> \$ 25.00 = int. for 1 yr. | $\$ 16 \times \frac{1}{360} \times \frac{5}{100} \times \frac{5}{60} = \frac{\$ 215}{6} = \$ 35.83$ |
| 10.41 $\frac{2}{3}$ = int. for 5 mo. | 72 |
| .41 $\frac{2}{3}$ = int. for 6 da. | 18 |
| <hr/> \$ 35.83 = int. for 1 yr. 5 mo. 6 da. | 6 |

(a) Explain the process in B. (b) What is the advantage of this method ?

Find the interest of:—

- | | |
|-----------------------------------|--|
| 8. \$ 600 for 60 da. at 4%. | 12. \$ 336 for 8 mo. 10 da. at 4%. |
| 9. \$ 250 for 1 mo. 15 da. at 6%. | 13. \$ 1728 for 2 mo. 17 da. at 9%. |
| 10. \$ 120 for 80 da. at 7%. | 14. \$ 2800 for 9 mo. at 4 $\frac{1}{2}$ %. |
| 11. \$ 372 for 36 da. at 10%. | 15. \$ 1000 for 93 da. at 4 $\frac{1}{2}$ %. |

To TEACHER. (If drill in interest is desired at this time, see page 156.)

1. Beginning with the shortest, name five units used in measuring lengths or distances. Give the table of length measures.
2. Compare an inch with a foot. An inch with a yard.
3. Compare a foot with a yard. With a rod. With a mile.
4. What part of a mile is a rod?
5. A yard is what part of a rod? Of a mile?
6. $\frac{5}{16}$ miles = — rods.
7. 91 in. = — ft. + — in.
8. $320 \times 5\frac{1}{2} \times 3$ ft. = 1 mi. = — ft.
9. Learn in some way, as measuring and counting your steps, the distance from your home to school in feet; in yards; in rods
10. Estimate in feet the dimensions of your schoolroom. Test your estimates by measuring.
11. Estimate and test longer distances as the dimensions of your playground in rods; estimate in yards.
12. At 1¢ per foot, what is the cost of 4 yards of picture wire?
13. At 12¢ a yard, picture molding for a room 25 feet long and 20 feet wide will cost what?
14. $\frac{1}{2}$ rd. = — ft.
15. 100 in. = x yd. + y ft. + z in.
16. 1000 rd. = x mi.
17. $\frac{1}{2}$ mi. less 50 rd. = x rd.
18. It is 884 feet around a square. How long is one side of it?
19. 32 rods is what part of a mile?
20. A 10-rod tape line is how many yards long?
21. $\frac{3}{8}$ of a foot is what part of a yard?
22. How many 4-inch bolts can be cut from a rod of iron $10\frac{1}{2}$ feet long?
23. A bicyclist travels 2 rods in a second. How long will he be in going a mile?

A **plane surface** or a **plane**, is a flat, level surface.

The boundaries of surfaces are **lines**. The sum of the lines bounding a surface is called its **perimeter**.

Surfaces bounded by straight lines are called **rectilinear** surfaces (*rect-* meaning *right* or *straight*).

1. What is the shape of the common units of surface measure?
2. Describe a square. Draw a square inch or a 1-in. square.
3. Name the five square measures of surface, beginning with the smallest and giving the length of each.
4. An acre is *not* a square unit of measure; it contains — square rods.
5. What is the length of a square yard in yards? In feet? In inches?
6. Give the length of a square rod in yards; in feet.
7. Give the length of a square mile in rods; in yards; in feet.
8. Draw a diagram of a square foot. If your paper is too small, draw to some scale as $\frac{1}{2}$, $\frac{1}{4}$, etc.; that is, represent 1 in. by $\frac{1}{2}$ in. or $\frac{1}{4}$ in.
9. Divide your diagram so as to show the number of square inches in a square foot. How many are there?
10. $\frac{1}{2}$ sq. ft. = — sq. in.; $\frac{1}{3}$ sq. ft. = — sq. in.; $\frac{1}{4}$ sq. ft. = — sq. in.
11. $\frac{1}{8}$ sq. ft. = —; $\frac{1}{6}$ sq. ft. = —; $\frac{1}{12}$ sq. ft. = —
12. Represent a square yard on a scale of $\frac{1}{6}$. Separate into square feet. How many are there?
13. Draw a diagram to represent a square rod. Let 1 in. represent a yard. What scale is this?
14. You will find — squares, — half squares, and — quarter squares. How many square yards in all?
15. Outline a square rod on the schoolroom floor or in the school yard. How many feet around it? How many square feet does it contain?

16. Draw a figure to represent an acre, 10 rd. wide and 16 rd. long. Let $\frac{1}{2}$ in. represent a rod. Divide your diagram to represent square rods. How many in an acre?

17. Lay off an acre in your school yard or in some field near by if convenient. Make it 10 rd. wide and 16 rd. long.

18. Lay off an acre 8 rd. wide and 20 rd. long.

19. How many acres, or what part of an acre, does your school lot equal?

20. $\frac{7}{8}$ A. = — sq. rd.

21. A square mile is called a **section**. How many rods long is a section?

22. How many acres in a strip a rod wide and a mile long?

23. How many such strips in a square mile? Then how many acres in a square mile or section?

24. A western **township** is 36 sq. mi. square. How many square miles does it contain? How many acres?

TABLE FOR SURFACE MEASURES

| | |
|--|---------------------------|
| 12 ² or 144 square inches (sq. in.) | = 1 square foot (sq. ft.) |
| 3 ² or 9 square feet | = 1 square yard (sq. yd.) |
| (5 $\frac{1}{2}$) ² or 30 $\frac{1}{4}$ square yards | = 1 square rod (sq. rd.) |
| 160 square rods | = 1 acre (A.) |
| 640 acres | = 1 square mile (sq. mi.) |

25. Change 20,000 sq. in. to square feet. 29. 1 sq. rd. = x sq. ft.

26. Change 12,371 sq. ft. to square rods. 30. 5 A. = x sq. ft.

27. Change 287 sq. rd. to square feet. 31. 200 sq. yd. = x sq. rd.

28. Change an acre to square feet. 32. 1900 sq. rd. = x A.

33. Bought $\frac{7}{8}$ A. for \$400 and sold it at 10¢ per square foot. How much did I gain or lose?

34. In 20,000 sq. ft., how many square rods?

1. Straight lines that lie in the same plane, and that cannot meet however far extended, are **parallel**, $A \text{ --- } B$
as AB and CD . $C \text{ --- } D$

2. When two lines meet, they are said to form an **angle**.

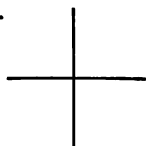
3. The lines are called the **sides** of the angle, and the point where they meet is the **vertex**.

4. In reading an angle, we read a point on one side, then the vertex and a point on the other side, reading around contrary to the motion of the hands $B \text{ --- } A$
of a clock. Thus the angle in the margin is read angle ABC . Or since there is but one angle at B , it may be spoken of as angle B .

5. To measure an angle is to measure the difference of direction of its sides. If BC were to start from the position BC and revolve about B as a pivot until it again came to the position BC , it has revolved through 360 degrees (360°), or made a complete revolution.

6. If two straight lines intersect, they form 4 angles.

If these angles are all equal, they are **right angles**.
How many degrees in a right angle?



7. Lines forming right angles with each other are **perpendicular**.

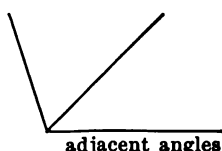
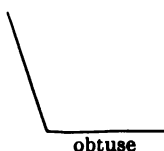
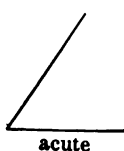
8. If you prolong the sides of an angle, do you increase the size of the angle?

9. Angles not right angles are **oblique**.

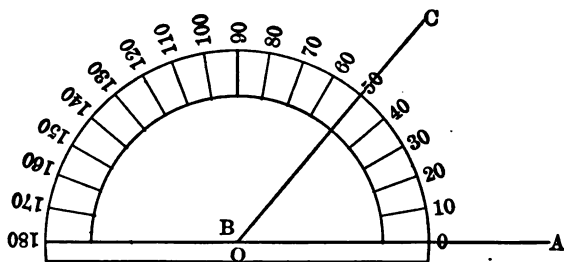
10. An oblique angle less than a right angle is **acute**.

11. An oblique angle greater than a right angle is **obtuse**.

12. Two angles having a side in common are **adjacent angles**.



1. Angles are measured by a protractor. The center O is placed at the vertex, and the magnitude is read on the scale between the two sides. Thus, in the figure, angle ABC is 50° .



2. Every degree contains 60 minutes ($60'$), and every minute contains 60 seconds ($60''$).

TABLE

| | |
|---------------------|--------------------------|
| 60 seconds ($''$) | = 1 minute ($'$). |
| 60 minutes | = 1 degree ($^\circ$). |
| 360 degrees | = 1 revolution. |
| 90 degrees | = 1 right angle. |

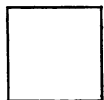
3. Draw angles of 90° ; of 45° ; of 60° ; of 120° ; of 105° .

4. Which of these are acute? Which obtuse? Measure them.

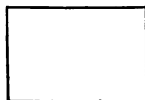
The Six Quadrilaterals

1. A figure bounded by four straight lines is a **quadrilateral** (*quattuor* four, *latus* a side).

2. There are three classes of quadrilaterals, namely, (a) those with *opposite sides parallel*; (b) *with one pair of sides parallel*; (c) *with no two sides parallel*.



A



B



C



D

3. A figure in which the opposite sides are parallel is a **parallelogram**. Are A, B, C, and D parallelograms?

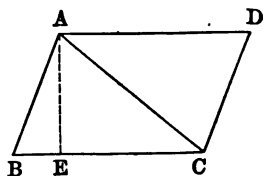
4. Describe A as to sides and as to angles.
 5. A figure whose *sides* are *equal* and whose *angles* are *right angles* is a **square**.
 6. Describe B as to sides and angles.
 7. A parallelogram whose angles are right is a **rectangle**.
 8. Is A a rectangle? Is B ? Is B a square?
- A square may be said to be an **equilateral rectangle**.
9. Describe C as to sides; as to angles.
 10. A parallelogram whose sides are equal, but whose angles are *not* right angles, is a **rhombus**.
 11. Describe D as to sides; as to angles.
 12. A parallelogram whose angles are *not* right angles is a **rhomboid**. Is C a rhomboid?

A rhombus might be called an equilateral rhomboid.

NOTE. One generally speaks of a figure like D as a parallelogram, and gives to any of the other three special parallelograms its proper name, as square, rectangle, or rhombus.

13. Which of the parallelograms are equilateral? Equiangular?
14. Any side upon which the parallelogram is supposed to stand is its *base*. That which shows the height of a parallelogram is its **altitude**.

In the figure BC is the *base* and AE the altitude.



15. The altitude and base are always *perpendicular* to each other.
16. The straight line AC joining the vertices of opposite angles is a diagonal. What other diagonal has the parallelogram $ABCD$?
17. By cutting, compare the two figures into which a diagonal divides a parallelogram.

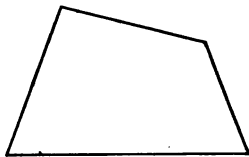
18. Prove by cutting accurately from cardboard that —

(1) *A diagonal bisects a parallelogram.*

(2) *The opposite angles of a parallelogram are equal.*

19. Prove by measuring with a protractor that —

The sum of the angles of any quadrilateral is equal to 360 degrees, or four right angles.



20. A quadrilateral having but one pair of sides parallel, as in Fig. *E*, is a **trapezoid**.

21. If no two of the sides of a quadrilateral are parallel, the figure is a **trapezium**.

The Measurement of Rectangles

1. A rectangle 1 in. wide and 5 in. long contains how many square inches? (See figure.)

2. A rectangle 4 in. wide and 5 in. long will contain how many rectangles 1 in. by 5 in.?

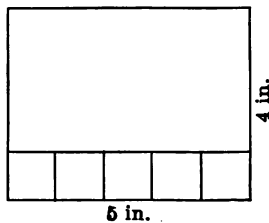
3. How many square inches in a rectangle 4 in. wide and 5 in. long?

4. What is the area of a rectangle 8 in. wide and 15 in. long?

5. A rectangle a foot wide and 20 ft. long contains how many square feet? A rectangle 12 times as wide and the same length will contain $12 \times$ — sq. ft.

6. Find the area of a rectangle 15 ft. long and 7 ft. wide.

7. Find the area of a rectangle 8 ft. by 15 ft.



8. Find the area of a square rod in square feet; in square yards.

9. A piece of land measures 20 rods one way and 25 rods the other. Find its area in square rods; in acres.

10. A kindergarten table 4 ft. 3 in. long and 20 in. wide is marked off in square inches. How many are there?

11. In a flag $10\frac{1}{2}$ feet long and $\frac{2}{3}$ as wide how many square yards of bunting, not allowing for seams?

12. A patchwork quilt 3 yards square is made of 4-inch silk squares. How many are there?

SUGGESTION. How many squares in a strip 1 square wide and 3 yards long? How many such strips in the quilt?

13. What is the area of $\frac{1}{2}$ mile of street 4 rods wide?

14. A lot of land 56 ft. by 125 ft. is sold at $49\frac{3}{4}$ ¢ a square foot. Required the proceeds of the sale.

15. A swimming tank is 40 ft. long and 18 ft. wide. What will it cost to cement the bottom at \$2.25 per square yard?

16. A city block is 600 ft. long and $\frac{2}{3}$ as wide. How many acres does it cover?

17. A half $\frac{1}{2}$ mile of 60-ft. street is paved with granite blocks. How many will be required at the rate of 36 to a square yard?

18. What will a sheet of zinc 8 feet long and 32 inches wide cost at $7\frac{1}{2}$ ¢ a pound if every square foot weighs 8 ounces?

19. Estimate the cost of the blackboards in your schoolroom at 75¢ a square yard.

20. What shall I pay Mr. Bates for concreting a wall 60 ft. long, 4 ft. wide half the length, and 3 ft. wide the rest of the way? The price is 75¢ a square yard.

21. If a city lot containing 3750 square feet has a frontage of 50 feet, how deep is it?

1. Ingrain carpets are generally woven in strips 1 yd. wide; other carpets $\frac{3}{4}$ yd. What two advantages come from running the strips lengthwise of the floor rather than across it?

2. On floors of the following widths, which of the two widths (1 yd. and $\frac{3}{4}$ yd.) could be used without cutting or turning under any strip?

12 ft. 15 ft. $22\frac{1}{2}$ ft. 27 ft. $13\frac{1}{2}$ ft. 20 ft. 18 ft.

3. How many strips of ingrain carpet are needed to carpet a room 18 ft. square? How many yards in each strip? In all, if there is no waste in matching?

4. How many strips of brussels or tapestry are needed for a room 15 ft. by 21 ft.? How many running yards in each strip? How many in all, if there is no waste in matching? Find the cost at \$1.25 per yard.

5. How many strips of ingrain will be needed to carpet a room 14 ft. by 20 ft., the strips to run lengthwise, no strip to be cut? How many yards in all, if there is no waste in matching?

6. On the same conditions, what will brussels $\frac{3}{4}$ yd. wide cost at \$1.50 per yard?

7. What will 4-foot oilcloth cost for the same room at \$1? In this case, would it be cheaper to put the strips crosswise? Why?

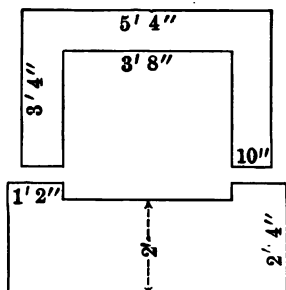
Find the cost of carpeting floors under the following conditions (strips that are cut cost as if whole):—

| Length of Room | Width of Room | Width of Carpet | Allowance for Matching | Cost per Yard |
|-------------------------|---------------------|-------------------|------------------------|---------------------|
| 8. 18 ft. | 14 ft. | 1 yd. | $1\frac{1}{2}$ yd. | \$0.90. |
| 9. 22 ft. | 18 ft. | $\frac{3}{4}$ yd. | $2\frac{1}{4}$ yd. | 4.25. |
| 10. $16\frac{1}{2}$ ft. | $13\frac{1}{4}$ ft. | 1 yd. | $\frac{7}{8}$ yd. | $0.87\frac{1}{2}$. |
| 11. $21\frac{1}{4}$ ft. | 20 ft. | $\frac{3}{4}$ yd. | $2\frac{3}{8}$ yd. | $1.37\frac{1}{2}$. |

1. Find the area of two surfaces represented at the right. ($5' 4'' = 5 \text{ ft. } 4 \text{ in.}$)

2. How many tiles 2 in. square will be required to cover them?

3. How many rolls of paper 8 yd. long and 18 in. wide will be required for a room $13\frac{1}{2}$ by 18, and 8 ft. high, if no allowance is made for doors, windows, and baseboards?



4. $9\frac{1}{2}$ squares of slating are required to cover a certain roof. This is equal to how many square yards? If the slates are 8×16 , and each course overlaps 10 in. of the one below it, find the number of slates used. (A square of roofing or flooring measures 10 ft. \times 10 ft.)

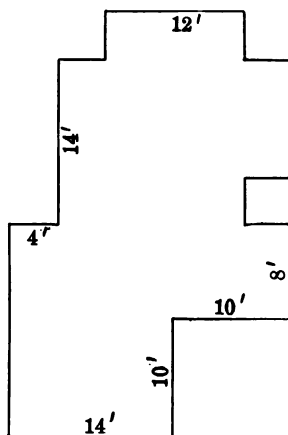
5. How many blocks 6 in. \times 4 in. will be used in paving a 4-rd. square?

6. How many tin plates 13 in. \times 19 in. must be used for 1 square of roofing; if they are lapped or folded $\frac{1}{2}$ in. on each side?

7. Three piazza roofs about a house measure in feet 30×8 , 24×7 , $7\frac{1}{2} \times 12\frac{3}{4}$. How much more than 5 squares do they contain?

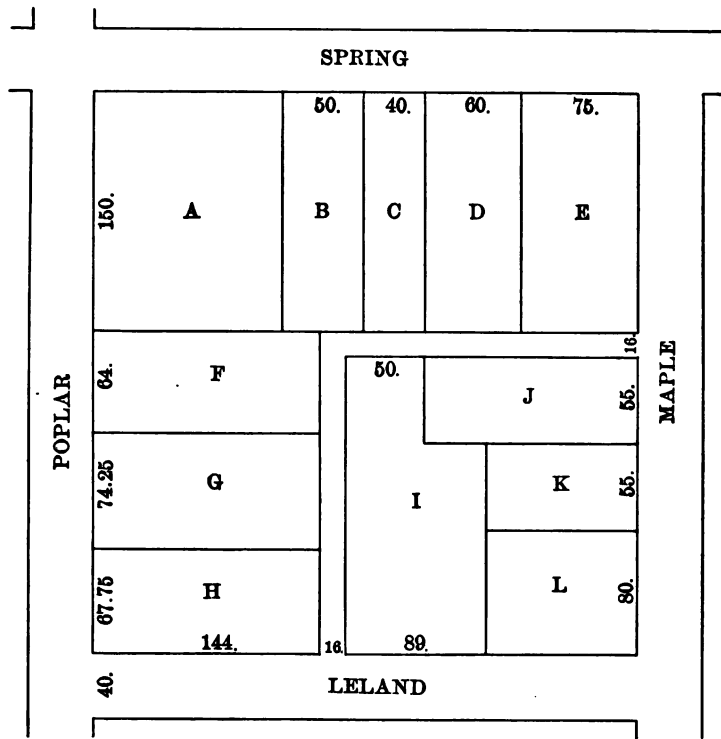
8. A house lot contains $\frac{1}{4}$ A. How many square feet? The house is $27\frac{1}{4} \times 40$. What would it cost to sod the remainder at \$1.50 a square?

9. Let this figure represent the outline of a cellar. Copy, and divide it into 5 rectangles. From the given dimensions find those of each rectangle.



10. How many square yards of cement would be required to cover the bottom of the cellar?

1. A chessboard contains 64 squares $1\frac{1}{2}$ in. long. What is its perimeter? If it has an inch-wide border, what is the entire perimeter?
2. In a 2-in. square how many $\frac{1}{4}$ -in. squares? How many $\frac{1}{2}$ -in.?
3. Compare the perimeter of a 4-ft. square and an equal surface 8 ft. long. (How wide is the latter surface?)
4. My sidewalk is 10 ft. wide besides the curb, and 100 ft. long. How many 4×8 bricks in it?
5. Compare a 4-in. square and a 12-in. square as to length and area.
6. A marble slab 4 ft. by $2\frac{1}{2}$ ft. was sold for \$4.50. Find the price per square foot.
7. What is the area of a square that can be set off with 200 ft. of rope?
8. How many boards 9 in. wide will make a close fence 8 ft. high around three sides of a square lot 180 ft. long?
9. A hall measures 12 ft. by 36 ft. How many breadths of yard-wide carpet would be needed? How many yards, allowing 3 yd. waste in matching the pattern?
10. A room 14 ft. by 18 ft. is to be covered with yard-wide carpet at \$1 a yard. Which is the cheaper way to run the strips? Why?
11. A rectangle twice as long as wide contains 512 square feet. What is the least number of squares into which it can be cut? What will be the area of each square? The length of each? What, then, are the dimensions of the given rectangle?
12. A rectangle 3 times as long as wide contains 108 sq. ft. Find its dimensions.
13. A rectangular room contains 192 sq. ft. The ratio of the length to the width is the same as 4 to 3. What are the dimensions?
Hint. What are the least number of squares into which the floor can be divided?



1. Mr. Sharp bought land bordering on Spring Street, between Poplar and Maple, at 3¢ per square foot, which he cut up into building lots. He first laid out a 40-ft. street in the rear, which he called Leland Street. What did he pay for the land?

2. He sold lot C to a civil engineer for his services in surveying and making plans, at a profit of only 2¢ per square foot. What did the lot sell for?

3. The grading of Leland Street cost him \$3.75 per square rod. What did the street cost him, including land and labor?

After reserving lot *A* for his own buildings, he sold the rest at the following prices per square foot:—

Find the proceeds of the sale of each lot.

4. Lot *B* for $12\frac{1}{2}\text{¢}$. 7. Lot *F* for $18\frac{3}{4}\text{¢}$. 10. Lot *J* for $19\frac{1}{2}\text{¢}$.

5. Lot *D* for 15¢ . 8. Lot *G* for 21¢ . 11. Lot *K* for $17\frac{3}{4}\text{¢}$.

6. Lot *E* for $22\frac{3}{4}\text{¢}$. 9. Lot *H* for $20\frac{1}{4}\text{¢}$. 12. Lot *L* for $25\frac{1}{8}\text{¢}$.

13. Before the sale, he opened and laid out a 16-ft. alley from Maple Street to Leland. What did the alley cost him, \$85 being paid for labor?

14. Mr. Sharp laid a sidewalk 8 ft. 8 in. wide on two sides of his own lot *A*. The 8-in. edgestones cost him 60¢ per running foot. The brick cost \$12 per thousand, and the labor \$58.25. The bricks were $8 \times 4 \times 2$, and laid flat. What did the walk cost him?

15. The owner of lot *J* paid \$3.30 per rod for fencing his lot. What did it cost if he paid for only half of the division fence?

16. The abutters combined and concreted the alley at $56\frac{1}{4}\text{¢}$ per square yard. What was the total cost?

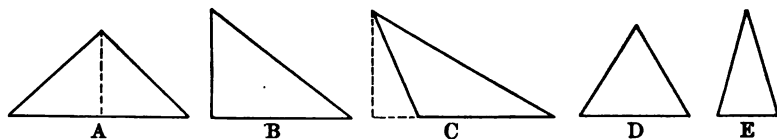
17. What part of the whole cost should the owner of lot *J* pay? How much did it cost him?

18. What does it cost the owner of lot *I*?

19. Leland Street is accepted by the city and paved at a cost of \$3 per square yard, the abutters agreeing to pay 25% of the cost of the part adjoining their property. What is the assessment on lot *L*?

20. What will a walk 8 ft. wide around the whole block cost at \$1.25 per square yard? Do not leave the corners without a walk.

21. What will it cost to fence lot *I* at \$1.25 a rod?



1. Surfaces bounded by three straight lines as those at the top of the page are **triangles** (*tri* = three).

2. Triangles are named from their largest angles. They are **acute**, **right**, and **obtuse** angled triangles. Name one of each shown above.

3. Triangles are also named from their sides as follows:—

Equilateral, with sides all equal;

Isosceles, with two sides equal;

Scalene, with no two sides equal.

4. Find a triangle of each kind at the top of the page. Draw others of each kind.

5. A triangle has three corners or **vertices**. A line drawn from any vertex of a triangle perpendicular to the opposite side, or opposite side produced as in *C*, is called an **altitude**, and the side is called the **base**. How many altitudes may a triangle have?

6. Show by measuring with a protractor or by cutting off and laying the angles together that —

The sum of all the angles of any triangle is equal to two right angles, or 180 degrees.

7. How many right angles may a triangle contain? How many obtuse? How many measuring 60° ?

Find the size of the third angle of a triangle when two angles measure as follows:—

8. 90° and 30° .

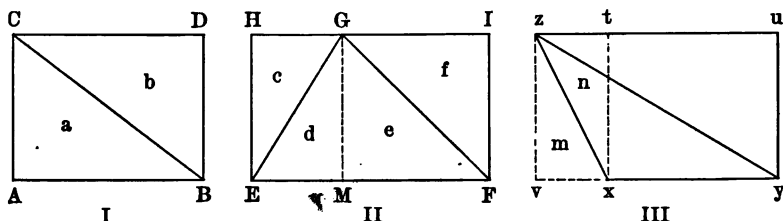
10. 65° and 35°

12. 75° and $37\frac{1}{2}^\circ$.

9. 120° and 40° .

11. $62\frac{1}{2}^\circ$ and $87\frac{1}{2}^\circ$.

13. 84° and $76^\circ 40'$



We have seen that any kind of an area having *straight* lines as sides, and *right angles* as angles, can be divided into rectangles, and thus the whole area found. We are now to deal with triangles and other areas bounded by straight lines, but whose angles are *not* all right angles.

1. By cutting, compare the two triangles into which a diagonal divides a rectangle.

2. What is the relation of a to b in the rectangle above? Then what part of the rectangle is the triangle?

3. If the dimensions of rectangle $ABDC$ are 10 in. by 6 in., what is the area of triangle ABC ?

4. In II compare c with d ; e with f . Compare triangle EFG with rectangle $EFIH$.

5. Compare the *base* and *altitude* of triangle EFG with the dimensions of the rectangle.

6. If the dimensions of the rectangle are 12 ft. and 8 ft., what are the *base* and *altitude* of the triangle? What is its area?

7. By cutting, show that in an obtuse-angled triangle, as in III, the triangle is also equal to half a rectangle having the same dimensions as the base and altitude of the triangle. It is now seen that—

Any triangle is equal to half a rectangle having the same dimensions as the base and altitude of the triangle.

8. What is the area of a triangle with base 20 ft. and altitude 12 ft.?

MEASUREMENT OF TRIANGLES, TRAPEZIUMS, ETC. 99

Oral and Written

1. What is the area of a triangle whose base is 12 rods and altitude 14 rods?

2. State your method of finding the area of a triangle.

Find areas of triangles of the following dimensions:—

3. Base=40 ft., alt.=18 ft.

7. $19\frac{1}{2}$ ft. and $2\frac{1}{2}$ yd.

4. Base=60 ft., alt.=25 ft.

8. 38 rd. and $22\frac{1}{2}$ ft.

5. Base=3 ft., alt.=9 in.

9. 64 yd. and 13 ft.

6. Base=4 rd., alt.=7 ft.

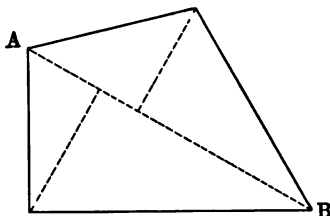
10. $3\frac{1}{2}$ rd. and 6 yd.

11. Draw or cut out a **trapezium**, or a quadrilateral no two of whose sides are parallel.

12. Separate it into two triangles along one of its diagonals, as *AB*.

13. Find the dimensions of each triangle and its area.

14. What will the area of the trapezium be?



15. The diagonal of a trapezium is 24 inches; the altitudes perpendicular to it are 18 inches and 15 inches respectively. What is the area?

16. The diagonals of a given trapezium cross at right angles. The point of intersection is 50 feet from the upper end of each diagonal. One diagonal is 100 feet long, the other 150 feet. Find the area. (Draw a diagram.)

17. In right triangles the two sides including the right angle, are called the *legs*. If one leg is the base, the other is the altitude. Why?

18. The legs of a right triangle are 10 and 15 ft. Find the area.

19. The diagonal of a trapezium is 14 in. The altitudes perpendicular to it measure 6 in. and 8 in. What is its area?

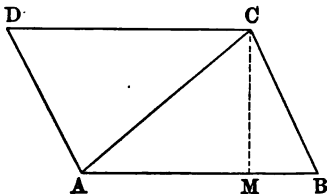
100 MEASUREMENT: OF PARALLELOGRAMS AND TRAPEZOIDS

Oral and Written

1. By cutting, find the relation between the two triangles into which any parallelogram is divided by a diagonal.

2. CM , the altitude of the triangle, is also the altitude of the parallelogram, and AB is the base of each.

3. Compare a parallelogram with a triangle having the same altitude and base.



4. What kind of parallelogram is shown in the figure?

5. Compare a rhomboid with a rectangle whose dimensions are the same as the base and altitude of the rhomboid.

6. How is the area of a rectangle found?

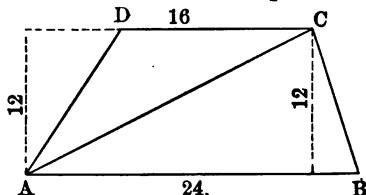
7. How, then, may the area of a rhomboid be found when its base and altitude are given?

Find areas of rhomboids with:—

8. Base $12\frac{1}{2}$ ft., altitude $7\frac{1}{2}$ ft. 10. $B = 2\frac{1}{2}$ yd., $A = 18$ in.

9. Base 16 rd., altitude 40 ft. 11. $A = 8\frac{3}{4}$ ft., $B = 5\frac{1}{8}$ in.

12. To find the area of a trapezoid whose two parallel sides are 24 in. and 16 in. and whose altitude is 12 in. If the trapezoid is divided as in the figure, what are the dimensions of triangle ABC ? Its area? If DC is taken as the base of triangle ACD , what is the altitude? The area? What, then, is the area of the trapezoid?



STATEMENT. $\frac{1}{2}$ of $24 \times 12 + \frac{1}{2}$ of 16×12 , or $\frac{1}{2}$ of $40 \times 12 = 20 \times 12$, or 240.

13. Notice that since the altitude is the same in each triangle, time may be saved by adding the two bases before multiplying by half the altitude. State the rule, then, for finding the area of a trapezoid.

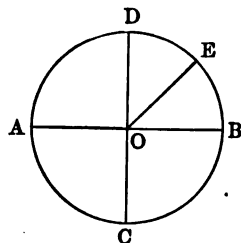
1. Name 2 quadrilaterals that are not parallelograms.
2. The base and the altitude of a rhombus are each 16 in. Area?
3. The altitude of a rhomboid is $\frac{3}{4}$ of its 2-ft. base. Area is x .
4. One angle of a parallelogram measures 90° . What do the other angles measure?
5. The parallel sides of a trapezoid are 25 and 37 ft., respectively, and the distance between them is 15 ft. Required the area.
6. The base of a triangle is $17\frac{1}{2}$ in., its altitude $8\frac{1}{2}$ in. What is its area?
7. A lot of land has a frontage of 50 ft. The parallel sides are perpendicular to this frontage. One of its parallel sides measures 10 rd. and the other 80 ft. What is its value at 45¢ a square foot?
8. A triangular park measures 600 ft. on one side, 300 ft. on the other which is perpendicular to it; how many square rods in the park?
9. Compare the area of a rhombus having a base of 16 ft., and an altitude of 12 ft., with that of a 16-ft. square
10. The diagonal of a trapezium is 42 in., and the perpendiculars dropped to it from the angles are 16 in. and 18 in. respectively. Required the area.

Find the areas of rhomboids:—

11. Base $13\frac{3}{4}$ ft., altitude $\frac{5}{8}$ ft.
12. Base 20 rd., altitude 50 ft.
13. Base $3\frac{1}{2}$ yd., altitude 20 in.
14. Base 2640 ft., altitude $\frac{1}{4}$ mi.
15. The diagonal of a trapezium is 7 yd. The altitudes perpendicular to it are $2\frac{1}{2}$ ft. and $12\frac{1}{2}$ ft. What is its area?
16. What is the area of one of the triangles into which a diagonal of a field 10 rd. square divides it?
17. What will it cost to fence a yard in a shape of an equilateral triangle 36 ft. long with 4 lines of wire weighing one pound to every 20 ft. and costing $3\frac{1}{2}$ ¢ a pound? There are 12 posts, costing 15¢ each.

1. An area bounded by a curved line, all points of which are equally distant from a point within called the *center*, is a **circle**.

2. The distance from the center to the curve is the **radius**, and the curve is called the **circumference**. In the figure how many radii are drawn? Name them.



3. Any straight line through the *center* terminating in the *circumference* is a **diameter**. Is a diameter shown in the figure? Which lines are diameters?

4. Into how many equal parts does a diameter divide a circle? One half of a circle is called a **semicircle**.

5. Into how many equal parts do two perpendicular diameters divide a circle? Such parts are called **quadrants**.

6. Any part of a circumference is called an **arc**. Any part of a circle bounded by two radii and an arc is a **sector**. Name some sectors in the figure. Is a quadrant a sector?

7. For convenience in measuring arcs, every circumference, whether large or small, is divided into 360 equal parts called degrees (360°). How many degrees in a semicircumference? In a quadrant?

8. Each degree is divided into 60 minutes ($60'$), and each minute into 60 seconds ($60''$).

$$15^\circ = x'. \quad 300' = x^\circ. \quad \text{An arc of } 30^\circ \text{ contains } x'.$$

$$10^\circ = x''. \quad 600'' = x'. \quad \frac{3}{4} \text{ of a circumference} = x^\circ.$$

9. Cut circles of different sizes from stiff cardboard or bring to the class several circular objects: plates, rings, covers, wheels, or coins. Measure very accurately the diameter and circumference of each.

NOTE. To get an accurate measurement of a circumference two pupils can work together to an advantage. Take two rulers, one standing edgewise on the other as a guide for the circle. Mark a point on the circumference and roll through one complete revolution, noticing the distance passed over on the bottom ruler, and holding against the second ruler to get a straight path.

RATIO OF CIRCUMFERENCE TO DIAMETER

10. In each case divide the circumference by the diameter, carrying the result to several decimal places.

11. Compare your results. Take all the results that are about alike and find the average.

12. If you have measured and divided accurately, the quotient will be 3.1416 *nearly*. What does this show?

13. If the diameter of a circle is 10 ft., what is the circumference?

14. 3.1416 is the ratio of the circumference to the diameter. It is represented by the Greek letter π (pi).

15. If D = diameter, R = radius, C = circumference, read the following:—

$$C = \pi \times D; D = C \div \pi, \text{ or } D = \frac{C}{\pi}; C = 2 R \pi, \text{ or } 2 \pi R.$$

Find the diameter or circumference or radius. Forecast the result.

16. $D = 20$ ft.; $C = x$.

19. $C = 4$ ft. 8 in.; $D = x$.

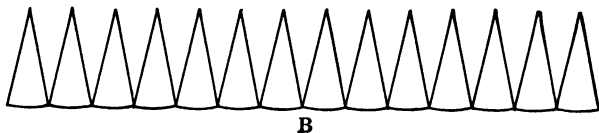
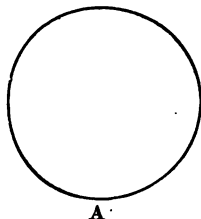
17. $C = 205$ ft.; $D = x$.

20. $D = 16\frac{1}{2}$ in.; $C = x$.

18. $R = 90$ ft.; $C = x$.

21. $C = 2$ yd. $1\frac{1}{2}$ ft.; $R = x$.

TO FIND THE AREA OF A CIRCLE



1. Draw a circle on stiff cardboard. Draw two perpendicular diameters. Divide each quadrant into two or more equal parts.

2. Arrange the parts as in *B*. What figures that you have studied do they most resemble? If these *sectors* were triangles, how would you find the area of each? Of all? Compare the sum of the bases of all the sectors with the circumference of the circle.

3. If these sectors were triangles, the area of all would equal the area of one triangle with a base equal to — and an altitude equal to the —, or $\text{Area} = \frac{C \times R}{2}$.

4. While the sectors are not triangles, yet the smaller they are made, the more like triangles they seem, and it is proven in geometry that the area of a circle is actually the same as you have found by supposing the sectors to be triangles, that is,

The area of a circle is the same as that of a triangle having a base equal to the circumference and an altitude equal to the radius; or $\frac{C \times R}{2}$ times the unit of measure.

5. Find the area of a circle whose diameter is 10 ft.

SOLUTION. $C = 10 \times 3.1416 = 31.416$.

$R = \frac{1}{2}$ of 10 = 5

$2 \overline{)157.080}$

78.54 = number of square feet.

NOTE. We must use C and R as abstract numbers representing the number of units. The product will then be an abstract number showing the number of units in the area.

Find the area of circles:—

6. $R = 6$ ft., $C =$ —.

8. $C = 100$ ft., $D =$ —.

7. $D = 12$ ft., $C =$ —.

9. $C = 50$ ft., $R =$ —.

Since $A = \frac{C \times R}{2}$ and $C = 2 \times R \times \pi$, we may multiply by $2 \times \pi \times R$ instead of C , then $A = \frac{2 \times \pi \times R \times R}{2} = \pi R^2$, the formula generally used when the radius or diameter is known.

Find the area of circles:—

10. $R = 15$ ft. 13. $R = 22\frac{1}{2}$ rd. 16. $C = 108$ ft. 19. $R = 182$ ft.

11. $D = 42$ ft. 14. $R = 16\frac{1}{2}$ rd. 17. $C = 1$ mi. 20. $D = 96$ in.

12. $C = 400$ in. 15. $D = 80$ yd. 18. $D = 38$ rd. 21. $D = 87$ ft.

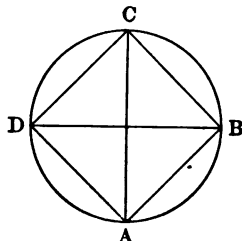
1. Draw a 2-in. circle. Draw two perpendicular diameters.

2. Compare the four arcs. Connect the points where the diameters cut the circumference as in the figure.

How do these lines compare?

3. Lines joining the extremities of arcs as AB are called **chords**. *Chords whose arcs are equal are also equal.*

4. Measure the angles of the quadrilateral $ABCD$.



5. What are the two parts into which AC divides the circle called?

6. The angle ABC is made, or **inscribed**, in a semicircle, when the vertex B is in the circumference and the sides pass through the ends A and C of the diameter. Measure the angle B with a protractor. How large is it?

Any angle inscribed in a semicircle is a right angle.

7. What kind of figure is $ABCD$? Why?

8. If you consider these figures made up of two triangles ABC and CDA , what is the length of the base of each? What is the altitude of each? Then what is the area of each? Of the whole square?

9. What is the area of the circle?

10. How much larger is the circle than the square?

11. Find the ratio of the square to the circle (4 decimal places).

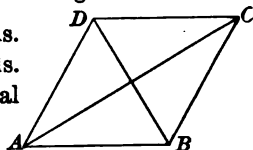
12. Observe Problem 8, and give the area of the largest square that can be drawn in a 3-in. circle. In a 4-in. circle.

13. What will be the diameter of the largest circle you can draw in a 6-in. square?

14. The top of a 4-ft. round table is what part of a 4-ft. square table?

1. Draw a 3-in. square. On the same base draw a 3-in. rhombus. Which has the greater altitude? The greater area?

2. Draw a rhombus. Draw the diagonals. Measure the angles made by the diagonals. Compare the lengths into which each diagonal divides the other.



The diagonals of a rhombus bisect each other at right angles.

3. If the diagonal $AC = 12$ in. and $DB = 8$ in., find the area by considering the figure two triangles.

4. The diagonals of a rhombus are 10 in. and 16 in. What is its area?

5. Using 1-in. lines, make a rhombus with an altitude of $\frac{1}{2}$ in. What is the area?

6. Using 1-in. lines, make a rhombus with an altitude of $\frac{1}{4}$ in. What is its area?

7. Using 1-in. lines, make as small a figure as you can. What is its shape?

8. The area of a rhombus, when the length of the sides is fixed, depends upon what?

9. Draw a trapezoid, the parallel sides being $1\frac{1}{2}$ in. and 2 in. One of the other sides 1 in. long is perpendicular to these. Find the area by dividing into a rectangle and a triangle; then find the area in the usual way.

10. Draw a rectangle. With the same sides draw a figure whose angles are not right angles. What is this figure called? How does it compare in size with the rectangle?

11. What dimensions would you need to know to find the area of the rhomboid you drew in Exercise 10?

12. Draw a trapezium. What dimensions must you know to find the area?

1. What objects before you are nearest in length to a yard? To a foot? To a rod?

2. How many degrees measure a \square (right angle)? Can you find as you look about you any but \square 's?

3. After going $\frac{1}{4}$ round a circle, how many degrees complete the circle?

4. Since the sum of all the angles of a triangle is equal to 2 \square 's, each of the three equal angles of an equilateral triangle measures how many degrees?

5. How may the area of a triangular park be found?

6. How can you find the area of any surface bounded by straight lines?

7. How much of an 8-in. square is not covered by a 7-in square? How much not covered by a 6-in. square?

8. An area containing 3 sq. yd. contains how many square feet?

9. Which is larger, a square or a rhombus with the same perimeter?

10. What part of a square yard is a mat $\frac{1}{4}$ yd. square?

11. A room is $\frac{2}{3}$ as wide as it is long. If it is 30 ft. long, how many square feet in the floor?

12. The area of a trapezoid is 350 sq. ft. The parallel sides are 30 ft. and 40 ft. What is the distance between them?

13. The area of a triangle is 25 sq. ft. The altitude is 5 ft. What is the base?

14. π , or 3.1416, is nearly $3\frac{1}{4}$ or $2\frac{1}{2}$. Using this value, what is the diameter of a tree if it takes a string 44 in. long to reach around it?

15. The diameter of a circle is $10\frac{1}{4}$ in. What is the circumference?

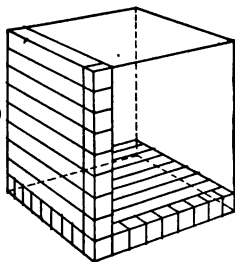
1. A button is 4.7124 in. round. How long a buttonhole is required?
 2. Find the circumference of the base of a lamp chimney that is $2\frac{3}{8}$ in. across.
 3. A circus ring is $414\frac{7}{10}$ ft. round. Find the distance to the center in rods.
 4. A hoghead is a little over $12\frac{1}{2}$ ft. round the middle. Will it go through a doorway that is 3 ft. 10 in. wide?
 5. If a mountain is 10 mi. round, what distance might be saved by tunnelling?
 6. A pie is cut accurately into 6 pieces. What is the ratio of the curved edge to the straight one?
 7. The hubs of two wheels are alike, but the spokes of one are 3 in. longer. How much greater is its circumference?
 8. If pails are 8 in. in diameter, how much strap iron will be required to make 100 hoops for them with 3-in. laps?
 9. In a lawn 100 ft. square the circular basin of a fountain is 40 ft. from each side. Draw a figure and find the area of the green-sward.
 10. A square is cut from a circle 12 in. in diameter. What is the size of each of the four equal segments cut from the circle?
 11. In Exercise 10, what per cent of the circle was cut away?
 12. A circular grass plot 2 rd. in diameter is surrounded by a walk 3 ft. wide. How many square feet in the walk?
- HINT. Find the areas of the two circles — the grass plot, and the one including plot and walk.
13. A circle 10 in. in diameter is cut from one 16 in. in diameter. What per cent of the large circle remains?

1. **Lines** have one dimension; viz.: —.
2. **Surfaces** have two dimensions; viz.: — and —.
3. **Solids** occupy space and have weight. They have three dimensions; viz.: —, —, and —.
4. Name the three common units of solid measure in the order of their size.
5. — cu in. = 1 cu. ft.; — cu. ft. = 1 cu. yd.; — cu. ft. = 1 cord.
6. State the method of finding the number of cubic feet in 20,000 cu. in.
7. 10 cu. ft. = x cu. in.
8. 20 cu. yd. = x cu. ft.
9. 10 cords = x cu. ft.
10. 720 cu. ft. = x cu. yd.

Rectangular Solids

1. What is a rectangle?
2. A solid bounded by *six* rectangles is a **rectangular solid**.
3. A solid bounded by *six squares* is a —.
4. How many corners and edges has a cube?
5. Is a cube a rectangular solid?
6. Describe an inch cube, or a cubic inch.
7. Describe a cubic foot; a cubic yard.
8. What is meant by a 2-ft. cube?
9. What is a 9-in. cube? How many cubic inches in a 9-in. cube?

(See the figure.) Along one edge of a cube 9 there is a row of x cu. in.; 9 such rows make a tier of $9 \times x$ cu. in. or y cu. in.; 9 such tiers contain $9 \times y$ cu. in., or 729 cu. in.



STATEMENT. $9 \times 9 \times 9$ cu. in. = x cu. in.

Find the contents or volume of:—

10. A 6-in. cube. 12. A 5-ft. cube. 14. A 10-yd. cube.

11. An 8-in. cube. 13. A 12-ft. cube. 15. A 20-in. cube.

16. Show that the volume of each cube is equal to as many units of measure as the product of the number of units in the edge taken as a factor three times.

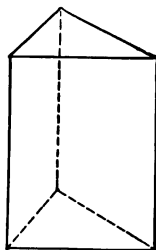
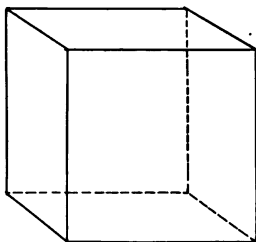
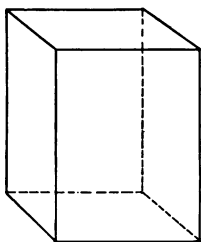
17. 5^3 means $5 \times 5 \times 5$. Since this is equal to the number of times 5 is taken as a factor to find the contents of a cube whose edge is five, 5^3 , or "5 to the third power," is called "the cube of 5," or "5 cubed."

18. 7^3 ? 9^3 ? 11^3 ? 12^3 ? 6^3 ? 10^3 ? 4^3 ? 20^3 ?

19. What is one of the three equal factors of 64, or $\sqrt[3]{64}$?

20. $\sqrt[3]{216}$? $\sqrt[3]{1728}$? $\sqrt[3]{512}$? $\sqrt[3]{1331}$? $\sqrt[3]{729}$? $\sqrt[3]{27,000}$?

Prisms



1. Solids whose sides are all rectangles and whose bases are bounded by straight lines are **prisms**.

2. Prisms are named from their bases: *square, rectangular, triangular, hexagonal*, etc. Name some objects that are square prisms; that are rectangular.

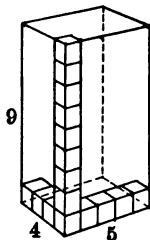
3. Is a square prism also rectangular? What kind of prism is a cube?

4. To find the contents of a rectangular prism whose dimensions are 4 in., 5 in., and 9 in.

Notice (a) the number of cubic inches in one row or square prism 1 in. by 1 in. by 9 in.

(b) The number of such rows in one tier or layer 4 in. wide.

(c) The number of such layers in the prism, and explain the statement $5 \times 4 \times 9$ cu. in. = x cu. in.



5. How many cubes may be put into a box 10 in. long, 8 in. wide, and 5 in. deep?

6. A trunk measures 3 ft. by 20 in. by 18 in. Find its cubical contents. Why multiply by 36 instead of 3?

Find the contents of rectangular prisms of these dimensions:—

| LENGTH | WIDTH | HEIGHT | LENGTH | BREADTH | DEPTH |
|-----------|--------|--------|-------------------------|---------------------|---------------------|
| 7. 16 ft. | 10 ft. | 8 ft. | 10. $42\frac{1}{2}$ ft. | 20 ft. | $13\frac{1}{2}$ ft. |
| 8. 1 yd. | 2 ft. | 9 in. | 11. $12\frac{1}{2}$ yd. | 10 ft. | 16 in. |
| 9. 24 ft. | 16 ft. | 4 yd. | 12. $20\frac{1}{2}$ ft. | $17\frac{1}{8}$ ft. | 6 in. |

13. How does the number of cubical units (cu. in., cu. ft., etc.) in any layer 1 unit thick compare with the number of units (sq. in., sq. ft., etc.) in the side of the layer?

14. If the base of a rectangular prism contains 30 sq. in., how many cubic inches in the bottom layer? If the prism is 10 in. high, how many cubic inches in the prism?

STATEMENT. 10×30 cubic inches = x cubic inches.

15. The floor of a cellar contains 36 sq. yd. If the cellar is 8 ft. deep, find its cubical contents.

16. A square prism is 16 in. wide and 4 ft. long. Find the volume.

17. A 5-in. cube is cut from the corner of a 20-in. cube. What part remains?

1. Wood for fuel is generally sold in what lengths? In what form is it usually piled?

2. Give the dimensions of a cord of wood; a half cord; $\frac{1}{8}$ of a cord, or a cord foot.

3. What is the unit of measure used in measuring wood?

4. What kind of solid does a half cord represent?

5. Explain: $8 \times 4 \times 4$ cu. ft. = x cu. ft. as applied to cord wood.

6. A pile of 4-ft. wood, 4 ft. high and 8 ft. long, contains a cord. How many cords if 16 ft. long? 24 ft.? 32 ft.? 96 ft.?

7. A pile of 4-ft. wood of the usual height must be how long to contain 10 cords? 12 cords? 25 cords?

8. How many cords in a pile of 4-ft. wood 4 ft. high and 18 ft. long? Explain the following statement, and show what may be cancelled:—

$$\frac{4 \times 4 \times 18 \text{ cu. ft.}}{128 \text{ cu. ft.}} = \text{the no. of cords.}$$

9. Bought a pile of 4-ft. wood 30 ft. long and 8 ft. high at \$6 per cord. Find the cost.

STATEMENT. $\frac{4 \times 8 \times 30}{128} \times \$6 = \$x.$

In the statement, what represents the number of cubic feet? The number of cords? The cost of all?

Find the value of piles of wood as follows:—

| LENGTH | WIDTH | HEIGHT | PRICE | LENGTH | WIDTH | HEIGHT | PRICE |
|------------|--------|--------|-------|-------------|--------|---------|--------|
| 10. 24 ft. | 4 ft. | 6 ft. | \$4 | 13. 24 ft. | 4 ft. | 7½ ft. | \$3.50 |
| 11. 40 ft. | 8 ft. | 8 ft. | \$5 | 14. 20 ft. | 3 ft. | 12½ ft. | \$5.00 |
| 12. 60 ft. | 10 ft. | 4 ft. | \$6 | 15. 16½ ft. | 44 in. | 2¾ ft. | \$4.25 |

16. A pile of tan bark 8 ft. wide and 10 ft. high contains 200 cords. How long is it?

1. Timber sawed for building purposes is **lumber**. What forms can you mention besides *boards, planks, joists, and beams*?

2. In measuring lumber no attention is paid to the thickness unless it exceeds an inch. A board 1 ft. square and an inch, or less, thick is called a **board foot**.

3. A board 12 ft. long, 1 ft. wide, and 1 in. thick contains 12 board feet, or usually spoken of as 12 ft. What would a board 12 ft. long, 10 in. wide and an inch thick contain?

4. A board 15 ft. long, 8 in. wide, and $\frac{3}{4}$ in. thick contains how many feet?

SUGGESTION. If the board were a foot wide it would contain — feet. Since it is but $\frac{3}{4}$ of a foot wide it contains —.

5. Ten 16-ft. boards averaging 9 in. in width contain how many feet? Explain the statement: $10 \times \frac{3}{4} \times 16$ board feet = x .

6. A board 10 ft. long, 1 ft. wide, and 1 in. thick is equal to x board feet. If $1\frac{1}{2}$ in. thick it is equal to $1\frac{1}{2} \times x$ board feet.

7. Find the contents of a 3-in. plank 15 ft. long and 10 in. wide. Explain: $3 \times \frac{5}{8} \times 15$ board feet = x .

8. How many feet of lumber in 12 joists, 16 ft. long, and 4 in. square?

9. Find the number of feet in 8 "3 × 4" joists each 12 ft. long.

NOTE. A "3 × 4" joist is one 3 in. thick and 4 in. wide.

Find the contents in board feet of lumber measuring as follows:—

10. 6 boards, 16 ft. long, $1\frac{1}{4}$ in. thick, and 14 in. wide.

11. Fifteen 3 × 4 joists, 18 ft. long.

12. A stick of timber 18 ft. long and 12 in. square.

13. A board 12 ft. long, 6 in. wide at one end and 10 in. at the other.

HINT. Find the average width.

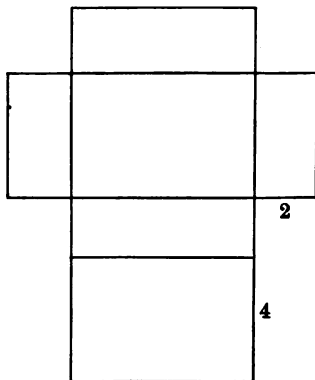
1. How many surfaces has a rectangular prism?
2. What name is given to a rectangular prism with equal faces?
3. Find the entire surface of a 5-in. cube.

Explain the statement: $6 \times 5 \times 5 \text{ sq. in.} = x \text{ sq. in.}$

4. The entire surface of a cube is 150 sq. in. How long is it?

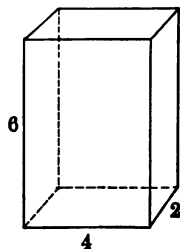
Find the entire surface of: — How long a cube has: —

5. A 9-in cube.
6. A cube 10 in. long.
7. A 16-in. cube.
8. An entire surface of 384 sq. in.?
9. An entire surface of 600 sq. ft.?
10. An entire surface of 294 sq. in.?
11. Compare the ends of a square prism with each other
12. Compare its four sides.
13. Find the entire surface of a square prism 8 in. long and 3 in. wide. (What is the area of each square? How many? Of each rectangle? How many?)
14. Compare the opposite faces of any rectangular prism.
15. Give the dimensions of each face in the figure at the right. What is its entire surface?



Find the entire surface of prisms: —

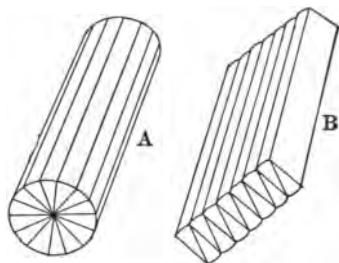
16. 10 in. by 6 in. by 4 in.
17. 12 ft. long, 9 ft. wide, 6 ft. high.
18. 20 ft. long, 4 ft. wide, 8 ft. high.
19. 16×18 by 4; 20 by 1 by 1.
20. 12 by 9 by 8; $2\frac{3}{8}$ by $\frac{1}{8}$ by 16.
21. 12 by 12 by 6; $\frac{2}{3}$ by $\frac{3}{4}$ by $7\frac{1}{2}$.



1. A solid having ends (or bases) which are equal, parallel circles, and having uniform diameters is a **cylinder**. Mention some common objects that are cylinders.

2. If the base of a prism is 6 sq. in., how many cubic inches in a section 1 in. thick? How many cubic inches in the prism if its height is 8 in.?

3. If a cylinder is cut as in Fig. *A* and arranged as in Fig. *B*, what does it nearly become?



4. If the circular end is the base, it has been changed into one nearly like a rectangle. Has the area of the base changed in *B*? Has the length of the cylinder changed?

5. In *B*, if the base were a rectangle and its area known, how would its volume be found?

6. If the radius of *A* is 1 in., what is the area of its base or the base of *B*? If the length of *B* is 8 in., what is the volume?

7. The volume that we have found by considering that the cylinder is nearly a prism with an equal base and the same height, is the volume that is found by geometry to be the true one, viz.:—

The volume of a cylinder is the same as that of a prism having a base of the same area and having the same height.

8. Find the volume of a cylinder whose radius is 8 in. and whose height is 15 in.

Explain the statement: $(15 \times 3.1416 \times 8 \times 8)$ cu. in. = x cu. in.

9. A cylindrical pail 6 in. in diameter inside and 12 in. deep contains how many cubic inches?

Explain the statement: $(12 \times 3.1416 \times 3 \times 3)$ cu. in. = x cu. in.

10. A cylindrical tank is 10 ft. deep and 8 ft. in diameter. Find the contents.

11. A gallon contains 231 cu. in. How many gallons will a cistern 8 ft. deep and 4 ft. in diameter hold?

12. A well is 32 ft. deep and 5 ft. in diameter. Find the contents.

13. To hold a gallon, a pail measuring 33 square inches on the bottom must be how deep?

To find the Surface of a Cylinder

1. In form, the ends of a cylinder are equal —s. The rest of the surface is **convex surface**.

2. Suppose the diameter of a cylinder to be 4 in.; its circumference = x , or $3.1416 \times D$.

3. The circumference of a cylinder is 8 in.; its diameter is x , or $\frac{C}{\pi}$.

4. A cylinder is 20 in. long and 4 in. in diameter. Find the area of its ends.

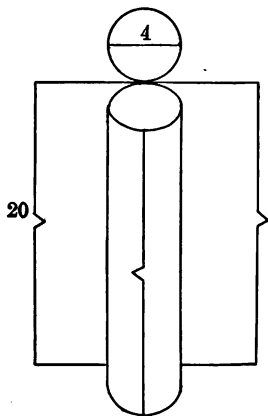
5. Roll an oblong paper to form a cylinder. Give the length and circumference of the cylinder thus made.

6. Unroll the paper and give the dimensions of a rectangle equivalent to the convex surface of the cylinder. Explain the diagram at the right.

7. The convex surface is equal to $C \times L$ square units, where C = circumference and L = length. Why is this?

8. A cylinder is 25 in. long, 4 in. in diameter. Its convex surface is x . Explain: $(3.1416 \times 4) \times 25$ sq. in. = x sq. in.

9. A cylinder is 20 in. long, 5 in. in diameter. Find the *entire* surface, or the convex surface plus the surface of the ends.



NOTE. In oral work use $3\frac{1}{2}$ for π .

1. An old tree is 22 ft. round; how far is it through?
2. Give the convex surface of a lead pencil $\frac{1}{4}$ in. in diameter and 7 in. long.
3. The sides that make the right angle of a triangle are each 10 ft. What is the area?
4. It is 28 ft. across a pond. How far is it around it?
5. Which takes more room, a cord of wood or a 5-ft. cube?
6. $\frac{1}{2}$ a circle = a rectangle having the radius for one side and — for the other.
7. A 20-ft. log averages one square foot in the cross section. The cubic contents are x .
8. How many cubic yards of earth will a bin hold that is 3 ft. \times 10 ft. \times 9 ft.?
9. About how many cubic yards does your schoolroom contain?
10. A horse tethered by a rope 10 yd. long can graze over how large an area?
11. A cellar 18 ft. by 15 ft. by $7\frac{1}{2}$ ft. deep contains how many cubic yards?

Written

1. The largest possible circle is cut from a cardboard measuring 16 by 24 in. What area of the cardboard remains?
2. What will it cost to cement the bottom of a circular cistern 8 ft. in diameter at \$ 2.50 per square yard?
3. The driving wheel of an engine is 6 ft. in diameter. How many revolutions does it make in going a mile if there is no slipping?
4. A large street roller is 5 ft. in diameter and 7 ft. long. How great an area does it cover in one revolution?

1. x times 6 = 144; $18 \times x = 198$.
2. Multiplicand = 25; product = 400. How is the multiplier found?
3. $186 + x = 31$.
4. Dividend and quotient being given, how is the divisor found?
5. When the product and multiplier are given, how is the multiplicand found?
6. What is the area of a rectangle 12 ft. long and $6\frac{1}{2}$ ft. wide?
7. A rectangle containing 108 sq. in. is 9 in. wide. How long is it? ($9 \times x$ sq. in. = 108 sq. in.)
8. A lot 200 ft. long contains 24,000 sq. ft. How wide is it?
9. A sidewalk 50 ft. long requires 50 sq. yd. of concrete. How wide is the walk?
10. One half an acre of land is taken for a new street 40 ft. wide. How long is the street?
11. The area of a triangle is 325 sq. in.; its base is 25 in. What is its altitude?
12. The altitude of an isosceles triangle is 14 ft.; its area is 126 sq. ft. What is its base?
13. At 30¢ a board foot a mahogany board 1 in. thick and 12 ft. long cost \$2.70. How wide was it?
14. The area of a rhomboidal field is 12 A. Its length being 20 rd., what is its altitude?
15. A square contains 400 sq. in. How long is it?
16. The perimeter of a square is 1000 ft. Its area?
17. A rectangular field 48 rd. wide contains 48 A. What is the other dimension? The perimeter?
18. A rectangle is 3 times as long as wide and contains 108 sq. ft. What are its dimensions?

SUGGESTION. Into how many squares can the rectangle be cut? What will be the size of each? The dimensions of each?

CONTENTS OF A SOLID AND TWO DIMENSIONS GIVEN TO FIND
THE THIRD

1. $13 \times 7 \times \$x = \910 .

2. I hired 15 men at \$2.50 per day each. At the completion of the work I paid them in all \$150. How many days did they work?

3. A box on my table holds 432 cu. in. It covers 72 sq. in. of the surface of the table. How high is the box?

4. The area of the floor of your schoolroom is 900 sq. ft. The room contains 10,800 cu. ft. How far is the ceiling from the floor?

5. A packing box is 48 in. long and 30 in. wide. How deep must it be to hold 10 cu. ft.?

STATEMENT. $\frac{10 \times 1728}{48 \times 30} = \text{depth.}$

Explain the statement, and show a short solution.

6. A closet 8 ft. high and 27 in. deep will contain 72 cu. ft. How wide is it?

7. A pile of 198 cu. ft. of 4-ft. wood covers 16 sq. rd. How long is it? How high is it?

Explain the statement:—

A. $\frac{16 \times 272\frac{1}{4}}{4} = x$.

B. $\frac{198 \times 128}{16 \times 272\frac{1}{4}} = y$.

8. A cylindrical oil tank holds 10 gal. Standing on the floor it covers 77 sq. in. How high must it be?

9. A bookcase holding 32 cu. ft. covers a wall space of 24 sq. ft. How far must it project into the room?

10. I have room in my stable for a grain bin 8 ft. by 4 ft. How deep shall I make it to have it hold 72 bu.?

11. A grindstone 4 ft. in diameter contains 6.2832 cu. ft. How thick is it?

Explain the statement: $6.2832 \div (2^2 \times 3.1416) = x$.

12. In digging a trench 3 ft. wide and $4\frac{1}{2}$ ft. deep 330 cu. yd. of earth are removed. How long was the trench?

1. I buy a corner lot 120 ft. by 50 ft. and use the earth obtained by digging a cellar 60 ft. by 30 ft. by 10 ft. to raise the grade, how many inches?

2. A circular standpipe 75 ft. high is 25 ft. in diameter. When $\frac{3}{4}$ full, how many gallons of water does it contain, reckoning $7\frac{1}{2}$ gal. to a cubic foot?

3. A speculator buys a field 600 ft. long and 500 ft. wide for \$2500. He runs a 40-ft. street through the center in each direction at an expense of \$425 for labor. He sells the land at 20 cents a square foot. How much does he make or lose?

4. At \$3.75 per square yard what will it cost to pave $\frac{3}{4}$ of a mile of street 81 ft. wide?

5. A reservoir supplies a town with 4,573,800 gallons of water daily. If its surface area is 7 acres, how much will the water be lowered in a week, providing $\frac{1}{2}$ as much runs in as runs out? Call 1 cu. ft. equal to $7\frac{1}{2}$ gal.

6. From a lot of land 40 rd. square I sold 40 sq. rd. What is the remainder worth at \$230 an acre?

7. The snow fall is 6 in. on the level. How many cubic feet rest on every acre of ground?

8. A watch chain cost \$28. This is $\frac{7}{8}$ of the cost of the watch. Find the cost of the watch.

9. Wood is bought at \$3.75 a cord. Transportation adds 15% to the cost, and storage 5% more. It is then sold at a profit of 20% on the total cost. What does it sell for?

10. A farmer sold 14,700 bushels of corn. He kept $33\frac{1}{3}\%$ of his crop. How many bushels did he raise?

11. A schoolroom 12 ft. high, 30 ft. long, and 28 ft. wide contains 40 pupils. How many square feet of floor space for each pupil? How many cubic feet of air for each one?

1. $6\frac{1}{2}$, 8, $4\frac{1}{2}$ are the dimensions of my coal bin in feet. Reckoning 90 lb. to the cubic foot, what will a bin full cost at \$5 per ton?
2. Quincy granite weighs $165\frac{3}{4}$ pounds to the cubic foot. What is the weight of 6 pieces of curbing 8 inches thick, 2 feet wide, and half a rod long?
3. Find the cost of carpeting a 9-foot hallway 22 feet long with three-quarter carpeting at \$0.87 $\frac{1}{2}$. Cut no strip, and allow $1\frac{1}{2}$ feet per seam for matching.
4. How many tons of 15-inch ice may be cut to the acre, a cubic foot weighing $57\frac{1}{2}$ pounds? Apply your knowledge of cancellation.
5. What is the capacity, in 42-gallon barrels, of a cylindrical oil tank $3\frac{1}{2}$ ft. in diameter, 22 feet long? Make a statement and cancel.
6. What is the area of a sector of 120° , its radius being 24 inches?
7. A ball ground 375 feet long and 280 feet wide is inclosed by a tight board fence 8 feet high. What will the boards cost at \$24 per M? Add 10% for waste.
8. Bought 12,000 long tons of coal at \$4 and sold it at the same price per short ton. What did I gain?
9. What will it cost to polish the visible portions of a shaft of red granite 6 feet by 2 feet by 22 inches at $6\frac{2}{3}$ ¢ per square inch?
10. Draw a 6-inch square, a rectangle 9 inches by 4 inches, and one 3 inches by 12 inches. Compare areas and perimeters. What inference do you draw?
11. A schoolroom 32 feet by 30 feet is lighted by 6 windows, each containing 15 panes of glass 12 by 16 inches. The lighting surface of the room is what per cent of the floor surface?
12. My garden is 80 ft. by 100 ft. What will a concrete walk around the inside cost at 80¢ a square yard, the width of the walk being 3 feet?

1. What decimal of a square prism becomes shavings when the largest possible cylinder is turned from it?
2. What number subtracted 88 times from 80,005 will leave 13 as a remainder?
3. A railroad company fences 13 miles of its road at $73\frac{3}{4}$ cents a rod. What is the cost?
4. How many square feet of zinc will line a cubical cistern 5 ft. 10 in. deep?
5. Bread sells for 10¢ with flour at \$5.00. Flour goes up to \$6.50. What should bread sell for on this basis?
6. In a city of 7200 school children there are 2720 cases of tardiness in a year, during which there are 400 sessions of the school. The average attendance is 6800. What is the rate of tardiness?
7. Find the cost of six 8×10 sills 18 ft. long at \$24.74 per M.
8. In a library every tenth book is a history. If there are 567 other books in the library, how many volumes in all?
9. A schoolroom measuring 32 ft. \times $28\frac{1}{2}$ ft. \times 13 ft. seats 49 pupils. Each one needs 1800 cu. ft. of fresh air an hour. The room full would last the class x minutes.
10. How many feet of wire will it take to fence a square field containing 625 sq. rd. if there are three rows of wire in the fence?
11. If I buy cloth at \$1.20 per yard, how many yards must I sell at a gain of 20% to gain \$20.40?
12. How many yards of $\frac{3}{4}$ yd. wide carpeting will be required to carpet a room 12 by $15\frac{1}{2}$ feet, allowing 5 inches to each strip for matching? The carpet selected was 85¢ per yard.
13. A cubic foot of mahogany weighs 45 lb. What will be the weight of a piece of mahogany 5 in. thick, 20 in. wide, and 16 ft. long?

1. What does the denominator of a fraction show? Which is the larger unit, $\frac{1}{8}$ or $\frac{1}{7}$? Why?

2. Which is larger, $\frac{2}{3}$ or $\frac{3}{4}$? Find this in a short way without changing to a like unit.

3. What is the ratio of $\frac{2}{3}$ to $\frac{5}{6}$? To what unit did you change both before you could compare them? Why?

4. Change $3\frac{2}{3}$ to 40ths. Explain the process.

5. Explain the process of changing $8\frac{2}{3}$ to $\frac{2}{3}$.

6. Explain the process of changing $\frac{2}{3}$ to $8\frac{2}{3}$.

7. Compare $16\frac{1}{4}$ with $3\frac{1}{4}$.

11. \$17 is $\frac{2}{3}$ of \$x.

8. Add $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{3}$.

12. \$6 is $\frac{2}{3}$ of \$x.

9. From $7\frac{1}{4}$ take $6\frac{1}{4}$.

13. $\frac{2}{3}$ of 48 is 80% of x.

10. $16 \times \$8\frac{1}{2} = \x .

14. $7 : 3\frac{1}{2} = 1\frac{1}{2} : x$.

15. What per cent of a 2-foot cube is 3 cubic feet?

16. A miller takes 4 quarts toll from every bushel. What per cent is this?

17. I spent \$27, or 30%, of my money. How many dollars had I remaining?

HINT. How many per cent remained? Compare it with 30%.

18. What is the rate of income on an \$8000 house rented at \$40 per month?

19. If wages are increased 10%, what will men now receive who received \$1.50 before?

20. Men who now receive \$2.75 after a 10% raise in wages received what before the raise?

21. What is $3\frac{1}{2}\%$ of 500? $5\frac{1}{2}\%$ of 600?

22. \$2 is what per cent of \$800? Of \$80? Of \$8?

23. $0.42 \div 70 = \text{what?}$

25. $0.006 \div 0.12 = \text{what?}$

24. $4.2 \div 700 = \text{what?}$

26. $0.06 \div 0.012 = \text{what?}$

1. A lot of land cost \$2800. This was $\frac{7}{25}$ of the cost of a house. The house cost x dollars

2. How many bags will be required for 1000 bushels of wheat, if each bag holds $2\frac{5}{8}$ bushels?

3. The snowfall is 5 inches on the level. How many cubic feet rest on $2\frac{1}{2}$ A. of ground?

4. The yachts start at precisely 2.15 P. M. The winner returns to the starting point at 5.36 P. M. and has averaged 12 miles an hour. How long was the course?

5. Give the day and hour when exactly $\frac{3}{8}$ of the month of March has passed.

6. 2800 mill operatives earn on the average \$1.68 a day. If their wages are reduced 10%, what will the weekly saving to the company be?

7. In a city with an average attendance of 10,000 in the public schools, which keep 360 sessions during the year, how many tardinesses would there have been to each thousand pupils if the whole number of tardinesses for the year had been 3600?

8. The interest on a mortgage is payable semiannually at $5\frac{1}{2}\%$. The face of the mortgage is \$2300. How large a check should be remitted to pay the interest as it falls due?

9. What will it cost to bronze a 2-foot cube at $1\frac{1}{2}\phi$ a square inch?

10. Hay is bought at \$12 a ton. Transportation adds 15% to the cost, and storage 5% more. It is then sold at a profit of 20%. What does it sell for?

11. I can buy alcohol for \$2.75. If I import it for school use, I can buy it for 90¢. This is a saving of $x\%$.

12. The number of children of school age in a certain city is 14,769. This is $33\frac{1}{3}\%$ of the whole population. How many not school age? Work a short way.

1. A pint of meal is put into a peck of flour. What per cent of the mixture is meal?
2. How many bullets, each weighing $\frac{1}{2}$ oz., can be molded from 2 lb. of lead?
3. May was sent to the store for $\frac{1}{2}$ pk. of apples. What did she pay for them, if they were \$3.20 a bushel?
4. How many pens can I buy for \$2, if I buy 2 for a cent?
5. What numbers between 30 and 50 are perfect squares?
6. What is the cost of 12 oranges at 3 for 5 cents?
7. $\frac{2}{3}\%$ of a number is 50. What is the number?
8. The difference between $\frac{1}{2}$ and $\frac{1}{3}$ of my money is \$30. How much have I?
9. Which is the better bargain, bananas at 20¢ a dozen or 16 for a quarter?
10. A mason works 8 hr. for \$3.00, and a carpenter 10 hr. for \$3.50. How much more does one earn than the other in 100 hr.?
11. What is 7 months' rent of a telephone at \$50 a year?
12. How long will it take a girl to earn \$5, if she works half the time for 18¢ and half the time for 12¢ an hour?
13. Three boys who are 10, 12, and 14 years old respectively are to share \$54 in proportion to their ages. What is the share of each?
14. Two men agree to cut lumber for \$200. One, with 3 men, works 5 days, and the other, with 4 men, works 6 days. How much of the money should each receive?
15. Paid \$21 for insuring my house for 5 yr., at $\frac{3}{4}\%$. What is it worth, if it is insured for $\frac{1}{2}$ of its value?
16. A teacher's salary is \$60 a month. If it is raised $8\frac{1}{2}\%$, what does she then receive?

1. Find the sum of:—

2496
3948
2769
5834
9625
3847
7859
6498
5936
4073
4569
3299
5678
6935
7894
9699
4678
3979
8462
9879
6432
9478
8997
9346

2. Find the total cost of the following: $3\frac{3}{4}$ lb. of butter at 28 cents a pound, 9 lb. 9 oz. ham at 16 cents a pound, 8 lb. 10 oz. cheese at 24 cents a pound.

3. A man sold $\frac{2}{3}$ of his farm for \$3900. What was $\frac{1}{3}$ of the farm worth at the same rate?

4. A builder bought 6500 brick at \$7.50 per thousand, 12,200 ft. of lumber at \$16.50 per thousand feet, and 975 lb. nails at \$3.80 per hundred pounds. What was the amount of his entire bill?

5. What will it cost to carpet a room 54 ft. long and 30 ft. wide with Brussels carpet $\frac{3}{4}$ of a yard wide at \$1.24 per yard, making no allowance for matching?

6. A man bought a house for \$2500 and sold it for \$1875. What per cent of the cost did he lose?

7. What is the interest of \$320, at 6 per cent for 2 yr. 10 mo. 12 da.?

8. A merchant sold goods for \$240, thereby losing $\frac{1}{3}$ of the cost. For what amount should he have sold them to gain 15%?

9. During the winter of 1902 and 1903 a ton of coal lasted a family 14 days, average time. What did the coal cost at \$5.25 a ton from Oct. 1 until March 31, inclusive? What would the coal cost for the same length of time at an increase of \$4.25 a ton?

10. A house which cost \$9600 rents for \$48 a month. This is $x\%$ income from the investment, if the yearly expenses amount to \$96.

11. A flour merchant sold 240 bbl of flour for \$1582, which was $\frac{1}{3}$ less than he paid for it. What was the whole cost? The cost per barrel?

12. If a cubic foot of granite weighs 165 lb., what will a 6-in. cube weigh?

1. How many yards of ingrain carpet will be needed for a room 9 ft. by 15 ft. ?

2. A building lot contains 3380 sq. ft. and is 100 ft. long. How wide is it ?

3. Divide 0.6 by 0.015.

$$4. 4 \times \sqrt{256} = x.$$

5. A house bought for \$3200 sold for \$3000. What per cent of the cost was lost ?

6. What part of anything is $12\frac{1}{2}\%$ of it ? If $12\frac{1}{2}\%$ of my money is \$25, how much have I ?

7. What will pay a note of \$500 that has been running $\frac{2}{3}$ of a year at 9% interest ?

8. Goods marked at \$1.50 per yard were sold at $33\frac{1}{3}\%$ discount. What did 5 yards cost ?

9. Bought 3 for 4¢ and sold 2 for 3¢. Did I gain or lose and how much ?

10. Bought for \$10 and sold for \$2.50. What per cent of the cost was lost ?

11. My weight increased from 150 lb. to 175 lb. What was the per cent of increase ?

12. Read as per cents :—

$$\frac{1}{16}, \frac{1}{12}, \frac{3}{16}, \frac{5}{12}, \frac{7}{8}, \frac{5}{16}, \frac{7}{12}, \frac{1}{24}, \frac{7}{16}.$$

13. Read in largest units :—

$$12\frac{1}{2}\%, 18\frac{3}{4}\%, 31\frac{1}{4}\%, 37\frac{1}{2}\%, 43\frac{3}{4}\%, 56\frac{1}{4}\%, 83\frac{1}{8}\%.$$

14. It is 160 rods around a square field. How many acres does it contain ?

15. At 15¢ a yard, picture molding for a room 12 ft. by 18 ft. will cost how much ?

16. Paid \$2.40 for 15 lb. of meat, 20% of which was bone. What did the meat really cost per pound ?

1. If $2\frac{3}{4}$ yd. are bought for \$23.10, what is paid for $15\frac{1}{2}$ yd. at the same rate?

2. $67.24 \times 82\% - 67.24\% \div 82 =$ what?

3. In a flag $17\frac{1}{2}$ ft. long and $\frac{2}{3}$ as wide, how many square yards of bunting, not allowing for seams?

4. How many cubic yards of earth will be thrown out in digging a cellar 54 ft. long, 2 rd. wide, and 9 ft. deep?

5. What per cent of its daily trip has the long hand of a clock accomplished at 3 P.M.?

6. I insure my house for \$2400. How much premium do I pay, the rate being $\frac{3}{4}\%$?

7. What is the interest of \$400 for 3 yr. 3 mo. 20 da. at 6%?

8. If I buy flour at $3\frac{5}{16}$ cents a pound and sell at $4\frac{5}{12}$ cents a pound, what part of the cost is the gain?

9. What per cent of a floor 16 ft. square is covered by a rug 12 ft. square?

10. A man sold $37\frac{1}{2}\%$ of his business for \$3900. What was $\frac{1}{3}$ of it worth at that rate?

11. A man built a double house at a cost of \$4500 on a lot valued at \$1500. If he receives \$30 a month from each tenant, and pays \$100 for taxes and \$20 for repairs, etc., what part of his investment will his net income be?

12. If flour sold for \$4.25 a barrel gains $6\frac{1}{4}\%$, at what price should it be sold to gain 15%?

13. If $\frac{1}{3}$ of $\frac{2}{3}$ of a ship is worth \$8600, what is $\frac{2}{3}$ of it worth?

14. Change 0.096 to a common fraction whose denominator is 375.

15. How do sixths and twenty-sevenths compare in size?

16. How many 63ds in $\frac{1}{2}\frac{1}{3}$?

[FOR REFERENCE]

Counting

| | |
|-----------|--------------------------|
| 12 things | = 1 dozen (doz.) |
| 12 dozen | = 1 gross (gro.) |
| 12 gross | = 1 great gross (G. gr.) |
| 20 things | = 1 score |

| | |
|-------------------|------------|
| 24 sheets (paper) | = 1 quire |
| 20 quires or | } = 1 ream |
| 480 sheets | |

Time

| | |
|---------------------------------|-------------------------------|
| 60 seconds (sec.) | = 1 minute (min.) |
| 60 minutes | = 1 hour (hr.) |
| 24 hours | = 1 day (da.) |
| 7 days | = 1 week (wk.) |
| 2 weeks | = 1 fortnight |
| 30 (31, 28, 29) days | = 1 month (mo.) |
| 3 months or | } = 1 quarter |
| 13 weeks | |
| 12 months or | } = 1 year (yr.) |
| 365 days | |
| 365 da. 5 hr. 48 min. 49.7 sec. | } = { 1 true or solar
year |
| 366 days | |
| 10 years | = 1 decade |
| 100 years | = 1 century (C.) |

Value*U.S. Money*

| | |
|--------------|-------------------------|
| 10 mills | = 1 ct. (ct., c., or ¢) |
| 10 cents | = 1 dime (di.) |
| 100 cents or | } = 1 dollar (\$) |
| 10 dimes | |
| 10 dollars | = 1 eagle |

Canadian Money

100 cents = 1 dollar = \$1

English Money

12 pence (d.) = 1 shilling (s.) = \$0.243 +
20 shillings = 1 pound (£) = \$4.8665

French Money

100 centimes = 1 franc (fr.) = \$0.193

German Money

100 pfennigs = 1 mark (M.) = \$0.238

Capacity*Liquid Measures*

| | |
|---------------|-------------------|
| 4 gills (gi.) | = 1 pint (pt.) |
| 2 pints | = 1 quart (qt.) |
| 4 quarts | = 1 gallon (gal.) |

1 gallon = 231 cu. in.

Dry Measures

(For grain, fruit, etc.)

| | |
|------------|---------------------|
| 2 pints | = 1 quart |
| 8 quarts | = 1 peck (pk.) |
| 4 pecks | = 1 bushel (bu.) |
| 10 pecks | } = 1 barrel (bbl.) |
| 2½ bushels | |

1 bushel = 2150.42 cu. in.

Weight*Avoirdupois Weight*

| | |
|------------------|-----------------------------------|
| 16 ounces (oz.) | = 1 pound (lb.) |
| 100 pounds | = { 1 hundred-
weight (cwt.) } |
| 2000 pounds or | } = { 1 ton (T.)
(short) |
| 20 hundredweight | |
| 2240 pounds | = 1 long ton |

| | | |
|------------|------------|---------------------|
| *60 pounds | = 1 bushel | { wheat or potatoes |
| *56 " | = 1 " | corn or rye |
| *32 " | = 1 " | oats |
| 196 " | = 1 barrel | flour |
| 200 " | = 1 " | beef or pork |

* In most States

Troy Weight

(For precious metals, jewels, etc.)

| | |
|-----------------|-----------------------------|
| 24 grains | { = 1 pennyweight
(pwt.) |
| 20 pennyweights | = 1 ounce |
| 12 ounces | = 1 pound |

| | | |
|-------------|-----------|--------|
| 437½ grains | = 1 ounce | { Av. |
| 7000 " | = 1 pound | |
| 480 " | = 1 ounce | |
| 5760 " | = 1 pound | { Troy |

Apothecaries' Weight

| | |
|-------------|-------------------------|
| 20 grains | = 1 scruple (sc. or ℥) |
| 3 scruples | = 1 dram (dr. or ℥) |
| 8 drams | = 1 ounce (oz. or ℥) |
| 12 ounces | { = 1 pound (lb. or lb) |
| 5760 grains | |

Length

| | |
|-----------------|------------------|
| 12 inches (in.) | = 1 foot (ft.) |
| 3 feet | = 1 yard (yd.) |
| 16½ feet or | { = 1 rod (rd.) |
| 5½ yards | |
| 320 rods | { = 1 mile (mi.) |
| 5280 feet | |
| 63,360 inches | |

4 inches = 1 hand

6 feet = 1 fathom

| | |
|----------------|---|
| 6086.7 feet or | { = { 1 knot
1.15 + com-
mon miles } = { 1 nautical mile
1 geographic mile |
| 3 knots | |
| = 1 league | |

Circular Measure

| | |
|---------------------|---|
| 60 seconds (") | = 1 minute (') |
| 60 minutes | = 1 degree (°) |
| 360 degrees | = 1 circumference |
| 69½ miles or | { = { 1° of latitude; or
1° of longitude
on the equator |
| 60 geographic miles | |

Surface or Square

| | |
|--------------------|---|
| 144 square inches | { = { 1 square foot
(sq. in.) (sq. ft.) |
| 9 square feet | = { 1 square yard
(sq. yd.) |
| 30½ square yards | { = { 1 square rod
(sq. rd.) |
| 272½ square feet | |
| 160 square rods | { = 1 acre (A.) |
| 43,560 square feet | |
| 640 acres | = { 1 square mile
(sq. mi.) |
| 1 mile square | = 1 section |
| 36 square miles | = 1 township |
| 100 square feet | { = { 1 square
(in roofs,
floors, etc.) |
| | |

Solid or Cubic

| | |
|-------------------|---|
| 1728 cubic inches | { = { 1 cubic foot
(cu. in.) (cu. ft.) |
| 27 cubic feet | = { 1 cubic yard
(cu. yd.) |

Wood Measures

| | |
|----------------|-------------------------|
| 16 cubic feet | = 1 cord foot (cd. ft.) |
| 128 cubic feet | { = 1 cord (cd.) |
| 8 cord feet | |

THE SOUTHWORTH-STONE ARITHMETIC

THIRD BOOK

PART II

PERCENTAGE

Computing by Hundredths

1. One hundred is the common standard of comparison. For example, the merchant may gain \$10 on every \$100 invested, or 10 per cent (10%). The rate of interest may be 6%, or \$6 on every \$100 used.

2. What is meant by saying:—

12% of the grain spoiled?

33 $\frac{1}{3}$ % of the month was stormy?

14% of the pupils were absent?

3. The phrase *per cent* (the short form of *per centum*) means *hundredths*, or *by the hundred*.

4. 25% of 400 is 25 times one of the 100 equal parts of 400, or —.

5. 100 is what part of 400? $\frac{1}{4} = \frac{1}{100}$, or —%.

6. If 25% of a number is 100, what is the whole number?

HINT. All of the number or $\frac{100}{100}$ of it = — $\times \frac{1}{100}$ of it, or — $\times 100$ = —.

It is seen in the preceding examples that there are three general classes of problems in percentage, viz.:—

I. The whole given to find a part.

II. A part given to find its relation to the whole.

III. A part given and its relation to the whole to find the whole.

The Whole given to find a Part*Find: —*

- | | |
|--------------------------------|--------------------------------|
| 1. $12\frac{1}{2}\%$ of 96 lb. | 6. $33\frac{1}{3}\%$ of 24 hr. |
| 2. 20% of 90 miles. | 7. 30% of 200 acres. |
| 3. 50% of 2000 lb. | 8. 14% of 300 pupils. |
| 4. 25% of \$ 60. | 9. 25% of 1200 votes. |
| 5. 75% of 400 yd. | 10. 6% of \$ 3000. |

A Part given to find its Relation to the Whole

1. Compare 2 and 4; thus, 2 is $\frac{1}{2}$ or 50% of 4; 4 is 2 times or 200% of 2.
2. Compare 3 and 6; 2 and 8; 3 and 12.
3. 5 is what part of 20? What $\%$ of it?
4. 16 is $\frac{1}{3}$, or — $\%$, of 48; \$ 24 is what part of \$ 36? What $\%$?
5. 12 ounces is what part of a pound? What $\%$?
6. 800 lb. is what part of a ton? How many 100ths of a ton, or $\%$ of it?
7. $16 = \text{what } \%$ of 40?
8. $48 = \text{what } \%$ of 64?
9. $96 = x\%$ of 144.
10. $35 = x\%$ of 105.

A Part given and its Relation to the Whole to find the Whole

1. $\frac{1}{2}$ of my age is 16 years. How old am I?
2. 50% , or $\frac{1}{2}$, of my money is \$ 80. How much have I?
3. $\frac{2}{3}$ of the price was \$ 36. $\frac{1}{3}$ of the price was $\frac{1}{3}$ of \$ 36; and $\frac{1}{3}$, or the whole price, was — \times \$ —, or \$ —.
4. 6% of my salary is \$ 72. 1% of my salary is \$ —, and my whole salary is —.
5. A whole flock is how many times 25% of it? If 25% of a flock is 40, the whole flock is — \times 40, or —.
6. What is the relation of all of anything to 50% of it? To 25% of it? To $12\frac{1}{2}\%$ of it? To $33\frac{1}{3}\%$ of it? To $66\frac{2}{3}\%$ of it?

1. 25 per cent = $25\% = \frac{25}{100} = 0.25 = \frac{1}{4}$. Which are fractions? Which are decimals? Which is most easily used?

2. $12\frac{1}{2}\% = \frac{1}{2}$ of $25\% = \frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$.

3. $6\frac{1}{4}\% = \frac{1}{2}$ of $\text{---}\% = \frac{1}{2}$ of $\text{---} = \text{---}$.

4. $37\frac{1}{2}\% = \text{---} \times 12\frac{1}{2}\% = \text{---} \times \frac{1}{8} = \text{---}$.

5. $50\% = \text{---}$; $62\frac{1}{2}\% = 50\% + \text{---}\% = \frac{1}{2} + \text{---} = \text{---}$.

6. $75\% = 3 \times \text{---}\%$; $87\frac{1}{2}\% = 75\% + \text{---} = \frac{3}{4} + \text{---} = \text{---}$.

7. $8\frac{1}{8}\% = \text{what fraction?}$ 12. $14\frac{2}{7}\% = \frac{14\frac{2}{7}}{100} = \frac{1}{7}$.

8. $16\frac{2}{3}\% = \text{---} \times 8\frac{1}{3}\% = \text{what?}$ 13. $28\frac{4}{7}\% = \text{what?}$

9. $41\frac{2}{3}\% = \text{---} \times 8\frac{1}{3}\% = \text{---}$. 14. $42\frac{5}{7}\% = \text{what?}$

10. $58\frac{1}{3}\% = 50\% + \text{---} = \text{---}$. 15. $57\frac{1}{7}\% = \text{what?}$

11. $83\frac{1}{3}\% = 75\% + \text{---} = \text{---}$. 16. $71\frac{3}{7}\% = \text{what?}$

17. $91\frac{2}{3}\% = 100\% - 8\frac{1}{3}\% = 1 - \frac{1}{12} = \text{---}$

18. $93\frac{1}{3}\% = 100\% - x\% = \text{---}$.

19. Find $14\frac{2}{7}\%$ of 280 feet. $14\frac{2}{7}\% = \frac{1}{7}$, then $14\frac{2}{7}\%$ of 280 feet = $\frac{1}{7}$ of 280 feet = what?

20. What is $58\frac{1}{3}\%$ of \$24? $\frac{7}{12}$ of \$24 = \$x.

21. If $16\frac{2}{3}\%$, or $\frac{1}{6}$, of my money is \$30, how much have I?

22. What is the relation of all, or 100% of anything, to $37\frac{1}{2}\%$ of it? If $37\frac{1}{2}\%$ of it is 6 ft., what is all, or 100% of the length?

The following fractions are used so often that we ought to know at sight that:—

| | | | |
|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| $\frac{1}{2} = 50\%$. | $\frac{2}{5} = 40\%$ | $\frac{1}{8} = 12\frac{1}{2}\%$. | $\frac{1}{10} = 5\%$. |
| $\frac{1}{3} = 33\frac{1}{3}\%$. | $\frac{3}{5} = 60\%$. | $\frac{3}{8} = 37\frac{1}{2}\%$. | $\frac{1}{25} = 4\%$. |
| $\frac{2}{3} = 66\frac{2}{3}\%$. | $\frac{4}{5} = 80\%$. | $\frac{5}{8} = 62\frac{1}{2}\%$. | $\frac{3}{10} = 3\frac{1}{3}\%$. |
| $\frac{1}{4} = 25\%$. | $\frac{1}{6} = 16\frac{2}{3}\%$. | $\frac{7}{8} = 87\frac{1}{2}\%$. | $\frac{1}{40} = 2\frac{1}{2}\%$. |
| $\frac{3}{4} = 75\%$. | $\frac{5}{6} = 83\frac{1}{3}\%$. | $\frac{1}{12} = 8\frac{1}{3}\%$. | $\frac{1}{50} = 2\%$. |
| $\frac{1}{5} = 20\%$. | $\frac{1}{7} = 14\frac{2}{7}\%$. | $\frac{1}{15} = 6\frac{2}{3}\%$. | $\frac{3}{50} = 6\%$. |

23. What is $62\frac{1}{2}\%$ (or $\frac{5}{8}$) of 40? Of 64? Of 100?

24. Subtract each per cent in the table from 100%.

Determine whether you are to find all, a part, or the per cent that a part is of the whole, and then find: —

1. $62\frac{1}{2}\%$ of 72.
2. $12\frac{1}{2}\%$ of a gross.
3. $14\frac{2}{3}\%$ of 35.
4. $33\frac{1}{3}\%$ of a sq. yd.
5. $12\frac{1}{2}\%$ of 96.
6. $14\frac{2}{3}\%$ of 70.
7. 50% of \$37.50.
8. 25% of \$48.60.
9. 7 is what part of 28? What per cent of 28? What part of 49? What per cent?
10. 13 doz. are $33\frac{1}{3}\%$ of — doz.
11. 25 qt. are $62\frac{1}{2}\%$ of — gal.
12. 30 pk. are $66\frac{2}{3}\%$ of — bu.
13. \$17 are 20% of \$——.
14. 16 is what % of 20?
15. 15 is what % of 60?
16. 20 is what % of 60?
17. 75 is what % of 300?
18. $62\frac{1}{2}\%$ of \$49.60 was $\frac{1}{2}$ of A's indebtedness. He paid $\frac{1}{4}$ of the amount. What is the balance?
19. A teacher pays \$6 per week for board and room, which is 40% of her salary. What is her salary for a school year of 40 weeks?
20. I pay for rent \$750 a year, which is $33\frac{1}{3}\%$ of my income. What do I receive annually?
21. A mechanic receiving \$72 a month spent \$60. What per cent did he save?
22. An agent collected a bill of \$6000, and received \$1500 commission. What was the rate per cent?
23. A farmer sold 64 bu. of apples, $87\frac{1}{2}\%$ of which were of the first quality. How many were of the second quality?
24. Bought 12 doz. buckets at 25¢ each. I wish to make $33\frac{1}{3}\%$ profit. How much must the marked price be per dozen?
25. A merchant having \$2000 paid \$500 for a team. What per cent remained?
26. A grocer received 60 bbl. of flour, and sold 12 of them the day they arrived. What per cent had he still?

What is given ? What are you to find ?

1. 45 is $83\frac{1}{3}\%$ of — ? 3. 5 oz. are $12\frac{1}{2}\%$ of — lb.
2. 3 is 2% of — ? 4. 6 dimes are 60% of \$—.
5. What per cent of a bushel is 4 quarts ?
6. What per cent of \$120 is \$10 ?
7. Dick caught 80 fish in a week, and Tom caught 16. What per cent of the whole did Tom catch ?
8. Paid \$75 for a watch, and sold it for \$50. I lost $x\%$.
9. A mechanic worked for \$3.50 per day. His helper received $14\frac{2}{3}\%$ of \$3.50 per day. What is the sum of their wages ?
10. A boy earned \$1.50 in a week, 60% of that amount the next week, and $66\frac{2}{3}\%$ of it the third. How much did he receive in all ?
11. A certain locomotive ran 990 miles without repairs. Another ran $66\frac{2}{3}\%$ of this distance farther. How far did the second one go ?
12. Find 50% of \$6200. 15. Find $\frac{1}{2}\%$ of 6200.
13. Find 25% of \$5200. 16. Find $\frac{1}{4}\%$ of 5200.
14. Find 75% of \$1600. 17. Find $\frac{3}{4}\%$ of 1600.

What is the difference between : —

18. $16\frac{2}{3}\%$ of 480 acres of land, and $\frac{1}{8}$ of 240 acres ?
19. $14\frac{2}{3}\%$ of \$105 and $\frac{1}{4}$ of \$105 ?
20. A trader sold a horse for \$175, which was $87\frac{1}{2}\%$ of its cost. How much did he gain or lose by the transaction ?
21. A number increased by 20% of itself is 72. What is it ?
22. One dollar and twenty cents is 20% of what number ?
23. A merchant sold goods for \$54 and lost 10% . The cost ?
24. C wrote a check for \$40, which was $3\frac{1}{3}\%$ of his bank balance. How much remained after the check was paid ?
25. A man's expenses are \$7.50 per week, which is $8\frac{1}{3}\%$ of his income from a small farm. What are the profits from the farm ?

1. A farm that cost \$5400 was sold for 75% of its value. What was the selling price?

$$\begin{array}{r}
 \text{A} \\
 75\% \text{ or } \frac{3}{4} \times \$5400 = \$4050.
 \end{array}
 \qquad
 \begin{array}{r}
 \text{B} \\
 \$5400 \\
 \underline{0.75} \\
 \$270.00 \\
 \underline{3780.0} \\
 \$4050.00
 \end{array}$$

2. Of the two methods, *A* and *B*, which seems preferable? Why?

3. When can method *A* be used to advantage?

4. When will it be better to use method *B*?

5. 43½% of my crop of 3290 bushels of wheat has been sold. How many bushels did I sell? (Which method? Why?)

6. I paid \$3400 for a house and sold it for 87% of what I paid. What did I get for it? (Which method? Why?)

7. Of the \$2700 paid for an estate, 12½% was in cash and the remainder in notes. What was the cash payment? (Method?)

8. Of 12,650 bushels of grain, 34% was in corn, 28% in oats, and the remainder in wheat. There were x bu. of corn, y bu. of oats, and z bu. of wheat. Explain the statement:—

$$[100\% - (34\% + 28\%)] \times 12,650 \text{ bu.} = z \text{ bu.}$$

How much is:—

Find a discount of:—

9. 25% of 3742 tons?

12. 15% on 6½ yards at \$2.50.

10. 7½% of 784 miles?

13. 37½% on a \$558 piano.

11. 16⅔% of 5733 acres?

14. 18⅓% on 42 tons at \$6.50.

15. Compare ¾ of \$400 with ¾% of it.

16. Read: $0.00\frac{5}{8}$; $\frac{5}{8}\%$; $\frac{5}{100}$. Explain: $\frac{\frac{5}{8} \times 6}{100 \times 6} = \frac{5}{600}$.

17. What is ⅞ of \$64,000?

18. Find ⅞% of \$64,000.

19. My property is assessed for \$24,800. Tax rate 1¾%. My tax?

1. I bought a house for \$3600, and sold it at a gain of \$540. What % did I gain?

$$\$540 = \frac{54}{360} \text{ of } \$3600.$$

$$\begin{array}{r} 0.15 = 15\% \\ 360 \overline{) 54.00} \\ \underline{36.00} \\ 18.00 \\ \underline{18.00} \end{array}$$

EXPLANATION. Comparing the gain with the cost, the ratio of the gain to the cost = $\frac{54}{360}$. This fraction expressed as a decimal is 0.15, or 15%. Observe that the % one number is of another is their ratio expressed as hundredths.

2. 6035 persons bought tickets to a fair. This was what per cent of the 8500 that attended?

3. 625 pupils belong in the Webster school. 600 of them are present, or $x\%$ of the whole.

4. 37%, or 11,100 tons, of an ice crop remained unsold. There must have been x tons in the whole crop.

5. The cargo of the *Sea King* was valued at \$38,475. The value of the cotton was $16\frac{2}{3}\%$ of the whole, that of the sugar $37\frac{1}{2}\%$. The miscellaneous part of the cargo was valued at x dollars, or $y\%$ of the whole. Take the shortest method.

6. I sold my bicycle for \$17. It cost me \$25. I must have lost what per cent of the cost?

7. If I had lost but 15%, I should have sold it for what?

8. 192 pages of a book of 432 pages are illustrated. That is what % of the whole?

9. If a retail dealer has liabilities amounting to \$1125, and owns property amounting to \$675, what per cent will he be able to pay his creditors?

10. A man owned \$175,000 worth of real estate in a certain city, but he has recently sold \$145,000 worth of it. What per cent of it does he still own?

11. A city increases 24% in 10 years; that is from 37,860 population to x .

1. I gained 15%, or \$540, when I sold my house. Find the cost.

$$\frac{100}{15} \times \frac{36}{\cancel{540}} = \$3600.$$

$$\begin{array}{r} \$3600 \\ \text{or } 15) \$54000 \\ \underline{45} \\ 90 \\ \underline{90} \\ 00 \end{array}$$

EXPLANATION. The whole cost is $\frac{100}{15}$ of 15% of it, or the ratio of 100% of the cost to 15% of it is $\frac{100}{15}$. Hence the house cost $\frac{100}{15} \times \$540$. *If it is not easy to cancel the terms, the division may be performed in the usual way.*

2. \$260.01 is 27% of \$——. 5. \$1406.25 is 45% of \$——.
3. \$533.40 is 84% of \$——. 6. 134.64 qt. = 51% of x gal.
4. \$368.93 is 79% of \$——. 7. 113.49 ft. = 39% of x yd.
8. The Indian population, according to the census of 1900, was about 145,000, which is only 58% of what it was in census of 1890. What was it in 1890, and what was the decrease, in round numbers?
9. There are 2844 school children in a certain city. This is 36% of the population. What is the population?
10. A man sold 20% of his interest in a mill for \$38,000. As he owned 20% of the mill, the mill was worth \$ x .
11. About 50,000,000 sq. mi. (or 25%) of the earth's surface is land and the remainder is water. What is the area of the water?
12. In a certain school 19 pupils are post graduates, 391 are regular students, and the remaining $37\frac{1}{2}\%$ are students taking special courses. How many are enrolled?
13. A man saved \$12,597 in ten years, and this amount was 5% of his whole property. What was he worth at the end of that time?
14. A Western ranchman sent 381 cattle to the Chicago market, which was 60% of the number he sent to the Eastern dealers. He found he had sold in all $\frac{1}{2}$ of his drove. How many had he at first?
15. A man owed \$470 in 1900, which was 40% of what he owed in 1898. How much did he owe in 1898?

1. Which is more profitable, a gain of 3 per dozen, 5 per score, 25 per cent, or 36 per gross? Why?

2. Compare $\frac{3}{4}$ of something with $\frac{3}{4}\%$ of it.

3. Six wrong out of 24 problems solved is x wrong out of a hundred, or $y\%$.

4. The Clevelands won 7 games in their series with the Pittsburg Club, the Pittsburgs won 4, and the one game was a tie. The winner's per cent was x .

5. Thirty-six hits in 80 times is a batting average of $x\%$.

6. The center fielder has 80 chances, and makes 4 errors. His fielding average is $x\%$.

7. The crew pulled 36 strokes to the minute at starting, but fell off to 30 at the finish. This was a loss of what per cent?

| 8. | 9. | 10. |
|--------------------------------|--------------------------------|---------------------------------------|
| $\frac{3}{4}\%$ of 600 = x . | $\frac{3}{4}\%$ of 800 = x . | $x = 87\frac{1}{2}\%$ of 128. |
| $\frac{3}{4}\%$ of $x = 2$. | $\frac{3}{4}\%$ of $x = 12$. | $x\%$ of 144 = 120. |
| $x\%$ of 1200 = 8. | $x\%$ of 200 = $\frac{1}{2}$. | $83\frac{1}{2}\%$ bad and $x\%$ good. |

| 11. | 12. | 13. |
|--------------------------------------|--|-------------------------------|
| 25% of $5\frac{1}{2}$? | 19 is $x\%$ 57. | 25 is 4% of x . |
| 35% of 400? | 70 is $x\%$ 2100. | 280 is 14% of x . |
| $8\frac{1}{2}\%$ of $2\frac{2}{3}$? | $16\frac{2}{3}$ is $x\%$ $66\frac{2}{3}$. | 8 is $\frac{4}{5}\%$ of x . |

14. I paid $\frac{3}{4}\%$ commission to my agent for selling a farm for \$1250. How much money did he have left to send me?

15. $2\frac{1}{4}\%$ was paid a collector who earned \$22.50 a month in this way. What were his annual collections?

16. Of a farm of 320 acres 108 acres are given to wheat, 96 acres to oats, and the remainder to corn. What per cent of the farm are the cornfields?

17. The drought destroyed 27 per cent of a crop, and only \$1660 was realized. What was the loss?

PERCENTAGE: TABLE FOR PRACTICE

The number of hundredths, or per cent, is sometimes called the **rate**. The number of which a part is to be found is called the **base**, and the part of the base required, when found, is called the **percentage**. For brevity these terms are used in the following table:—

Find the value of x .

| ORAL | | | | WRITTEN | | | |
|------|------------------|----------------------|----------------------|---------|--------------------------|-----------------------|------------------|
| | Rate % | Base | Percentage | | Percentage | Base | Rate % |
| 1 | 16 $\frac{1}{2}$ | \$9.30 | x | 1 | 32 $\frac{1}{10}$ mi. | 267 $\frac{1}{2}$ mi. | x |
| 2 | 12 $\frac{1}{2}$ | x | 123 tons | 2 | 170.40 | x | 17 $\frac{1}{4}$ |
| 3 | x | 75 yd. | 18 $\frac{1}{2}$ yd. | 3 | 36 yr. | 130 yr. | x |
| 4 | 37 $\frac{1}{2}$ | x | 57 | 4 | 18 $\frac{1}{2}$ A. | x | 45 $\frac{1}{2}$ |
| 5 | 40 | 9000 mi. | x | 5 | 4857.6 ft. | 5280 ft. | x |
| 6 | x | \$0.50 | \$0.31 $\frac{1}{2}$ | 6 | \$5.76 | x | $\frac{1}{3}$ |
| 7 | $\frac{2}{3}$ | x | 14 da. | 7 | \$5400 | \$9600 | x |
| 8 | 87 $\frac{1}{2}$ | 1 $\frac{1}{2}$ tons | x | 8 | x | 1500 cd. | 15 |
| 9 | x | 160 | 106 $\frac{1}{3}$ | 9 | 20 $\frac{1}{2}$ sq. ft. | 5200 sq. ft. | x |
| 10 | 6 $\frac{1}{4}$ | x | 15 sq. mi. | 10 | 6500 T. | x | 83 $\frac{1}{3}$ |
| 11 | 75 | 725 | x | 11 | 14 $\frac{1}{2}$ da. | 365 da. | x |
| 12 | x | a century | 8 mo. | 12 | \$13,651.56 | \$75,842 | x |
| 13 | 90 | 608 bu. | x | 13 | 5 | 36 | x |
| 14 | 18 $\frac{1}{4}$ | x | 18 bales | 14 | 84 | x | $\frac{7}{8}$ |
| 15 | x | 75 rd. | 6 $\frac{1}{4}$ rd. | 15 | x | 31 $\frac{1}{2}$ | 16 $\frac{1}{2}$ |
| 16 | $\frac{3}{4}$ | \$12,000 | x | 16 | 328 $\frac{2}{5}$ | x | 9 $\frac{1}{2}$ |
| 17 | 31 $\frac{1}{4}$ | x | 35 cords | 17 | \$349.06 | \$9006 | x |
| 18 | x | 726 gal. | 605 gal. | 18 | \$18 | x | $\frac{2}{3}$ |
| 19 | $\frac{5}{8}$ | \$120,000 | x | 19 | x | \$8100 | $\frac{2}{10}$ |
| 20 | $\frac{7}{8}$ | x | 3 $\frac{1}{2}$ lb. | 20 | x | $\frac{1}{4}$ | $\frac{1}{4}$ |

1. Sold a house that cost \$5000 at a profit of 30%. Proceeds of sale?

2. A merchant's sales for January amounted to \$28,000, but 12% was lost in bad debts. The net proceeds of the sales for the month were x dollars.

3. Gained \$12, or 20% in selling a Century Dictionary. It cost me x dollars, and I sold it for y dollars.

4. A sewing machine cost me \$24. I sold it for \$32. I gained $x\%$.

Explain the statement: $\frac{32 - 24}{24} = x\%$.

NOTE. The gain or loss is always reckoned on the cost.

5. A conductor's wages are \$72 a month. They are reduced to \$60. This is a cut down of $x\%$.

Explain the equation: $\frac{72 - 60}{72} = x\%$.

6. Cost, \$8000; selling price, \$6000; loss per cent, x .

7. Cost, \$6000; selling price, \$8000; gain per cent, x .

8. Which is more profitable, to buy cloth for \$3 and sell it for \$3.50, or to buy for \$4, and to sell it for \$4.80?

9. Gas is reduced from \$2 to \$1.60 per 1000 cu. ft. How much do I save on \$45 worth of gas?

10. Last winter my coal cost me \$6 a ton. This winter, I pay \$8. This is an increase of what per cent?

11. A man sold two city lots for \$5600 each. On one he gained $14\frac{1}{2}\%$, on the other he lost $12\frac{1}{2}\%$. Find the loss or gain.

12. Bought silk at \$1.75 a yard. I marked it to sell at a gain of 20%, but sold it at $33\frac{1}{3}\%$ less than the marked price. What per cent did I gain or lose?

13. 160 is 10% more than what?

1. Bought wood at \$4 a cord and sold it at a gain of 20%. What did I sell it for?

(a) $\$4 + 20\%$ of $\$4 = x$. (b) 120% of $\$4 = x$.

Explain the two statements. Which suggests the shorter solution?

2. Sold a typewriting machine that cost me \$80 at a loss of 10%. What did I receive for it?

(a) $\$80 - 10\%$ of $\$80 = x$. (b) 90% of $\$80 = x$.

Explain the two statements. Which is preferable?

3. Bought a house for \$4800 and sold it at a gain of 16%. Find the selling price. Which method?

4. Sold a dwelling house for \$7500 at a profit of 25%. It cost me x dollars.

5. An epidemic decimated a certain village, leaving it with but 639 inhabitants. How many died? (What does decimate mean?)

6. A farmer who owned 390 acres increased his farm 30% within 2 years. How much did he own at first?

7. A speculator lost \$3000 or 6% of his property. What was it then worth?

8. A sold a yacht for \$800 at a loss of 60%. Required its cost.

9. A piano that cost \$450 was sold for \$292.50. What was the per cent of loss?

10. At a "special sale" goods were sold for 10% less than the market price. If the dealer still makes a profit of 20%, find the market price of dress goods that cost \$1.20 a yard.

11. How shall I mark goods that cost me \$2 so that I can sell at a discount of 20% and yet make 40% on the purchase?

12. The cost was $\frac{2}{3}$ of the selling price. What was the gain per cent?

1. In $\frac{25}{8}$ what per cent is the numerator of the denominator?
In $\frac{5}{8}$?
2. What is $\frac{2}{11}$ of a rod in feet? In inches?
3. 231 cubic inches = 1 gallon. Separate 231 into its prime factors. Give the dimensions of a tin pan that will hold a gallon.
4. What part of a year are the longest three months?
5. What is $\frac{1}{8}\%$ of 21,000?
6. $33\frac{1}{3}\%$ of 60% of $\frac{1}{2}$ of the money remained. What part did the thieves take?
7. My property is assessed for \$2500. The rate of taxation is $2\frac{1}{2}\%$. What is my tax?
8. What per cent of the surface of a 4-inch cube is on five sides of it?
9. Bought thread for 4 cents a spool and gained 300%. It sold for x cents.
10. $8\frac{1}{3}\%$ of a yard = $x\%$ of a foot.
11. Three sides of a square = $x\%$ of its perimeter.
12. $x\%$ of the day has passed at 9 P.M.
13. $(\frac{1}{3} + 30\% + \frac{2}{3})$ of 64 is 25% of x .
14. 16 is $\frac{2}{3}$ of x and $\frac{2}{3}\%$ of y .
15. Gave \$24 to James and \$30 to Lucy. Lucy had $x\%$ more than James, and he had $y\%$ less than Lucy.
16. Paid the price of a pound for 14 ounces. I thus lost $x\%$.
17. $\sqrt{9} = x\%$ of $\sqrt{144}$.
18. In a series of ball games the Alphas won 40% and the Omegas 50%. Two games were drawn. How many were played?
19. $\frac{1}{2}$ a mile is what per cent of two leagues?

1. \$12, or $12\frac{1}{2}\%$, (of cost) is gained; cost = \$ x .
2. \$8, or $16\frac{2}{3}\%$, is lost; cost = \$ x .
3. \$24 = cost; $33\frac{1}{3}\%$ is gained; selling price = \$ x .
4. \$35 = cost; $14\frac{2}{3}\%$ is lost; selling price = \$ x .
5. \$36 = selling price, which includes the cost and a gain of 20% of the cost. \$36 = cost + $\frac{1}{5}$ of cost, or $\frac{6}{5}$ cost; \$36 is $\frac{6}{5}$ of \$ x .
6. \$28 = selling price, which is the cost less a loss of 20%. What part of the cost is the loss? The selling price is $\frac{x}{5}$ of the cost; \$28 is $\frac{x}{5}$ of \$ y .
7. Bought a bicycle for \$80 and sold it for \$100. My gain per cent was x .
8. If I had sold it for \$60 I should have lost \$ y , or x per cent of cost.
9. An importer bought silk at \$2.50 a yard and sold it to a retailer for \$3, who sold it to the wearer for \$3.50. What per cent of profit did each make?
10. Sold a watch for \$119 and gained $16\frac{2}{3}\%$. How much should I have gained or lost if I had sold it for \$100?
11. A thrifty clerk resolves to live on 60% of his salary. He spends \$48 more than he intends, but still saves \$300. What was his salary?
12. I paid \$125 for what I thought was 4-foot wood. It proved to be but 45 inches long. What deduction should be made in the settlement?
13. Sold telephone stock for \$25,000 at an advance of 25% on what I paid for it. What did I gain?
14. I purchased a patent for \$8000. The seller lost 84% of its original value. What was its original value?

1. Which is more profitable, buying meat at 16¢ and selling at 19¢, or selling potatoes at 64¢ that cost me 56¢?
2. Butter sold at 28¢ yields no profit. What would be gained on what cost \$140 if sold at 30¢?
3. Milk bought at 20¢ a gallon is sold at 8¢ a quart. What per cent is gained if 25% of the quantity bought spoils?
4. A 5% increase in wages means \$200 more a month for the employer to pay. What was his annual pay roll before and after the increase?
5. Mr. H. earns \$1200 a year selling carriages at 15% commission, all expenses paid. The manufacturer makes a net profit of 14½%. If 50 carriages are sold, what is their average cost?
6. A dishonest dealer buys 50 gallons of alcohol at \$2.50 a gallon, adds 14 gallons of water, and sells the mixture at a price per gallon 10% below actual cost. What per cent does he gain?
7. I am offered a 10% discount on a suit of clothes marked to sell at \$60. I know that even then the dealer will make 12½%. I offer \$50 and get the suit. What per cent does the dealer gain?
8. I sell $\frac{3}{4}$ of a lot of land at $\frac{3}{4}$ the cost and get \$200 for the remainder. The original cost being \$1200, what is my per cent of loss?
9. What per cent is gained by selling coal at the rate of $\frac{1}{3}$ of a ton for what 1000 pounds cost?
10. A farmer's sheep cost him \$200. One out of every seven dies and he sells those that remain for \$275. What was the gain per cent, the cost of keeping being \$40?
11. A merchant sold a stock of goods for \$3042 and gained 17%. What per cent would he have gained or lost had he sold it for \$2392?
12. For what should he have sold it to gain 100%?
13. I bought a \$6 umbrella at 16⅔% discount. The dealer made 25% profit. What did it cost him?

1. If I pay 6¢ for a year's use of a borrowed dollar, what is the rate of interest?
2. What does the expression "6 per cent interest" mean?
3. At 6%, what is a year's interest of \$300?
4. What part of a year is 2 months? If the interest for 1 year is \$18, what should it be for 2 months?
5. What is the interest of \$400 for 1 year and 6 months at 5%?
6. At 7%, what is the interest of \$200 for 2 years, 6 months?
7. What is the first step in finding the interest of any principal? The second step?
8. Explain: $2\frac{1}{2} \times 0.07 \times \$200 = \$x$.

This is called the **general method** of finding interest. It may always be used, but the work may be shortened by some of the special methods.

At 6% the interest of any principal for:—

12 months = 6% of it.

2 months = 1% of it. (Why?)

20 months = 10% of it. (Why?)

200 months = 100% of it, or the principal itself.

9. Find the interest at 6% of \$380 for 2 yr. 7 mo. (31 mo.).

PROCESS

| | |
|------------------------------------|-----------------------------------|
| Interest for 20 months = \$38.00 | Observe that the time was so |
| Interest for 10 months = \$19.00 | separated as to avoid multiplying |
| Interest for 1 month = <u>1.90</u> | by anything except 10. |
| Interest for 31 months = \$58.90 | |

10. Into what convenient parts would you separate the time if it were 26 mo.? 37 mo.? 3 yr. 7 mo.? 5 yr. 8 mo.? 3 yr. 11 mo.? 1 yr. 7 mo.? 8 yr. 4 mo.?

Explain the following process of finding the interest at 6%:—

1. Of \$725 for 3 yr. 11 mo.

Interest for 47 mo. of \$725.

$$\text{Int. for } \left\{ \begin{array}{l} 20 \text{ mo.} = \$72.50 \\ 20 \text{ mo.} = 72.50 \\ 5 \text{ mo.} = 18.125 \\ 2 \text{ mo.} = 7.25 \\ \hline 47 \text{ mo.} = \$170.375 \end{array} \right.$$

2. Of \$278 for 1 yr. 7 mo.

Interest for 19 mo. of \$278.

$$\text{Int. for } \left\{ \begin{array}{l} 20 \text{ mo.} = 27.80 \\ 1 \text{ mo.} = 1.39 \\ \hline 19 \text{ mo.} = \$26.41 \end{array} \right.$$

Find the interest at 6%:—

3. Of \$280 for 2 yr. 8 mo.

4. Of \$640 for 3 yr. 7 mo.

5. Of \$95 for 4 yr. 11 mo.

6. Of \$73.50 for 1 yr. 4 mo.

7. Of \$649 for 7 yr. 8 mo.

8. Of \$750 for 8 yr. 4 mo.

9. Of \$295.75 for 5 yr. 11 mo.

10. Of \$641.86 for 3 yr. 3 mo.

TIME IN DAYS: INTEREST 6%

Oral and Written

1. How many days in an interest month? In an interest year?

2. 60 days is what part of a year?

3. Since at 6% the interest for a year is 6% of the principal, the interest for 60 days is $x\%$ of the principal.

4. The interest for 6 days is what part of the interest for 60 days?

At 6% the interest of any principal for:—

$$60 \text{ days} = 1\%, \text{ or } \frac{1}{100} \text{ of it.}$$

$$6 \text{ days} = 0.1\%, \text{ or } \frac{1}{1000} \text{ of it.}$$

5. Find the interest at 6% of \$720 for 75 days.

PROCESS

Interest for 60 days = \$7.20

Interest for 15 days = 1.80

Interest for 75 days = \$9.00

6. How was 15 days' interest found from 60 days' interest being known?

Explain the process of finding the interest at 6% :—

7. Of \$196 for 115 days.

Int. for 115 da. of \$196.

$$\text{Int. for } \left\{ \begin{array}{l} 60 \text{ da.} = \$1.96 \\ 30 \text{ da.} = 0.98 \\ 20 \text{ da.} = 0.6533+ \\ 5 \text{ da.} = 0.1633+ \\ \hline 115 \text{ da.} = \$3.7566 \end{array} \right.$$

8. Of \$119 for 89 days.

Int. for 89 da. of \$119.

$$\text{Int. for } \left\{ \begin{array}{l} 60 \text{ da.} = \$1.19 \\ 20 \text{ da.} = 0.3966+ \\ 6 \text{ da.} = 0.1190 \\ 3 \text{ da.} = 0.0595 \\ \hline 89 \text{ da.} = \$1.7651 \end{array} \right.$$

What shall I pay at 6% for the use of:— Find the interest at 6% :—

9. \$780 for 67 da. ?

14. \$94 for 200 da.

10. \$640 for 93 da. ?

15. \$762 for 5 mo. 14 da.

11. \$92 for 3 mo. 12 da. ?

16. \$815 for 86 da.

12. \$87.50 for 117 da. ?

17. \$924 for 8 mo. 11 da.

13. \$106 for 2 mo. 17 da. ?

18. \$17.84 for 17 da.

In distinction from the *general method*, this is sometimes known as the **bankers' method**.

INTEREST AT ANY RATE: BANKERS' METHOD

1. If the interest at 8% = \$18, the interest on the same sum and for the same time at 1% = \$x. At 5% = 5 × \$x = —.

2. If the interest at 6% = \$42, the interest at 7% = \$x.

To find the interest of \$105 for 75 da. at 5%.

PROCESS

6% int. for 75 da. of \$105.

6% int. for 60 da. = \$1.05

6% int. for 15 da. = 0.2625

$$\begin{array}{r} 6) \$1.3125 \\ \underline{0.2187} + \\ \$1.0938 \end{array} \begin{array}{l} = 6\% \text{ int.} \\ = 1\% \text{ int.} \\ = 5\% \text{ int.} \end{array}$$

3. Show how the interest at 6% is found.

4. At 1% ; at 5%.

5. What if the rate had been 7% ? 10% ? 12% ?

Find the interest of:—

6. \$ 640 for 1 yr. 8 mo. at 1%. ($\frac{1}{3}$ of 6%.)
7. \$ 270 for 3 yr. 10 mo. at $1\frac{1}{2}$ %. ($\frac{1}{4}$ of 6%.)
8. \$ 382 for 1 yr. 9 mo. at 2%. ($\frac{1}{3}$ of 6%.)
9. \$ 927 for 6 mo. 4 da. at 3%. ($\frac{1}{2}$ of 6%.)
10. \$ 864 for 2 mo. 7 da. at 4%. (6% - 2%.)
11. \$ 318 for 1 mo. 13 da. at 5%. (6% - 1%.)
12. \$ 725 for 29 da. at 7%. (6% + 1%.)
13. \$ 649 for 67 da. at $7\frac{1}{2}$ %. (6% + $1\frac{1}{2}$ %.)
14. \$ 84 for 54 da. at 8%. (6% + 2%.)

THE ONE DOLLAR METHOD

1. What two methods of computing interest have previously been resented?

2. In which one did you first find 6% interest?

NOTE. *Any method in which the interest at 6% is first found may be called a "6% method."* 6% is a very usual rate of interest. It is the legal rate in 30 or more states. It is the rate that is understood when no other is mentioned.

3. What is the interest of \$1 for 1 year at 6%? For 2 mo.? For 6 days?

At 6% the interest of \$1 for:—

| | |
|----------|------------|
| 1 year | = \$ 0.06 |
| 2 months | = \$ 0.01 |
| 6 days | = \$ 0.001 |

4. Find the interest of \$1 for 2 yr. 8 mo. 18 da.

Interest for 2 yr. = \$ 0.12

Interest for 8 mo. = 0.04

Interest for 18 da. = 0.003

Interest for 2 yr. 8 mo. 18 da. = \$ 0.163

5. Compared with the interest of \$1, what will be the interest of \$75? Of \$183? Of \$240? Of \$360?

1. I hire \$ 48.96 at 7% for 3 yr. 7 mo. 19 da. What shall I pay on settlement?

2. *Explain the four steps in the process:—*

I. Finding the interest of \$ 1 at 6%.

II. Finding the interest of the given principal at 6%.

III. Finding interest at the given rate.

IV. Finding the amount.

3. What will discharge a debt of \$ 475 which has been drawing 5% interest for 2 yr. 11 mo. 24 da.?

4. Find the amount of \$ 7000 at 4% for 3 yr. 3 mo. 13 da.

5. I hold two notes of \$ 731 each, one at 5% interest, the other at 8%. They have been running 4 yr. 8 mo. 17 da. What shall I receive at settlement?

| PROCESS | |
|-------------------------|---|
| Interest of \$ 1 at 6%. | |
| For 3 yr. = | \$ 0.18 |
| For 7 mo. = | 0.035 |
| For 19 da. = | 0.0031 $\frac{1}{2}$ |
| I. | <u>\$ 0.218$\frac{1}{2}$</u> |
| | 48.96 |
| | <u>816</u> |
| | 39168 |
| | <u>4896</u> |
| | 9792 |
| II. 6) | <u>\$ 10.68144</u> at 6% |
| | 1.7802 at 1% |
| III. | <u>\$ 12.46</u> at 7% |
| | 48.96 = Prin. |
| IV. | <u>\$ 61.42</u> = Amount. |

Find the amount under the following conditions:—

6. Principal, \$ 84.75; rate of interest, 4%; time, 3 yr. 15 da.

7. Principal, \$ 942; rate of interest, 5%; time, 4 yr. 1 mo. 7 da.

8. Principal, \$ 193; rate of interest, 7%; time, 18 mo. 27 da.

9. Principal, \$ 64.50; rate of interest, 8%; time, 5 yr. 5 mo. 5 da.

10. Principal, \$ 712.10; rate, 9%; time, 7 yr. 4 mo. 29 da.

11. 4 yr. 6 mo. 21 da. \$ 425.50, 3%.

12. What is the difference between the interest of \$ 810 for 5 yr. 7 mo. 6 da. at 6%, and the interest of \$ 865 for 5 yr. 7 mo. 6 da. at 6%? Use a short method.

13. What is the interest of \$ 5000 for 6 yr. 4 mo. at 4%?

1. Find the time from June 24, 1903, to Aug. 13, 1907.

FIRST PROCESS

Explain the process.

1907 yr. 8 mo. 13 da.

1903 yr. 6 mo. 24 da.

4 yr. 1 mo. 19 da.

NOTE. This method is generally used when 30 da. is considered a month. When the exact number of days is wanted, the method is shown below.

SECOND PROCESS

A

B

From June 24, 1903 to June 24, '07 = 4 yr., or 6/03 to 6/07 = 4 yr.

From June 24, '07 to July 24, '07 = 1 mo., or 6/07 to 7/07 = 1 mo.

From July 24, '07 to Aug. 13, '07 = 20 da., or 7/24 to 8/13 = 20 da.

2. Find the time (exact) from Sept. 14, 1902 to Mar. 11, 1906.

Explain each process.

PROCESS A

PROCESS B

From Sept., '02 to Sept., '05 = 3 yr., or 9/02 to 9/05 = 3 yr.

From Sept. 14 to Feb. 14 = 5 mo., or 9/14 to 2/14 = 5 mo.

From Feb. 14 to Mar. 11 = 25 da., or 2/14 to 3/11 = 25 da.

3. What advantage has Process B over Process A?

4. Why is it well to know the months by their numbers as well as names?

5. In Process B, what is found at the left of the inclined line? At the right of it?

PROCESS

8/1769 to 8/1820 = 51 yr.

8/15 to 4/15 = 8 mo.

4/15 to 5/5 = 20 da.

6. Napoleon was born Aug. 15, 1769, and died May 5, 1821. How long had he lived?

7. Find the time from May 22, 1898, to June 12, 1904.

8. Find the time from Dec. 25, 1901, to Mar. 3, 1905.

9. How long from 4/18, '04, to 3/11, '07?

10. Find your exact age to-day

11. Find the time from May 9, '03, to June 12, '05.

12. How long to-day since Aug. 20, 1903?

Three methods of computing interest : —

- I. A general method, page 146.
- II. The bankers' method, page 148.
- III. The one dollar method, page 149.

Any of these three methods may be used exclusively, but as no one method is always the best, it is well to learn to choose the one that will give an accurate result most quickly.

Solve the following problems by each method, compare results, and tell which method you prefer, and why.

1. Find the interest of \$360 at 7% for 207 da.
2. What is the amount of \$75 at 8% for 3 yr. 4 mo.?
3. What shall be paid for the use of \$723.70 for 85 da. at 10% interest?

What is the interest under the following conditions?

| PRINCIPAL | TIME | RATE | PRINCIPAL | TIME | RATE |
|-----------|---------|------|--------------|--------------------|------|
| 4. \$648 | 111 da. | 4%. | 9. \$432. | 1 yr. 8 mo. 7 da. | 3%. |
| 5. \$324 | 167 da. | 5%. | 10. \$767.80 | 3 yr. 11 mo. 9 da. | 5½%. |
| 6. \$750 | 200 da. | 9%. | 11. \$ 50.40 | 10 mo. | 4½%. |
| 7. \$427 | 93 da. | 12%. | 12. \$ 87.75 | 114 da. | 6%. |
| 8. \$865 | 48 da. | 4%. | 13. \$137.77 | 4 yr. 9 mo. 25 da. | 7½%. |

14. Interest is the product of what three factors?
15. Which method of finding interest is best when principal, rate, or time is divisible by 4? By 4½? 6? 9 or 12? Why?
16. Which method uses the aliquot parts of the time?
17. Which are "6% methods"? Why so called?
18. Which is the best method when there are years, months, and days in the time, and when cancellation is impossible?
19. What is the interest of \$400 at 10% for 2½ yr?

1. In computing interest for parts of a year we commonly consider 30 days a month and 360 days a year. In taking $\frac{1}{360}$ of a year's interest to find the interest for 1 day, do we take too much or too little, considering the actual length of a year?

2. **Exact or accurate interest** is reckoned for the actual number of days in the given time, and counting 365 days to the year. It is used by the United States government and sometimes by others in business transactions. It differs from common interest only as applied to *parts of a year*. What part of a year is August? February? The last three months of 1896?

3. $\frac{1}{365}$ is what part of $\frac{1}{360}$? Explain this process:—

$$\frac{1}{365} \div \frac{1}{360} = \frac{1}{365} \times \frac{360}{1} = \frac{360}{365} = \frac{72}{73}.$$

4. If 1 day's accurate interest is $\frac{72}{73}$ of 1 day's common interest, what is the accurate interest when the common interest is \$146?

5. If from the common interest I deduct $\frac{1}{73}$ of itself I shall have the exact interest. Explain.

Common interest decreased by $\frac{1}{73}$ of itself is exact interest.

6. Find the accurate interest of \$500 for 90 days at 4%.

7. Find the exact interest of \$1000 at 3% from May 9 to Sept. 4.

NOTE. The exact number of days must be found; that is, 22 + 30 + 31 + 31 + 4 = 118.

Find the exact or accurate interest of—

8. \$800; 6%; Aug. 11 to Oct. 9.

9. \$720; 8%; Jan. 4 to Mar. 15.

10. \$1200; 3 mo. 12 d.; 5%.

11. \$1500; 72 d.; 10%.

12. What is the exact interest of \$1000 for 2 yr. 249 d. at 6%?
(Find common interest for 2 yr. + exact interest for 249 d.)

NOTE. The method to be employed in the solution of the following problems is shown by the Roman numerals I, II, III (page 152).

1. What is the interest of \$840 for 9 mo. 17 da. at 4%? I.
2. Find the amount of \$722 for 156 da. at 12%. II.
3. What will settle an account of \$425 that has been drawing interest of 5% for 5 yr. 5 mo.? III.
4. May 17, 1903, I borrowed \$248 at $2\frac{1}{2}\%$. Aug. 15, 1905, how much interest had accrued? III.
5. In $4\frac{1}{2}$ yr. how much will be received on a \$5000 railroad bond paying 2% semi-annually? I.
6. May 27, 1904, I paid a note of \$475.28 that had been drawing 4% interest since Dec. 31, 1900. III.

Find by inspection the best method of solving the following problems, and use it in finding the interest. Try to forecast the result:—

7. \$90,000 on interest 7 mo. 24 da. at 4%.
8. \$728 draws 5% interest for 20 months.
9. $3\frac{1}{2}\%$ interest of \$900 from Jan. 15 to Nov. 2.
10. Principal, \$72.59; time, 125 days; rate, $12\frac{1}{2}\%$.
11. What shall I pay for the use of \$500 for 50 days at 5%?
12. \$320; $7\frac{1}{2}\%$; July 7, 1898, to Aug. 4, 1902.
13. \$720; 8%; Oct. 19, 1900, to May 11, 1903.
14. \$472 3% 112 da. 20. 3% \$872 4 yr. 8 da.
15. \$648 $4\frac{1}{3}\%$ 8 mo. 21. 5% \$5000 16 mo.
16. \$800 $2\frac{1}{2}\%$ 180 da. 22. $2\frac{1}{2}\%$ \$178.91 104 da.
17. \$2000 7% $19\frac{1}{2}$ mo. 23. 1% \$3294 $17\frac{5}{8}$ mo.
18. \$4000 4% $4\frac{2}{3}$ yr. 24. $4\frac{1}{2}\%$ \$700 $41\frac{2}{3}$ yr.
19. \$950 9% 248 da. 25. 8% \$64.87 295 da.

Find amount as required: —

| | PRINCIPAL | TIME | | | RATE | | PRINCIPAL | TIME | | | RATE |
|-----|-----------|------|-----|-----|------|--|-----------|-----------|-----|-----|------|
| | | Yr. | Mo. | Da. | | | | Yr. | Mo. | Da. | |
| 1. | \$ 485.75 | 2 | 4 | 18 | 6 | | 11. | \$ 478.95 | 3 | 5 | 4½ |
| 2. | \$ 147.50 | 1 | 6 | 19 | 7½ | | 12. | \$ 25,648 | 3 | 23 | 12 |
| 3. | \$ 64.90 | 1 | 11 | 27 | 5 | | 13. | \$ 54.50 | 6 | 9 | 16 |
| 4. | \$1298.25 | | 3 | 20 | 4½ | | 14. | \$ 29.78 | 2 | 8 | 5 |
| 5. | \$2576.50 | | 4 | 26 | 3 | | 15. | \$ 846.20 | 10 | 21 | 10 |
| 6. | \$ 832.60 | 1 | 5 | 15 | 7 | | 16. | \$9724.70 | 4 | 29 | 9 |
| 7. | \$5620.75 | | 2 | 19 | 3½ | | 17. | \$256,780 | 6 | 13 | 8 |
| 8. | \$ 75.40 | 5 | 7 | 17 | 5 | | 18. | \$ 45.25 | 4 | 1 | 24 |
| 9. | \$ 12,760 | | 1 | 25 | 4 | | 19. | \$ 738.20 | 2 | 6 | 22 |
| 10. | \$ 255.80 | | 8 | 7 | 5½ | | 20. | \$1287.75 | 1 | 1 | 14 |

21. What is the interest of a note for \$275, dated July 27, 1902, and paid May 12, 1903, bearing interest at 7%?

22. What must I pay Nov. 17, 1903, to take up my note for \$387.60 given Mar. 25, 1901? Interest at 5%.

23. What is the value, on the 5th of Jan., 1904, of a 7½% note for \$475.50, dated June 18, 1902?

24. What must I pay on the 8th of December, 1904, to redeem my note for \$360.75, dated Feb. 19, 1901? Interest at 6%.

25. How much is due July 20, 1905, on a note for \$725, given Oct. 6, 1898, with interest at 5%?

26. Find interest of \$7685, Apr. 25 to Nov. 11, 1905, at 4½%.

27. What is the amount of \$2850 investment at 12% from July 2, '03, until Dec. 25, '03?

28. I took a 10% mortgage of \$6000 on Feb. 28, '01. How much interest shall I receive by Nov. 30, '05?

29. What is the interest of \$375.50 at 8½% for 3 yr. 6 mo. 29 da.?

30. Interest on \$1775.80 invested for 5 yr. 7 mo. 15 da. at 4½%.

| | PRINCIPAL | RATE % | MO. | DA. |
|----|-----------|--------|-----|-----|
| a. | \$4500 | 4½ | 1 | 1 |
| b. | 7695 | 1 | 2 | 2 |
| c. | 8910 | 2 | 3 | 3 |
| d. | 520 | 2½ | 4 | 4 |
| e. | 750 | 6 | 5 | 5 |
| f. | 896 | 3½ | 6 | 6 |
| g. | 4000 | 4 | 7 | 7 |
| h. | 487 | 4½ | 8 | 8 |
| i. | 384 | 5 | 9 | 9 |
| j. | 4835 | 5½ | 10 | 10 |
| k. | 650 | 3 | 11 | 11 |
| l. | 8496 | 6½ | 12 | 12 |
| m. | 700 | 7 | 13 | 13 |
| n. | 576 | 7½ | 14 | 14 |
| o. | 648 | 3 | 15 | 15 |
| p. | 567 | 8 | 16 | 16 |
| q. | 794 | 9 | 17 | 17 |
| r. | 2345 | 10 | 18 | 18 |
| s. | 678 | 12 | 19 | 19 |
| t. | 9150 | 3½ | 20 | 20 |
| u. | 375 | 4½ | 21 | 21 |
| v. | 829 | 7½ | 22 | 22 |
| w. | 1475 | 5 | 23 | 23 |
| x. | 645 | 4 | 24 | 24 |
| y. | 486 | 3¼ | 25 | 25 |
| z. | 1728 | 15 | 26 | 26 |

| A | B | C |
|----------|------|-------|
| 1. also | arms | tough |
| 2. bank | boat | shoes |
| 3. city | crow | watch |
| 4. down | dust | heavy |
| 5. edge | elms | sober |
| 6. flax | flew | drunk |
| 7. gold | grow | roses |
| 8. hunt | hate | anger |
| 9. jerk | just | knoll |
| 10. king | kept | parch |
| 11. lazy | lisp | petal |
| 12. most | mend | shone |
| 13. nail | nice | black |
| 14. ours | oven | thief |
| 15. pity | pink | wedge |
| 16. rung | rock | vowel |
| 17. self | very | tulip |
| 18. trim | ugly | union |
| 19. yarn | lynx | tansy |
| 20. whom | zeal | gnash |

EXPLANATION. Each word above furnishes an exercise in interest, the letters standing in order for principal, rate, and time in months and days. Thus, "also" asks for the interest of \$4500 at 4½% for 19 mo. 15 da. Other words may be used at will.

Find interest from conditions in:—

- | | |
|------------------------------------|--------------------------------------|
| 1-20. Col. A. | 81-100. Col. A, spelling backwards. |
| 21-40. Col. B. | 101-120. Col. B, spelling backwards. |
| 41-60. Col. C, first four letters. | 121-147. The table taken by lines. |
| 61-80. Col. C, last four letters. | |

1. Show the difference between grower or producer, importer, wholesaler, retailer.

2. Do you buy from wholesalers or retailers?

3. With whom do wholesalers have to deal?

It is the general custom of wholesale dealers, manufacturers, and publishers to fix a price on their goods and then allow a certain per cent **discount** from this price to "the trade"; that is, to retail dealers handling their kind of goods.

4. Pear trees are listed at \$1.50 each. As I am dealing in trees I buy from the nursery at \$12 per dozen. What *discount* or reduction do I get on each tree? What per cent is this of the **list price**?

5. A man is trusted for goods billed at \$100. He is to pay in 3 months. How long is the **term of credit**?

6. The dealer offers to sell the same goods for \$98 *cash*. Why is this? How much does he *discount for cash*? What per cent of the list price is this?

7. The price of knives per dozen is \$2. By buying 30 dozen I get them for \$50. What discount did I get by buying in the larger quantity? What per cent was this?

8. If I had bought 100 dozen the *net price*, or what I actually paid, would have been \$1.60 per dozen. Is this a larger or smaller rate of discount than offered on 30 doz.? Why is this?

9. The discount on a car load of coal is 10%, or \$8. What is the gross price of the car of coal?

10. The list price of a bill of goods is \$40. The net price was \$32. What was the discount? What per cent was this of the list price?

11. The list price of hats was \$36 a dozen. The retailer sold them at \$4 each and doubled his money. What rate of discount did he receive on the list price?

1. A \$4000 house is offered at \$3500 cash. What discount was offered for cash? What per cent is this?

2. Bought \$200 worth of flour. If I need not pay for 6 months, what do I save by not paying until the end of 6 months?

3. Which customer should receive the larger discount, one who pays cash, one who pays in 3 months, or one who pays in a year?

4. I buy \$400 worth of goods. The dealer does not require me to pay for 6 months, but gives me a *time discount* of 2% if I pay within 3 months from the date of purchase, or 4% *cash discount* if I pay cash. What will the goods cost me if I pay cash? If I pay within 3 months? If I pay at the end of 6 months?

5. On the bill heads of wholesale houses there is usually a note showing what discounts are allowed. Thus, "Terms: 60 days, 2% 10 days." Explain what is meant.

6. Explain: "Terms: 90 da., 4% 30 da., 6% 10 da."

7. I bought \$400 worth of goods May 1. "Terms: 3 mo. or 4% off 60 da. or 10% off 30 da." What will settle the bill May 28?

Discount is always a part, or per cent, of the price which it reduces.

Written

Find the missing terms:—

| | AMT. OF BILL | % OFF FOR CASH | NET COST | | DISCOUNT FOR CASH IN 30 DAYS | LIST PRICE | % |
|---|--------------|----------------|----------|----|------------------------------|------------|----------|
| 1 | \$2000 | 5 | <i>x</i> | 8 | \$24 | \$600 | <i>x</i> |
| 2 | \$900 | <i>x</i> | \$810 | 9 | <i>x</i> | \$150 | 1 |
| 3 | <i>x</i> | 2 | \$490 | 10 | \$20 | <i>x</i> | 4 |
| 4 | \$1200 | 6 | <i>x</i> | 11 | \$36 | \$1800 | <i>x</i> |
| 5 | \$1850 | <i>x</i> | \$1786 | 12 | <i>x</i> | \$1400 | 5 |
| 6 | <i>x</i> | 5 | \$1900 | 13 | <i>x</i> | \$2200 | 2 |
| 7 | \$2400 | <i>x</i> | \$2000 | 14 | \$18 | \$900 | <i>x</i> |

It frequently happens that wholesale merchants send out expensive catalogues containing a description of their goods, in which is quoted a certain *list price*. The retail dealer gets a discount from this list price. When a further discount can be given, it is the custom to quote another discount that is to be taken from the former discounted price, rather than a new discount on the list price.

1. I buy a bill of goods listed at \$ 400. The discounts are 25% and 10%. What is the net price?

Explain: $\$ 400 - 25\% \text{ of } \$ 400 = \$ 300$; $\$ 300 - 10\% \text{ of } \$ 300 = \$ 270$.

2. I buy an organ listed at \$ 150. Being a music teacher, I get a discount of 20%, and a further discount of 5% for cash. What does the organ cost me?

3. I am offered my choice of 10% discount, or two successive discounts of 5% each on a bicycle listed at \$ 100. How much shall I gain by taking the better offer?

4. I can buy \$ 480 worth of pressed bricks of Henry Clay & Co. at the discounts of 15% and 5%. Another firm offers me two 10% discounts. Shall I accept the first or the second offer? Why?

5. I receive the following estimates for a miscellaneous lot of hardware:—

\$ 476 with three 5% discounts.

\$ 508 with two 10% discounts.

\$ 492 with a 15% and a 5% discount.

Which is the best offer?

Find net prices:—

| LIST | % OFF | LIST | % OFF |
|-------------|------------|---------------|----------------|
| 6. \$ 15.40 | 20 then 5. | 9. \$ 14.85 | 60, 10, and 2. |
| 7. \$ 49.50 | 50 then 2. | 10. \$ 320.15 | 20, 5, and 1. |
| 8. \$ 600 | 45 then 3. | 11. \$ 4000 | 30, 12, and 3. |

1. One buyer gets 30% off, another gets 25% and 5%. Give net cost to each on a shipment of \$ 2000 gross value.

2. A furniture maker allows 15% from the list price. Find the net cost on an order amounting to \$ 12,438, including \$ 118 for carting, which is without discount.

3. Tubing listed at \$ 10,000 is billed less 60% and 2% for cash. Net cost = \$ x .

4. The trade discount on certain goods is 70%. Large buyers receive a second discount of 10%, making the total discount $x\%$.

5. List price, \$ 500; net, \$ 425; discount, \$ x ; rate, $y\%$.

6. List price, \$ 488.90; net, \$ 391.12; discount, \$ x ; rate, $y\%$.

7. 1800 ft. of moulding at 20¢, less 12% to the trade and 1% for cash, cost \$ x .

8. A shipment of sugar invoiced at \$ 11,000 is subject to a rebate or reduction of 5%. Terms: 15 days. $\frac{1}{2}\%$ off for cash makes the net cost \$ x .

9. The discounts on a \$ 1000 invoice are, not 45%, but 30%, 10%, and 5%. Find the net cost.

10. On an invoice of flour amounting to \$ 648, I receive three successive discounts of 5%. What was the net cost of the flour?

11. A dealer in agricultural implements sends me his price list, and offers discounts of 40, 15, 7, and 5%. I order a bill amounting to \$ 1500 according to the price list. What shall I actually pay?

12. I order of the White Publishing Co.:—

13 books listed at \$ 1.50

10 books listed at 1.75

15 books listed at 1.10

1 set Dickens at \$ 18, and 12 vols. at 90¢.

I receive a dealer's discount of $16\frac{2}{3}\%$ and 2% for cash. Find the net cost

1. If the owners of a hundred ships agree to share the loss if one is wrecked, who might profit by the arrangement?

2. Mr. Smith owns a \$3000 house. By spending \$30 he can be sure that $\frac{3}{4}$ of his loss by fire will be made good. Many others do the same and from their money losses are paid. If Mr. Smith's house is burned, what will he receive?

3. An Insurance Company agrees to pay part of the loss of those having paid a certain per cent or premium on the insurance of their property. What will it cost a man to insure for \$250 at $1\frac{1}{4}\%$?

4. 100,000 persons pay 25¢ each to an accident insurance company. If it pays \$15,000 in claims for injuries, and \$4000 for expenses, the profit is \$x. Who is the insurer? Who are insured?

5. The agreement to make good a loss on certain conditions is printed in a policy made by the underwriter or insurer, and held by the insured. The cost of a \$25,000 policy at $\frac{3}{4}\%$ a year is \$x.

6. By paying an annual premium a person may be assured that at his death or at a certain age, his family, or he himself, will receive a certain sum. How will this money have been obtained?

7. A ship, costing \$210,000 is insured for $\frac{3}{4}$ of its value at 2%. If lost, the owner receives \$x. The underwriters lose \$y.

8. \$40 pays for five years' insurance on a brick store which cost \$5000. The insurance valuation is \$4000. What is the annual rate?

9. A wooden tenement house two miles from a fire-engine is insured for $\frac{3}{4}\%$, but only for one year. If the valuation is \$4000, the cost for five years is \$x. The property insured is called a risk. Compare the last two risks.

10. Insurance provides for sharing loss due to what causes?

11. A schoolhouse is insured for five years at $\frac{1}{2}\%$ premium, which is \$300. The insurance valuation is $\frac{3}{4}$ of the cost of the house. What is the underwriter's loss if it burns?

1. A stock of goods is worth \$12,000. The premium for a year is 1%, or \$100. What is the insurance valuation? If destroyed what will the underwriters pay? What will the owner lose besides the premium?

2. Why is property usually insured for less than its full value? A \$7500 house is insured at $1\frac{1}{4}\%$ for \$62.50. What was the insurance valuation?

Supply values for x:—

| INSURANCE | RATE % | PREMIUM | INSURANCE | RATE % | PREMIUM |
|-------------|---------------|---------|-----------|----------------|---------|
| 3. \$15,000 | x | \$200 | 6. x | $\frac{1}{4}$ | \$10 |
| 4. \$7000 | x | \$84 | 7. \$7500 | $1\frac{1}{4}$ | x |
| 5. x | $\frac{7}{8}$ | \$60 | 8. \$4740 | $\frac{3}{8}$ | x |

9. A factory worth \$60,000 is insured for $\frac{3}{4}$ its value at $1\frac{1}{8}\%$. The possible loss to the owner, including the premium, is \$ x .

10. \$175 was the premium on $\frac{3}{4}$ the value of a collection of paintings, at 1% a year. What was the value of the paintings?

11. \$1960 is 98% of the insurance. \$40 was the premium. What was the rate of premium?

12. A house was insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$ premium, the premium being \$27. Required the value of the house.

13. The cargo of the *Neptune* is insured for $\frac{3}{4}$ of its value, at $\frac{3}{8}\%$. What is the value of the cargo? The premium paid is \$3750.

14. At the age of 32 a gentleman insures his life for \$8000. The annual premium is \$120. If he dies at the age of 44, what per cent of the insurance will he have paid as a premium?

15. A manufacturing company pays an average premium of $\frac{7}{10}\%$ on its plant. In the annual report of expenses, \$280 is entered as cost of insurance. What is the value of the plant if insured for 80% of its value?

16. A manufacturer pays \$800 premium, at $\frac{3}{4}\%$. The property is partly destroyed by fire, and the adjuster settles with him for 25% of the insurance. What does he receive?

Selling through an Agent

Farmers, fruit growers, and other producers, as well as the merchants of the cities and towns away from commercial centers, usually ship their *produce* to **agents** or **commission merchants** in some city, rather than take it there themselves and then seek a buyer.

When an agent sells goods for another without actually receiving the goods, he is usually called a **Broker**.

The commission merchant usually charges a per cent of the amount sold as his fee or **commission**.

The one who sends the merchandise to be sold is the **principal**, **shipper**, or **consignor**.

1. I send a shipment of berries to a commission merchant in Chicago. He sells them for \$100. His rate of commission is $1\frac{1}{2}\%$. What does he keep, and what does he send to me?

2. A commission merchant received 3000 melons which he sold for 50¢ each. What was his commission at 2%?

3. My agent, a commission merchant in New York, charges me 5% for selling a *consignment* of fruit. What were the sales if his commission was \$20?

4. I receive \$14 for selling \$700 worth of flour. What was my rate of commission?

5. I sold through a commission merchant 100 bu. of potatoes at 50¢ per bushel, allowing my agent 2%. What were my *net receipts* from the sale?

6. I shipped 100 baskets of peaches to my agent. He sold them at \$1 per basket. His commission was 5% and the freight twice as much. What did I receive from the sale?

7. A commission merchant charged me 15¢ a crate for selling strawberries. What per cent was this if they sold for \$3.00 per crate?

8. My commission at 3% was \$18. What were the sales?

Selling through an Agent

1. How much does a broker receive for selling 1500 barrels of pork at $\frac{3}{4}\%$? The pork sells at \$7.83 per barrel.

2. A dealer buys 1000 doz. of eggs at 13¢, and sells them through a commission merchant at 19¢. What were his profits if the commission was 2%? Expressage \$3.40.

3. A western merchant received a commission of 20% for selling harvesting and mowing machines. During one season he sold 6 harvesting machines at \$115 each, and 4 mowing machines at \$85 each. The cost of selling and delivery was \$37.50. What was his net gain?

4. Sold goods at auction, amounting to \$11,450. The charges were: commission, $2\frac{1}{2}\%$; storage, $1\frac{1}{2}\%$; and advertising, \$21. What were the net receipts?

What is the rate of commission when an agent receives:—

5. \$100 on \$15,000?

9. \$4.50 on \$300?

6. \$29.43 on \$981?

10. \$128 on \$896?

7. \$1517.92 on \$85,896?

11. \$21.50 on \$615?

8. \$112.50 on \$4500?

12. \$2.58 on \$86?

13. An agent received \$54.50, including \$4.50 for surveying, deed, etc., for selling a house valued at \$2500. What was the rate of his commission?

14. A purchased 350 bu. of corn at \$1 per bushel, and B sold it at a commission of 3%. If the selling price was \$1.15 per bushel, what were B's commission and A's profit?

15. The selling price was \$6000, the commission was \$240, and the other costs of the sale were \$18; the net gain was \$742. What was the rate of commission? The cost?

16. During the coal strike of 1902, a man sold 760 tons of coal as follows: 10% of the whole amount at \$12 per ton, 25% of it at \$11 per ton, and the remainder at \$10 per ton. *Brokerage* on the whole amount 2%. What were his net receipts?

Buying through an Agent

1. If I employ an agent, or **correspondent**, to buy goods for me, I must pay him a per cent of whatever amount he expends for my benefit. If he buys \$1000 worth of corn for me at 3% commission, what must be my **remittance** to him for the corn and his commission?

2. An agent buys a horse for me at \$200. His commission is 2%. How much money shall I send him?

3. My agent buys 250 bbls. of cement for me at \$2 per barrel. What is the amount of my remittance to him if his commission is 1%?

4. I telegraph my agent to buy 10,000 bushels of wheat at 90¢. I allow him 2%. How large a check should I send him to cover the whole transaction?

5. An agent netted \$80 per month after paying \$5 for office expenses. What were his sales if his rate of commission was 2%?

6. A collector sends his principal \$180. He collected \$200. What rate did he charge?

7. A boy earns \$6 per week collecting gas bills at $\frac{1}{2}\%$. What are the net proceeds to the gas company?

8. An agent buys \$2000 worth of copper, charging 1% commission. What shall the principal remit to the agent to pay for the copper and his commission?

9. I remitted \$102.50 for buying umbrellas at $2\frac{1}{2}\%$. What had the agent spent for umbrellas?

10. An agent charges $1\frac{1}{2}\%$ for buying chair stock. How large a check shall I send him if I wish him to buy \$1000 worth of stock for me?

11. A broker buys pork at 1% commission. How much did he buy if his commission was \$17.50?

12. I remit my agent \$2040 for buying \$2000 worth of hay for me. What was his rate of commission?

1. A remittance of \$669.50 is the purchase price of wool, and a commission of 3%. What is the commission?

2. I have remitted to an agent \$1332.50 for flour at \$6.50 per barrel. His commission was $2\frac{1}{2}\%$. How many barrels did he buy?

3. A broker received a sum of money to expend, after deducting his brokerage of $1\frac{1}{4}\%$. He spent \$2400. What was his brokerage?

4. An agent received 5% for selling wool. His commission amounted to \$208.50. How much did he pay for the wool?

5. Sent a broker in Minneapolis a draft for \$603.75 for the purchase of flour. Brokerage 5%, and the flour \$5 a barrel. How many barrels were bought?

6. I received a remittance of \$2639 for a bill of coal and my commission of $1\frac{1}{2}\%$ for buying. Required my commission.

7. What are the net proceeds from a sale of 1000 tons of hay at \$11.75 per ton? Commission $1\frac{1}{2}\%$.

8. My agent sells cotton at a commission of $5\frac{1}{2}\%$. He remits to me \$2929.50. What was his commission?

9. My agent sells 500 bbls. of flour at \$10 a barrel, and sends me a check for \$4750. What rate of commission did he charge?

10. Received a consignment of 3000 bu. of corn, with instructions not to sell for less than 56¢, and that all received over 60¢ was to cover the commission. I sold at 65¢. The rate of commission was $x\%$.

11. The remittance for the purchase of lumber was \$3668.25. The rate of commission was $\frac{1}{2}\%$. How many thousand feet of lumber were bought at \$36.50 per M.?

12. A promoter received \$50,000 for effecting a combination of furniture manufacturers whose united capital was \$1,800,000. What was the rate of commission paid for the promoter's services?

Discount, Commission, and Insurance

1. On a bill for hardware amounting to \$480 I received four successive discounts of 10% each. What was the amount paid?

2. My residence is insured for $\frac{3}{4}$ of its value in the Provident Insurance Co. at $\frac{1}{2}\%$. The premium is \$40. What is the value of my property?

3. My agent in Mobile bought 40,000 lb. of cotton at $9\frac{1}{8}\phi$. His commission is $\frac{3}{4}\%$ and his expenses are \$143.75. What shall I remit him?

4. A real estate broker sells a farm for \$8000 at a 5% commission. What are the net proceeds of the sale, and what is his commission?

5. I can buy 1000 bbl. of oil at \$1.12 $\frac{1}{2}$ with 3% off in 30 days, or 5% off for cash. What shall I save by accepting the better offer, money being worth 6%?

6. The estimated loss of property at a large fire was \$275,000. The insurance received was \$180,000. How much must be taken in new risks at an average of 2% to cover this loss to the underwriters together with \$5000 expenses?

7. I receive from my agent in London a draft for \$3860, the net proceeds of a sale of flour at $3\frac{1}{2}\%$ commission. What were the gross proceeds?

8. A salesman earns \$2500 annually. \$1000 is a guaranteed salary; the remainder is his commission of 5%. What are his annual sales?

9. A broker negotiates a loan of \$6500 on a real estate mortgage. His commission is 2%, and the expenses of examining title, etc., are \$72.37. What does the mortgager receive?

10. Bought 1000 gross of screws at 27 cents, with a discount of 15, 10, and 5. I sold the lot at cost plus 30%. What was my gain?

Supply values for x and y (dealing with selling agents):—

| | GROSS
PROCEEDS | EXPENSES | RATE OF
COMMISSION | COMMISSION | NET PROCEEDS |
|----|-------------------|----------|-----------------------|------------|--------------|
| 1. | \$ 437 | 0 | 2% | x | y |
| 2. | x | \$ 47.50 | 1% | \$ 27.50 | y |
| 3. | \$ 250 | x | y | \$ 25 | \$ 200 |
| 4. | x | 0 | 2% | y | \$ 900 |
| 5. | \$ 1200 | \$ 100 | x | \$ 18 | y |
| 6. | \$ 1680 | x | 4% | y | \$ 60 |
| 7. | x | 0 | y | \$ 13.52 | \$ 437.18 |

8. \$ 430 is received from a sale of linen. After retaining $1\frac{1}{2}\%$ commission and paying \$ 2.50 for advertising the sale, what is the balance to be remitted?

9. What value of goods can be bought on 5% commission from a remittance of \$ 577.50, allowing \$ 24.50 for advance charges of forwarding the purchase?

10. A correspondent retains $4\frac{1}{2}\%$ on the receipts of a certain sale, and after paying \$ 4.37 for carting, etc., remits \$ 200. The gross receipts include \$ x + \$ y + \$ z .

11. A dealer sends to his agent \$ 20,500. This includes a commission of 2% on what the agent will spend, and \$ 100 for insurance and sundries. He spends the balance, \$ x , for wool.

Supply values for x and y (dealing with purchasing agents):—

| | REMITTANCE | AM'T OF PURCHASE
+ FREIGHT, ETC. | RATE OF
COMMISSION | COMMISSION |
|-----|------------|-------------------------------------|-----------------------|------------|
| 12. | \$ 595.80 | x | y | \$ 6.60 |
| 13. | \$ 179.76 | \$ 148.30 + \$ 27.94 | x | y |
| 14. | x | \$ 755 | y | \$ 22.65 |
| 15. | x | y | 5% | \$ 9.90 |
| 16. | \$ 1293.75 | x | $3\frac{1}{2}\%$ | y |
| 17. | x | \$ 684.10 + \$ 88 | $2\frac{1}{4}\%$ | y |

1. Aug. 15, 1903, Edward Hale lends to George Sharp \$ 500 to be repaid when the lender asks it, together with interest at 5 %; as evidence of the loan and security for its payment, the lender receives from the borrower a **promissory note** like the following: —

| | |
|--|---------------------------------|
| \$ 500--- | Chicago, Ill., August 15, 1903. |
| On demand, after date, I promise to pay to the | |
| order of----- | Edward Hale----- |
| Five Hundred----- | Dollars |
| with interest at five per cent. | |
| Value received. | George Sharp. |

2. Who is the **maker**, or the one who promises to pay ?
3. Who is the **payee**, or the one to whom the promise is made ?
4. What is the **face** of the note, or the sum named in it ?
5. When is the note **payable** ?
6. Why is such a note called a **demand note** ?
7. Why is it called an **interest-bearing note** ?
8. A promissory note is a kind of property, and may be bought and sold like other property, and hence is called **negotiable paper**.
9. Whenever the payee of a note transfers it to the ownership of another person he first **indorses** it; that is, he places his signature on the back of it. Who would be the **indorser** of this note should it be sold ?
10. Write a note to fulfill the following conditions: maker, E. L. Price; payee, A. T. Holmes; face, \$ 300; rate of interest, 6 %; demand note dated Chicago, Sept. 26, 1904. Indorse it.

A*Edward Hale.***B***Pay to the order of
Henry Hall.**Edward Hale.***C***Without recourse.**Edward Hale.*

11. The payee of a note may *indorse it in blank* as in A, or he may make a *special indorsement*, as in B.

12. A **blank indorsement** makes a note payable to the holder. A **special indorsement** makes it payable to the person named by the indorser as payee. Copy and indorse the note on page 169.

13. Every indorser of a note is responsible for its payment unless the words "without recourse" precede his signature, as in C.

14. If the holder of the note on the preceding page demands payment on Nov. 15, 1903, what amount does Mr. Sharp owe him?

15. What will Mr. Hale receive if he demands payment on Feb. 15, 1904?

16. Write a demand note from the following data: face, \$ 300; date, Sept. 20, 1904; maker, H. E. King; payee, A. L. Hayes; place where you live; rate, 6%.

17. Indorse the note you have written.

18. Suppose the note sold to H. E. Smith; write a special indorsement.

19. Suppose Mr. Hayes is not to be held responsible for the payment in case that Mr. King cannot pay Mr. Smith; write the indorsement.

20. What is due on the note March 20, 1905? To whom is it due? Who pays it?

21. If the words "one year," "four months," "sixty days," etc., were substituted for the words "on demand," when would the note (p. 169) be payable? The note is then a **time note**.

NOTE. In some states *three extra days* after the expiration of the time named in the note are allowed the maker for its payment. They are called **days of grace**. Interest is exacted, however, for the days of grace.

22. If the note (p. 169) were a four months' note, at what date would it be payable without grace? With grace? When, if it were a 2 months' note? A six months'?

23. Notes **mature**, or are **legally payable**, on the day when the time named in them expires, or on the third day thereafter, when **grace** is allowed.

24. All notes that contain the words "with interest" draw interest from date unless otherwise specified. All other notes draw interest from maturity. When no rate of interest is specified, the legal rate is understood.

NOTE. It is quite usual to name in a note the place at which it is payable. The place named is generally some bank (see p. 185).

WRITTEN EXERCISES

Make interest-bearing notes answering to the following conditions, and compute the amount due at settlement. In finding the day of maturity, allow three days of grace if that is the custom in your State.

| | DATE | FACE | TIME TO RUN | PAYER | RATE | SETTLED |
|----|-----------|---------|-------------|-------------|------|------------|
| 1. | 3/17, '04 | \$ 240 | On demand | A. P. Rice | 6% | 9/14, '05 |
| 2. | 8/12, '03 | \$ 800 | One year | E. F. Foss | 7% | 12/18, '05 |
| 3. | 4/21, '03 | \$ 725 | Four months | Wm. Ward | 5% | Maturity |
| 4. | 6/15, '04 | \$ 1800 | Six months | Thos. True | 4% | 9/21, '06 |
| 5. | 1/19, '04 | \$ 610 | Two years | A. M. Bates | 8% | 12/25, '07 |
| 6. | 2/24, '05 | \$ 280 | On demand | R. E. Nye | 4½% | 7/16, '06 |
| 7. | 1/1, '04 | \$ 75 | Sixty days | E. B. Hale | 12% | Maturity |

1. A note of \$300 draws 10% interest. What amount would discharge the note at the end of the first year? Suppose that instead of the note being paid in full at the time, a partial payment of \$100 were made, what would then be due?

2. Would the \$100 pay all the interest due? How much of the face or principal would it also pay?

3. How much of the original \$300 does the maker of the note continue to keep? On what sum, therefore, should he pay interest?

4. If the remaining \$230 should be used another year, the interest on it at 10% would be \$ x , and the amount due would be \$230 + \$ x , or \$ y .

5. If another partial payment of \$100 should then be made, a remainder of \$ y - \$100, or \$ z , would still be left at interest, and yet to be paid by the maker of the note.

6. Give the values of x , y , and z in the solution of the following problem:—

On my note, payable to you for \$300, I make a partial payment of \$100 at the end of each year for three years. What is then due you, 10% interest being charged?

SOLUTION

| | | |
|------|---|-----------------|
| I. | Of your money I have for use . . . | \$300 |
| | For a year's use of it at 10% I owe . . . | z |
| | At the end of the year I owe you . . . | <u>\$330</u> |
| | I make a partial payment to you of . . . | <u>100</u> |
| II. | This leaves a balance for me to use of . . . | \$230 |
| | For a year's use of this sum I owe you . . . | <u>23</u> |
| | At the end of the second year I owe you . . . | <u>\$y</u> |
| | I make a second partial payment of . . . | <u>100</u> |
| III. | I now have of your money only . . . | <u>\$z</u> |
| | For a year's use of this sum I owe you . . . | <u>15.30</u> |
| | I owe you at the end of the third year . . . | <u>\$168.30</u> |
| | If I pay you a third | <u>100</u> |
| IV. | I shall still owe you | <u>\$68.30</u> |

When partial payments of a note are made, the holder records the amount and date of each payment on the back of the note.

\$720---

Sioux City, August 14, 1904.

On demand, after date, I promise to pay to the order of.....Henry Howard & Co.....

Seven Hundred Twenty.....Dollars

with interest at six per cent.

Value received.

Epa Stevens.

INDORSEMENTS ON NOTE

Received on the within note:

Dec. 26, 1905, \$200

Sept. 14, 1908, 175

Dec. 31, 1909, 400

Settled, Dec. 31, 1910.

1. Who puts these indorsements on the note?

2. Will any other receipt be requested for the \$200 paid Dec. 26, 1905, and if so, by whom?

3. What needs to be done before the note can be transferred to a third person?

The Supreme Court of the United States has decreed that:—

Partial payments of notes must first be used to cancel the interest due. Any balance remaining may be used to lessen the principal.

This decree gives what is called "The United States Rule." Sometimes other rules are used. If your state uses a different rule, find what it is and use it. The special rules will not be discussed in this book.

A SOLUTION

Find the amount due at settlement, Dec. 31, 1910, on the note on the preceding page.

From date of note to 1st payment

| | | |
|--|---------|-----------------|
| 8/14, '04, to 8/14, '05 = 1 yr. . . . | \$0.06 | \$720 |
| 8/14, '05, to 12/14, '05 = 4 mo. . . . | 0.02 | 0.082 |
| 12/14, '05, to 12/26, '05 = 12 da. . . . | 0.002 | 1 44 |
| Int. of \$1 | \$0.082 | 57 6 |
| Interest due when 1st payment is made | | \$59.04 |
| Face of note, or 1st principal | | 720. |
| Amount due at time of 1st payment | | \$779.04 |
| 1st payment, which cancels interest due, and more | | 200. |
| Remainder which continues to draw Int. ; 2d Prin. | | \$579.04 |

From 1st payment to 2d payment

| | | |
|--|----------|-----------------|
| 12/26, '05, to 12/26, '07 = 2 yr. . . . | \$0.12 | 9650 |
| 12/26, '07, to 8/26, '08 = 8 mo. . . . | 0.04 | 1 73712 |
| 8/26, '08, to 9/14, '08 = 19 da. . . . | 0.0031 | 34 7424 |
| | \$0.1631 | 57 904 |
| Interest due at time of 2d payment | | \$94.48002 |
| 2d principal | | 579.04 |
| Amount due at time of 2d payment | | \$673.52 |
| 2d payment cancels interest and part of principal | | 175. |
| Remainder which still draws Interest ; 3d Prin. | | \$498.52 |

From 2d payment to 3d payment

| | | |
|---|----------|-----------------|
| 9/14, '08, to 9/14, '09 = 1 yr. . . . | \$0.06 | 6)249260 |
| 9/14, '09, to 12/14, '09 = 3 mo. . . . | 0.015 | 41543 |
| 12/14, '09, to 12/31, '09 = 17 da. . . . | 0.0021 | 3 48964 |
| | \$0.0771 | 34 8964 |
| Interest due at time of 3d payment | | \$38.80147 |
| 3d principal | | 498.52 |
| Amount due at time of 3d payment | | \$537.32 |
| 3d payment cancels interest and part of principal | | 400. |
| Remainder which still draws Interest ; 4th Prin. | | \$137.32 |

From 3d payment to settlement

| | | |
|---|-------------|-----------------|
| 12/31, '09, to 12/31, '10 = 1 yr. . . . | \$0.06 Int. | \$8.2392 |
| 4th principal | | 137.32 |
| Amount due at Settlement | | \$145.56 |

1. How much interest was cancelled by the first payment? How much of the principal?

Payments too Small to cancel Interest Due

A SOLUTION

Commonly the partial payment of a note will not only cancel all interest, but will also pay a portion of the principal, and thus reduce the amount upon which the borrower has to pay interest. It sometimes happens, however, that the payment is too small to cancel even the interest due. In such cases —

The interest must not be used to increase the principal, which must never represent more than the money actually and previously due to the holder or payee, and in use by the maker.

1. What is due at settlement on a note of \$ 600 dated Aug. 15, 1902, drawing 6%, and indorsed as follows: —
Dec. 15, 1903, \$ 25? Sept. 15, 1905, \$ 200? Settled Aug. 15, 1907?

SOLUTION

From date to 1st payment

| | | |
|---|-------------|-------------|
| 8/15, '02 to 8/15, '03 = 1 yr. | \$ 0.06 | |
| 8/15, '03 to 12/15, '03 = 4 mo. | <u>0.02</u> | \$ 600 |
| | \$ 0.08 | <u>0.08</u> |

The payment of \$ 25 will not pay the interest due,
Hence we compute the interest: —

From date to 2d payment

| | | |
|---|--------------|------------|
| 8/15, '02 to 8/15, '05 = 3 yr. | \$ 0.18 | |
| 8/15, '05 to 9/15, '05 = 1 mo. | <u>0.005</u> | \$ 0.185 |
| Interest of \$ 1 | \$ 0.185 | <u>600</u> |
| Interest due at 2d payment | | \$ 111.0 |
| Face of note on interest | | <u>600</u> |
| Amount due at 2d payment | | \$ 711 |
| Sum of 1st and 2d payments (\$ 25 + \$ 200) | | <u>225</u> |
| New Principal on Interest | | \$ 486 |

From 2d payment to settlement

| | | |
|---|--------------|--------------|
| 9/15, '05 to 9/15, '06 = 1 yr. | \$ 0.06 | <u>0.115</u> |
| 9/15, '06 to 8/15, '07 = 11 mo. | <u>0.055</u> | \$ 2.430 |
| | \$ 0.115 | <u>4.86</u> |
| Interest due at settlement | | \$ 55.890 |
| Principal due at settlement | | <u>486.</u> |
| Amount due at Settlement | | \$ 641.89 |

1. Study the solution on the preceding page. Why do we not add the first interest, \$48, to the principal, subtract the payment, \$25, and compute the interest on the remainder, \$623, as in the example on page 174 ?

2. Is it fair that the maker of the note should pay interest on more than he has hired and used ?

3. Will the two payments together cancel the interest due when the last one, \$200, is paid ? How much of the principal besides ?

4. How must we proceed when the payment will not cancel the interest due ?

5. Which is better for the one that owes the money, to subtract the amount paid from the amount due when payment is made, regardless of the amount of the payment, or do as in the last solution ?

6. Show why the debtor might object to the arrangement proposed below ?

| | |
|---|--------------|
| I loan Mr. James | \$600 |
| A year's interest at 5% is | 30 |
| He then owes me | <u>\$630</u> |
| He pays me | 20 |
| I ask him to pay me interest on | <u>\$610</u> |

7. What will settle a 5% note for \$1000 Aug. 1, 1907, on which \$50 was paid 2 years after date, and \$500 4 years after date ? The date was Aug. 1, 1899.

8. What was due May 1, 1905, on a note for \$3000, on which \$100 was paid at the end of each year for 3 years ? The rate was 4%, and the note was given May 1, 1901.

9. Cyrus Drew gave Frank Watson his note for \$800 at 3% interest, Aug. 17, 1900. Dec. 23, 1902, he paid \$300, and May 29, 1904, he paid \$40. What was due 6 years from date ?

1. A note for \$500, dated Aug. 10, 1903, drawing 6 per cent interest, is paid Aug. 10, 1905. A payment of \$100 was made Aug. 10, 1904. How much was due at time of settlement?

2. A note dated May 16, 1904, for \$1200, 4 per cent interest, has two partial payments of \$300 each indorsed upon it, 6 months and 18 months after date respectively. What will cancel the note 3 years from date?

3. The face of a note is \$1500. It draws 5% interest. 8 months after date \$500 is paid. 20 months after date \$600 more is paid. At the end of 4 years, what will pay what is due?

4. A note of \$2000 drawing 6 per cent interest, is dated Jan. 1, 1904, and is paid Jan. 1, 1907. What is due, provided \$500 was paid on New Year's day in 1905 and in 1906?

5. Aug. 20, 1904, Gordon Cook gave Thomas Swan his note for \$5000 with interest at 8 per cent. Feb. 20, 1905, he paid \$2000. Aug. 20, 1906, he paid all but \$600 of what was due. What sum cancelled the note Dec. 26, 1907? Study the problem and use a short method.

6. One half of a note of \$7500, with interest at 4%, was paid March 17, 1904. What was due 4 yr. 6 mo. from date of the note, which was Dec. 12, 1903?

7. I paid a note that had been running just 5 years. It drew 8 per cent interest, and its face was \$900. It was given Dec. 11, 1905, and \$300 had been paid on it 2 yr. 8 mo. after date. What cancelled it?

8. Face, \$600. Rate, 6 per cent. Date, May 12, 1903. First payment, \$50, May 12, 1905. Second payment, \$600, June 19, 1907. What was due Aug. 12, 1909?

9. What is due Aug. 15, 1905, on a note for \$1600 drawing 5% interest, given May 15, 1903, a payment of \$25 having been made Dec. 15, 1904?

The expenses of towns, cities, counties, and states are met by levying **taxes** annually on all owners of property. In addition to this in most states all male citizens over 21 years of age are required to pay a **poll tax** (*poll* = head).

1. Why are some persons taxed more than others ?
2. Mention several ways in which the taxes collected in your city are expended.
3. What need have county and state to raise money by taxation ?
4. Find the amount of poll taxes in your city or town.

Property is divided into two classes for taxation : —

(a) **Real Estate**, regarded as immovable, as land and buildings, including mines, quarries, forests, railroads, etc.; and

(b) **Personal Estate**, which is usually movable.

5. Give examples of valuable personal property.
6. How large a tax on property is assessed in your city or town ?

Suppose a town has to raise a certain sum of money for the expenses of the coming year. Officers, called **assessors**, first estimate the value of the property to be taxed, and then **assess** each owner in proportion to what he has.

7. If the amount to be raised is \$20,000, and the poll tax amounts to \$1600, how large a property tax must be assessed ?

8. The assessed valuation of the property of a certain town is \$200,000. The tax to be raised is \$4000. What is the *rate of taxation* ?

9. What shall Mr. Smith pay who owns \$5000 worth of property in the town ?

10. My property is assessed at \$3000. The tax rate is $1\frac{1}{2}\%$. What is my tax if I pay a poll tax of \$2 ?

11. The tax to be raised is \$15,000. There are 250 polls, that is, male citizens over 21. What must be raised by a property tax if the poll tax is \$2 each ?

1. Assessors find the value of real estate in their city to be \$15,000,000, and of personal estate \$5,000,000. They assess a tax for state, county, and city. If the rate is \$15 tax on \$1000 valuation, the total to be collected will be how much?

2. The rate is \$12½ on \$1000. What will be assessed on \$4,000,000 valuation? A man who pays \$125 is assessed on how much?

3. A man's tax, including \$2 poll, is \$122. The rate is 1½%, or \$x on \$1000, \$y on \$1. His property is valued at how much?

4. 900 is what per cent of 60,000? If \$12 is the tax on \$1000, what is the tax on \$1? What the rate per cent?

5. Find the rate when the tax on \$120,000 is \$1770. On \$100 the tax would be \$x.

Find the rate of taxation on a thousand dollars under the following conditions:—

| ASSESSED
VALUATION | | TAX TO BE RAISED
ON PROPERTY | ASSESSED
VALUATION | | TAX TO BE RAISED
ON PROPERTY |
|-----------------------|-------------|---------------------------------|-----------------------|---------------|---------------------------------|
| 6. | \$48,000 | \$1600 | 10. | \$51,000,000 | \$800,000 |
| 7. | \$650,000 | \$1300 | 11. | \$49,000,000 | \$630,000 |
| 8. | \$1,650,000 | \$110,000 | 12. | \$135,000,000 | \$900,000 |
| 9. | \$2,470,000 | \$190,000 | 13. | \$215,000 | \$2700 |

14. The valuation of property in a certain town is \$2,306,000. The tax raised on the property is \$39,202. What is the rate?

15. If I own \$6500 worth of property in the same town, what will be my tax, including a poll tax of \$2?

16. In the city of B the rate of taxation is \$15.50 on a thousand; polls, \$1.75. If a mill owner's tax is on \$250,000 worth of real estate, what is his total tax?

17. The taxable property in the town of Pleasant Valley is estimated at \$3,000,000; the number of polls, 1800; the state tax, \$1044; the county tax, \$4500; the town tax, \$36,300. If the poll tax is \$2.75, what must be the rate of property tax on a thousand dollars' worth of property?

1. The expenses of the national government are not paid from a tax upon property and polls, as are the expenses of the city, county, and state, but are paid from : —

I. The **Internal Revenue**, chiefly taxes of a fixed sum on the right to make or sell spirituous liquors, tobaccos, etc.

II. The **Customs Revenue** (tariff, duty), taxes collected on goods imported from foreign countries.

2. What need of money has the government ?

NOTE. The expenses of the government are over \$ 1,000,000 per day.

3. Mention several articles commonly imported.

4. Merchandise brought into this country is either : —

I. On the **free list**, that is, exempt from duty ; or

II. Subject to an **ad valorem** duty, a per cent on the cost of the goods where they were bought, as shown by the invoice ; or

III. A **specific duty**, fixed according to number, quantity, weight, etc., without reference to value ; or

IV. There may be both an *ad valorem* and a *specific duty*.

5. Find the duty at 15% on 200 T. of coke invoiced at \$ 1.50 a ton. Is the duty specific or ad valorem ?

NOTE. Invoices are made out in the money of the country where the goods were bought. When changed to United States money, the duty is computed on the nearest dollar, 50 cents counting as \$ 1.

Imported goods must be brought to a **port of entry**, a place where the government has established a **custom house** with officers for the collecting of the duties.

6. What is meant by **smuggling** ?

7. A gross of leather pocketbooks invoiced at \$ 12.50 a dozen pays 30% ad valorem. What is the duty ?

8. What is the duty on an importation of \$ 5600 at an ad valorem rate of $12\frac{1}{2}\%$?

Tare, Leakage, and Breakage are allowances for boxes, bags, etc., used in packing, and for liquids lost from barrels or bottles, etc.

9. A dealer withdraws from the custom house 4 T. (2240 lb.) of rice, 8% tare. On what weight must he pay duty?

10. What will this amount to at $1\frac{1}{2}$ ¢ a pound? Is this specific or ad valorem duty? Why?

11. Define *gross weight* and *net weight*.

12. An importation of velvet invoiced at 5181.35 francs weighs 400 lb. 10% tare. Which would be more, a duty \$ 1.50 a pound or 50% ad valorem? (A franc = \$ 0.193.)

13. 20 bbl. vinegar of 42 gal. each pay a specific duty of $7\frac{1}{2}$ ¢ a gal., leakage 30 gal. Find the whole duty and the per cent of leakage.

14. The duty on cut nails is $22\frac{1}{2}$ % ad valorem. \$ 183.75, the cost including duty on a certain importation, is what per cent of the amount of the invoice? They were invoiced at \$ x.

15. In 1903 the duty on Brussels carpet was 28¢ a square yard and 40% ad valorem. Find the total cost of 200 running yards invoiced at 6 s. a yard, $\frac{3}{4}$ yd. wide. (1 s. = \$ 0.24 $\frac{1}{2}$.)

16. The gross cost of a lot of calfskins is \$ 740, including \$ 20 freight and \$ 120 duty. Find the ad valorem rate.

17. If the cost of gloves is doubled by importing, what will be the profit on a pair invoiced at 60 fr. a dozen, and sold for \$ 2.25?

18. Find the duty on 50 dozen rubber coats at £ 1 $\frac{1}{4}$ per dozen. Duty, 40%. (The value of £ 1 = \$ 4.8665.)

19. What is the duty on 12 dozen suits invoiced at £ 36? Rate, 50%.

20. What is the duty on 3500 lb. cheese at 6¢ specific duty? If the cheese is invoiced at 9¢, to what ad valorem duty is this equivalent?

1. Jan. 1, 1904, Mr. Dale borrows \$500 of Mr. Coe, agreeing to pay 6% interest at the end of every year. How much interest is due at the end of the first year?

2. Who is entitled, under the agreement, to use this \$30 interest for the second year?

3. If Mr. Dale uses the overdue interest during the second year, instead of paying it to Mr. Coe, is it just that he should pay for the use of it?

4. On how much then should the debtor (Mr. Dale) pay interest the second year, including both principal and overdue interest?

Interest reckoned on both principal and overdue interest added to the principal as often as due is called **compound interest**.

NOTE. When interest is added to the principal, or "compounded," it is done yearly, unless otherwise stated, as half-yearly, quarterly, or oftener.

5. What is due on a debt of \$600, which has been standing 2 yr. 6 mo. 18 da., interest at 6% compounded annually? Explain the following process.

| | |
|---|-----------------|
| Principal used for 1st year | \$600 |
| Interest due at end of 1st year | 36 |
| Principal used for 2d year | <u>\$636</u> |
| Interest due at end of 2d year | 38.16 |
| Principal used for 6 mo. 18 da. . . . | <u>\$674.16</u> |
| Interest due at end of 6 mo. 18 da. . . . | 22.25 |
| Amount due at settlement | <u>\$696.41</u> |
| First principal | 600 |
| Compound interest | <u>\$96.41</u> |
| Simple interest would have been | 91.80 |
| Interest on all overdue interest | <u>\$4.61</u> |

6. What is due on a note of \$1000 standing 3 yr. 6 mo., interest due annually at 6%?

7. Find the compound interest, that is the amount due less the principal, on a note of \$800 at 5% that has run 2 yr. 8 mo., interest due annually.

1. Find the compound interest of \$ 500 for 3 yr. at 5 %.
2. What is the compound interest at 4% of \$ 2000 for 2 yr. 6 mo.?
3. Interest compounds semiannually on \$ 400, at 8% a year. For 1 yr. 6 mo., what is the amount due?

NOTE. Compound interest is not in general use. The collection of compound interest on notes and debts cannot be enforced, even when agreed upon; it may happen, however, that large investors wish to compute the result of reinvesting all interest when due. It is then computed by tables. Savings banks generally allow compound interest on deposits.

COMPOUND INTEREST TABLE

AMOUNT OF \$ 1

| Yr. | 2 PER CENT | 2½ PER CENT | 3 PER CENT | 3½ PER CENT | 4 PER CENT | 5 PER CENT | 6 PER CENT |
|-----|------------|-------------|------------|-------------|------------|------------|------------|
| 1 | 1.020000 | 1.025000 | 1.030000 | 1.035000 | 1.040000 | 1.050000 | 1.060000 |
| 2 | 1.040400 | 1.050625 | 1.060900 | 1.071225 | 1.081600 | 1.102500 | 1.123600 |
| 3 | 1.061208 | 1.076891 | 1.092727 | 1.108718 | 1.124864 | 1.157625 | 1.191016 |
| 4 | 1.082432 | 1.103813 | 1.125509 | 1.147523 | 1.169859 | 1.215506 | 1.262447 |
| 5 | 1.104081 | 1.131408 | 1.159274 | 1.187686 | 1.216653 | 1.276282 | 1.338226 |
| 6 | 1.126162 | 1.159693 | 1.194052 | 1.229225 | 1.265319 | 1.340096 | 1.418519 |
| 7 | 1.148686 | 1.188686 | 1.229874 | 1.272279 | 1.315932 | 1.407100 | 1.503630 |
| 8 | 1.171660 | 1.218403 | 1.266770 | 1.316809 | 1.368569 | 1.477455 | 1.598848 |
| 9 | 1.195093 | 1.248863 | 1.304773 | 1.362987 | 1.423312 | 1.551328 | 1.689479 |
| 10 | 1.218994 | 1.280085 | 1.343916 | 1.410599 | 1.480244 | 1.628895 | 1.790848 |

NOTE. The compound interest on any amount for 5 yr. at 6% payable semiannually is evidently the same as upon the same amount for 10 yr. at 3% payable annually.

Find by the table the amount at compound interest of:—

1. \$ 500 for 2 yr. at 5% ; semiannual dividend.
2. \$ 600 for 3 yr. 6 mo. at 5% ; semiannual dividend.
3. \$ 320 for 1 yr. 9 mo. at 8% ; quarterly dividend.
4. \$ 800 for 2 yr. 4 mo. at 4% ; semiannual dividend.

One of the sources of income of banks is from lending money. If I wish to borrow money from a bank, I give a promissory note. (See page 169.)

A promissory note given a bank is usually in the following form:—

\$65---

Ypsilanti, Mich., Sept. 11, 1904.

*Three months, without grace, after date, I promise
to pay to the order of-----R. W. Hemphill, Cashier,
Sixty-five-----Dollars
at Ypsilanti Savings Bank, Ypsilanti, Mich.*

Value received.

Residence, 314 A. Adams st.

John Doe.

Due, 12/11/04. Discount, \$0.98.

According to the custom of banks, the interest (called by the banks, *discount*) on the face of the note is deducted from the face when the money is borrowed and the **proceeds**, that is, the difference between the *face* of the note and the *interest*, is given the borrower.

1. If John Doe gives his note for \$65 and the interest for the time is \$0.98, what does he receive?
2. At the end of three months what does he pay the bank?
3. If Mr. Doe had given his note for \$65 to some one, not a banker, what would he have received at the time the note was given?
4. If the interest of this for 3 months had been \$0.98, what would have settled the note at the end of 3 months?
5. The rate being the same, how did the interest compare in each case?
6. How much money had Mr. Doe the use of in each case?

1. I borrowed \$100 of a bank. I gave my note due in 6 mo. without grace. Rate of discount (interest) is 6%.

How much money did I actually get at the bank?

What did I pay at the end of 6 mo.?

2. How much more money could I have hired for \$3 of some one not connected with the bank?

3. State the difference between bank discount and interest.

4. What must be done before Mr. Alden can transfer the ownership of the following note to a third person?

\$500---

Milwaukee, Wis., Aug. 12, 1903.

Two months after date I promise to pay to the order of ----- John T. Alden -----

Five Hundred ----- Dollars
at the Third National Bank.

Value received.

Horatio Long.

5. What responsibility does the indorser of a note assume?

6. In what case may Mr. Alden be called upon to pay this note?

7. What risk does a bank take in buying this note of Mr. Alden if he is a reliable person?

8. When does this note **mature** or become due, no grace being allowed?

9. Under what conditions will it draw interest?

10. What is this note worth Oct. 12, 1903? Oct. 15, 1903?

11. Why is it not worth \$500 at date?

12. How much could Mr. Alden get at the Third National Bank for his own note, that is, one of which he is the maker, of \$500 due in 2 months, the rate of interest charged by the bank being 6%?

13. Why should Mr. Alden receive from the bank just as much for a note which he has indorsed as for one in which he is the maker, if it is to run the same time and is worth the same at maturity?

14. What will the Third National Bank pay for this note on Aug. 12, 1903? The rate of interest charged by the bank is 6%.

15. When does the bank get back the \$495? How much more does it get on Oct. 12, 1903?

Should Mr. Long fail to pay the note before the closing of the bank on the day of maturity, immediate notice is given the indorser and he is held for payment.

Whether you are the maker or indorser of a note given to a bank, the transaction is called **selling** or **discounting** the note, the interest is called **bank discount**, and what you receive for it is the **proceeds**.

Written

Most of the notes discounted at banks, or by brokers or others, are given for short times, 30, 60, 90 days, or 2, 3, 4, or 6 months.

NOTE. Compute bank discount as if it were interest on the face of a note for the time the bank's money is used; and take the shortest method. Allow three days of grace if such is the lawful custom in your state. Answers are given both with grace and without it.

1. What will a bank pay me for a note of \$800 payable in 3 mo., the rate being $4\frac{1}{2}\%$?

2. How much of its money will a bank permit me to use in return for a note of \$720? Money is worth 5%, and the note runs 60 days.

3. What allowance shall Mr. Strong make to the Exchange Bank for its prepayment of a 4-mo. note for \$875 at a discount rate of 3%?

The *term of discount* is the time for which the bank's money is used. It extends from the day of discount to the day of maturity.

Find the bank discount and the proceeds of notes made under these conditions, and discounted at date :—

| FACE | TIME
TO RUN | RATE OF
DISCOUNT | FACE | TIME
TO RUN | RATE OF
DISCOUNT |
|-----------|----------------|---------------------|-----------|----------------|---------------------|
| 1. \$ 525 | 30 da. | 6% | 4. \$ 800 | 90 da. | 7½% |
| 2. \$ 917 | 3 mo. | 4½% | 5. \$ 960 | 60 da. | 4% |
| 3. \$ 324 | 6 mo. | 8% | 6. \$ 721 | 75 da. | 5½% |

When a note is discounted *at date*, the time named in the note (with grace or without, as the case may be) is the term of discount, showing how long the discounter's money is used. A note, however, may be sold or discounted at any time between date and maturity.

The term of discount extends from day of discount to the day of maturity.

NOTE. The method of reckoning the time from the day of discount to the day of maturity is not uniform among banks.

The two more common methods are the following :—

I. When the time is less than two months, the exact number of days is counted ; but when it is more than two months, the time is reckoned in months and days. (See page 151.) Or,

II. The exact number of days is taken in all cases.

In general, the latter method is used, when notes are large, being to the advantage of the bank.

Thus, a 4-month note dated June 30 matures, grace allowed, Nov. 2. If discounted July 10, interest may be computed either for 115 days or 3 mo. 23 da., the difference being \$ 3½ in favor of the bank on a \$ 10,000 note.

The student should conform to the custom of his own vicinity. Answers to examples are given for both methods.

Find date of maturity and term of discount by each method :—

| DATE OF
NOTE | TIME
TO RUN | DAY OF
DISCOUNT | DATE OF
NOTE | TIME
TO RUN | DAY OF
DISCOUNT |
|-----------------|----------------|--------------------|-----------------|----------------|--------------------|
| 1. May 8 | 60 da. | June 10 | 5. April 14 | 4 mo. | June 8 |
| 2. Nov. 17 | 90 da. | Dec. 14 | 6. Jan. 25 | 30 da. | Jan. 29 |
| 3. Aug. 5 | 2 mo. | Aug. 29 | 7. Sept. 19 | 4 mo. | Nov. 10 |
| 4. Mar. 17 | 3 mo. | May 1 | 8. Feb. 10 | 90 da. | Mar. 6 |

Notes discounted after Date

1. I get a 3-mo. note discounted 27 days after date. What is the term of discount?

2. What if it were sold 24 days before maturity?

3. A 6-mo. note for \$450 dated Aug. 11 is discounted Sept. 4, at 6%. Reckon months and days with grace. Find proceeds.

4. A 90-da. note for \$1000 is discounted 37 da. before maturity. Find the proceeds, the rate being 5%.

5. A 5-mo. note for \$800 was sold at 3% discount 80 days after it was made. Proceeds without grace?

6. 60 days; \$450; 4%; date June 15; day of discount July 1. What does the borrower have for immediate use out of his note?

Using these data, find the proceeds of notes. Conform to the custom of your own vicinity in allowing grace and finding the term of discount.

| | FACE | DATE | TIME | DATE OF DISCOUNT | RATE |
|-----|----------|--------------|--------|------------------|------|
| 7. | \$875 | Jan. 16, '03 | 90 da. | Feb. 24, '03 | 5% |
| 8. | \$984 | Aug. 8, '04 | 5 mo. | Oct. 27, '04 | 6% |
| 9. | \$696 | June 15, '03 | 60 da. | Aug. 4, '03 | 8% |
| 10. | \$842.50 | Oct. 31, '02 | 1 mo. | Nov. 1, '02 | 5½% |
| 11. | \$1250 | July 9, '04 | 60 da. | Aug. 29, '04 | 4½% |

Notes that are discounted commonly bear no interest. *When an interest-bearing note is discounted, the maturity value of the note must be made the base of discount.*

1. An interest-bearing note of \$1000 is payable in 1 yr. Interest 8% What is it worth at maturity?

2. This note is discounted at a bank 3 months before maturity at 6%. Find the proceeds.

SUGGESTION. Find the proceeds on \$1080. Why?

1. A 4-mo. note for \$1200, drawing 9% interest, is discounted at 4% 3 months before maturity, without grace. Required the proceeds.

2. Face, \$500; rate of interest, 5%; time to run, 60 da.; term of discount, 48 da.; rate of discount, 6%. Proceeds?

3. A 90-da. note for \$720, dated May 15, 1904, drawing 8% interest, is discounted June 12, 1904, at 6%. Proceeds?

4. Write an interest-bearing note and find the proceeds.

5. Sold a \$400-note to the Merchants' Bank at 8% discount. It had 40 days to run. Proceeds?

6. What shall the Farmers' Bank pay for a note of \$1000; the rate of discount is 7%; the note matures 4 months from April 10, 1904, and is bought 25 days after date?

7. An interest-bearing note for \$1200 is payable in one year from date. What will pay it at maturity?

8. If this note is discounted 6 months before maturity at 5%, what are the proceeds?

9. What will it bring if sold to a bank at 6% discount when it is 3 months old?

10. What will the same bank pay for it at date?

11. A bank bought a \$600 note for \$591. What was the rate of discount if the note had 60 days to run?

Find proceeds of notes under these conditions, each bearing 6% interest.

| | FACE | RATE | DATE OF NOTE | RUNS | DISCOUNTED |
|-----|--------|------|--------------|--------|------------------------|
| 12. | \$450 | 4% | May 10, '03 | 60 da. | 28 da. after date |
| 13. | \$720 | 5% | Aug. 15, '04 | 90 da. | 48 da. before maturity |
| 14. | \$958 | 7% | Oct. 12, '04 | 45 da. | at date |
| 15. | \$800 | 5% | Aug. 21, '05 | 90 da. | Oct. 1, '05 |
| 16. | \$278 | 8% | Dec. 20, '04 | 3 mo. | 16 da. after date |
| 17. | \$5000 | 3% | Aug. 19, '04 | 60 da. | Sept. 19, '04 |

When a business requires more **capital**, or money invested, than a single individual or a few partners wish to furnish, a **stock company**, or **corporation**, may be formed with any number of partners, who choose a **board of directors** to conduct the business as one person.

The capital stock of a stock company is divided into **shares** of a fixed amount, usually \$100. The value of the shares in different corporations varies, and is determined by the persons forming the corporation.

1. A corporation is formed with a capital of \$250,000, shares \$100 each. How many shares are there?

2. If there were only 500 shares, what would be the value of each?

Each owner or **stockholder** receives a **certificate of stock** bearing the seal of the corporation and giving the number and size of the shares.

Since shares are often sold, stockholders are often changing.

Stock Certificate

| | |
|---|-----------------------|
| Incorporated under the Laws of the State of New York. | |
| No. 212. | 12 shares. |
| EASTERN PACKING COMPANY. | |
| This certifies that <i>Julius Holm</i> | |
| is the owner of <i>Twelve</i> shares of One | |
| Hundred Dollars each of the full paid Capital Stock of | |
| the Eastern Packing Company. | |
| Transferable only on the books of the company in person or by attorney upon | |
| surrender of this certificate. | |
| New York, July 12, 1904. | |
| <i>Carl Jacobs,</i> | <i>M. S. Kendall,</i> |
| Secretary. | President. |

1. From whom does the Eastern Packing Company get the right to carry on business as a corporation ?

2. Who is the owner of the certificate ?

3. What is the *face value* of each share ?

The income or profit of the business is called a **dividend**, because it is divided and paid yearly, half-yearly, or quarterly to the stockholders as a per cent on the **par value** or **face value** of each share ; that is, the value named in the certificate of stock.

4. About what income does one receive from \$ 100 loaned on an interest-bearing note ?

5. If the yearly dividend on a \$ 100 share in a gas company is \$ 20, which would you prefer, stock in the company, or to loan your money ?

6. Could you afford to give more than \$ 100 for a share in such a company ?

When a business is prosperous and paying large dividends, the stock usually sells **above par**, and is then said to be at a **premium**.

7. Which would pay you the better income, money loaned at 6% interest, or stock paying an annual dividend of \$ 4 on a share (\$ 100) ?

8. In that case could you afford to pay par value for the stock ?
Stock selling **below par** is said to be at a **discount**.

9. I own 10 shares (\$ 100 each) of mining stock. The company declares a 10% dividend. What is my share ?

10. If such a dividend is declared semiannually, to what rate of interest is it equivalent ?

11. What will three \$ 100 shares of railroad stock cost at a discount of 10% ? At a premium of 20% ?

12. Bought 5 shares of mill stock at 103 and sold it at 108. What did I gain ?

13. Would one care to buy stock at 97% discount ?

A company may issue two kinds of stock, viz: —

Preferred stock, which entitles the holder to dividends which are a certain per cent of the par value of the stock held; and

Common stock, which entitles the holder to a share of part or all of the remainder of the gains.

Sometimes an **assessment** is levied on the stockholders to pay the debts of the company.

Look up **stock quotations** in some newspaper.

A newspaper quotation of 165 means that \$100 of stock, that is, stock whose par value is \$100, is selling for \$165.

Stock is usually bought and sold through a **stock broker** who belongs to some **stock exchange**. The usual brokerage charged is $\frac{1}{8}\%$ of the *par value*, or \$0.12 $\frac{1}{2}$ for buying 1 share of \$100 stock.

1. Stock quoted at 120 will cost you what if you buy through a broker?

2. When stock is selling at 15% below par, what is the *market value*, or at what is the *stock quoted*?

3. I buy 10 shares of railroad stock quoted at 104 $\frac{7}{8}$, brokerage $\frac{1}{8}$. What do they cost me?

4. I own 20 shares in a paper company. What do I get for them if I sell direct (*i.e.* not through a broker) at 108?

5. I received \$80 dividend on stock paying an 8% dividend. How many \$100 shares have I?

6. If stock is selling at 200 and paying a 25% dividend, what rate of income is this?

7. How many shares of 5% stock will yield an annual income of \$1000?

8. A broker receives \$20 for a sale of stock. How many \$100 shares were sold?

9. The market value of a \$50 share of stock is \$65. What is the per cent of premium?

1. 15 boys organize a ball club with a capital of \$120. What does each boy contribute if they share equally?

2. Suppose this capital is divided into 25¢ shares, how many shares will there be?

3. Tom Jones, the captain, takes 80 shares at par. What is the value of his stock in the club?

4. After the members of the club have subscribed for all the stock they care to take, 16 shares remain. They are bought by an outsider at 10% above par. What does he pay for them?

5. Harry Irving, one of the original stockholders, is obliged to retire from the club, and sells his stock to the catcher at a discount of 15% from par. He had 8 shares. How much did he lose by his investment?

6. At the end of the season, \$40 remained in the treasury, over and above expenses. The club voted to divide this among the stockholders in proportion to their holdings. Ed Sprinter had 16 shares. What was his part of the dividend?

7. The next season the club started out by levying an assessment of $16\frac{2}{3}\%$ for an outfit. What was paid on each share?

8. An electric railroad pays a semiannual dividend of 4% on \$5,000,000 of capital. What is the annual dividend?

9. What will the market value of the stock be at 20% premium?

10. Bought 10 shares of Pennsylvania Railroad stock at 75, and brokerage of $\frac{1}{8}$. What was the cost?

11. Sold 20 shares of stock in the Massachusetts Mills at $106\frac{1}{2}$. I bought it for 105. What was my gain?

12. Bought telephone stock at 120, and received an annual dividend of \$8. What per cent of the par value was this?

13. Which is more profitable, to buy stock at 80 and sell it at 90, or to buy it at 150 and sell it at 175?

14. A mining stock pays 12% annually. My dividend is \$240. How many \$100 shares of the stock do I own?

1. What is the market value of 35 shares of North Western stock at $43\frac{1}{2}\%$ above par?
2. What would be the value of the same stock if sold at the same rate below par?
3. I receive a stock dividend of \$1728. This is at the rate of $14\frac{2}{3}\%$ on the par value of my investment. How much of the stock do I own?
4. A man exchanges 170 shares of stock worth 103 in the market for a cottage at the seaside valued at \$8510. The difference was made up in mill stock at a par value of 50 per share. How many shares were there? Leave brokerage out of the account.
5. A gentleman bought 1200 shares of railroad stock at 115, and was glad to sell it at 58. What did he lose?
6. The Atlantic Steamship Company is capitalized at \$8,000,000. The receipts for the year are \$16,400,000. The expenses are \$14,800,000. \$600,000 is put into a reserve fund, and the remainder distributed as a dividend. What rate per cent of dividend was declared?
7. How many hundred-dollar shares of mining stock can be bought at 118 for \$3750, and what sum will remain? (Parts of a share are not sold.)
8. I receive \$2133 as the net profits of stock bought at par and sold at 107. How many hundred-dollar shares were sold, allowing $\frac{1}{4}$ brokerage?
9. My broker paid me \$8595 which he had received for Old Colony railroad stock sold for me at $\frac{1}{8}$ brokerage. How many shares did he sell at 215?
10. In the New York stock market 50,000 shares of railroad stock were bought at a premium of $22\frac{7}{8}$, and sold the next day at a premium of $26\frac{1}{4}$. The brokerage in each case being $\frac{1}{8}$, required the profits.

When corporations, or national, state, or city governments, borrow large sums of money, they usually give a series of **bonds**, or promissory notes, for one or more hundred or thousand dollars each, and to run several years at a fixed rate of interest. Instead of finding some one that will lend the money needed, the bonds are issued and offered for sale.

Registered bonds are recorded by number on the books of the corporation with the name and address of the holders. They can change owners only through the office of the treasurer. The interest is sent to the holder when due.

Coupon bonds bear small, detachable coupons or certificates of the amount of interest regularly due.

These coupons are paid by the treasurer on presentation, or they may be deposited at one's bank for collection.

1. If this coupon is attached to a 7% bond, what is the face value of the bond?

One of several interest coupons attached to a bond.

| |
|--|
| <p>The Northern Loan Association
 OF ST. PAUL, MINNESOTA.
 <i>Will pay to bearer at the office of the
 Company.....Thirty-five.....Dollars
 on the...15...day of...April..., 1904,
 being one year's interest on coupon
 bond No. 137.</i></p> <p style="text-align: right;"><i>Edward James,</i>
 Secretary.</p> |
|--|

2. When is the next payment of interest due?
3. Where can it be collected, and by whom?
4. If the bonds sell at 140, what would this one cost?
5. What would the annual rate of income be?
6. My 4½% bonds yield me \$180 annually. What is their par value?
7. I get only 3% on what I paid for them. At what were they quoted when I bought?

1. I receive an income of \$200 from 4% bonds. What is their face value?
2. I receive \$30 each year as interest on a 6% bond. What is its face?
3. I wish to secure an income of \$1200. How much shall I spend for 4% bonds at par to do this?
4. The semiannual coupon of a government 2% bond is worth \$50. What is the face of the bond?
5. A father gives his son sufficient 5% bonds to pay his college expenses with their income. His expenses amount to \$800 a year. What is the face value of the bonds?
6. I receive \$36 a year as interest on bonds whose face value is \$600. What is the rate of interest?
7. If a five-thousand-dollar bond yields \$250 annually, what must the rate of interest be?
8. \$75 is the interest on a thousand-dollar city bond. What per cent does the investment pay?
9. Which pays the larger rate of interest, a thousand-dollar bond that yields \$50 annually, or a five-hundred-dollar bond that yields \$12.50 semiannually?
10. If a 5% bond pays \$75 annually, what will a 6% bond of the same size pay?
11. Bought a U.S. bond for \$540, and sold it for \$630. What was my per cent of profit?
12. A coupon evidently cut from an 8% bond called for the payment of \$40 interest. If the interest was payable semiannually, what was the face of the bond?
13. A bankrupt corporation pays 70% of the face value of its bonds. I hold \$3500 worth. What shall I lose?

1. I receive \$5 on a hundred-dollar bond that cost me \$80. This is $\frac{5}{80}$ of the investment, or x per cent?

2. What rate of interest does a 4% bond pay when bought at par? Does it pay more or less if bought below par? If bought above par?

3. What rate of interest would a 4% bond pay if bought at 200? At 50? At 120? At 110? At $107\frac{1}{2}$?

4. Which pays the greater rate of income, a 4% bond bought at par, or a 3% bond bought at 60?

5. Bought a 5% bond at 105. What per cent did I secure on my investment?

6. I traded 50 shares of manufacturing stock listed at $108\frac{1}{2}$ for a five-thousand-dollar bond which is selling in the market at $109\frac{1}{2}$. How much did I gain or lose?

7. Paid \$640 for a hundred-dollar share of copper stock which paid quarterly dividends of 12%. What per cent did I secure on my investment?

8. Sold \$6000 of the bonds of a bankrupt road for $18\frac{1}{2}$. I bought them at par and received no interest. What did I lose on that investment?

9. Which is more valuable, an 8% stock at 200, or a 6% bond at 150?

10. Bought 6 thousand-dollar bonds at $108\frac{1}{2}$ and $\frac{1}{8}$ brokerage, and sold them for $112\frac{1}{2}$ and brokerage $\frac{1}{8}$. Required my gain.

11. Sold 90 shares of insurance stock at 98, and with the proceeds bought mining stock at 110. How many shares did I buy, and how much money remained?

12. A thousand shares of Calumet and Hecla stock bought at 118 are sold at a premium of 841. The gain per cent is x .

13. I am offered either \$1500 cash or 7 shares of 8% stock whose market value is 220, for 15 shares of P. W. & B. stock. Which is the better offer, and how much?

Payment at a Distance without sending Money

1. If you should send to the Peoria Bicycle Company for a bicycle, in what ways could you send the payment?

2. Mention some objections to sending coin or paper money by mail or express.

The postal service and some express companies keep large sums in many offices. If you pay from \$1 to \$100 to a postmaster or express agent, he can write an *order* directing the postmaster or agent at another office to pay the same sum to any person you name. For this accommodation you pay him from 3 to 30 ¢, no matter what the distance; or, if you send to a foreign country, from 10 ¢ to \$1. This is the *cost of exchange*.

NOTE. Post-office orders are payable at an office named; express orders, at any office of the same company.

The charges for postal money orders payable in the United States are as follows:—

| | | | |
|-------------------------------|------|-----------------------------------|------|
| Not over \$2.50 | 3 ¢ | Over \$30.00 to \$40.00 | 15 ¢ |
| Over \$2.50 to 5.00 | 5 ¢ | Over 40.00 to 50.00 | 18 ¢ |
| Over 5.00 to 10.00 | 8 ¢ | Over 50.00 to 60.00 | 20 ¢ |
| Over 10.00 to 20.00 | 10 ¢ | Over 60.00 to 75.00 | 25 ¢ |
| Over 20.00 to 30.00 | 12 ¢ | Over 75.00 to 100.00 | 30 ¢ |

NOTE. The maximum amount for which a single money order may be issued is \$100.

3. What will it cost me to send \$70.50 to Mandell Bros., Chicago, including the cost of the money order?

4. I buy some books in New York. The cost is \$7.35. What will a money order cost me?

5. What must you pay in New York for an international money order for 200 m. payable in Hamburg, the rate being 10 ¢ on each \$10 or fraction of \$10? A mark = \$0.238.

If a person, say Edward Bacon, keeps his money deposited at a **national bank**, or with a **banking company**, he may write a check for payment as on the next page:—

*Chicago, July 12, 1904.**No. 348.***State Bank of Chicago**

LA SALLE & WASHINGTON STS.

Pay to the order of-----*Simmonds & Newton*-----*\$98²⁴/₁₀₀**Ninety-eight & ²⁴/₁₀₀*-----*Dollars**Edward Bacon.*

Simmonds & Newton, on receiving this check will **indorse** it; that is, sign the firm name "Simmonds & Newton" across the back, and deposit it at the bank with which they are doing business. The bank will then collect the amount, usually without charge to a regular depositor. If one has no bank account, a small fee of 10¢ to 25¢ is generally charged for collecting. This fee is called the **exchange**.

If Simmonds & Newton "indorse in blank," that is, simply sign their name, the check may be collected in Chicago by any one known to the State Bank of Chicago.

If it is indorsed "in full" as, "Pay to the order of James Gray, Simmonds & Newton," it may be paid to James Gray or his order as soon as he indorses it.

Most debts are paid in this way.

Indorsed in blank

Simmonds & Newton.

Indorsed in full

*Pay to the order of
James Gray.
Simmonds & Newton.**James Gray.*

If a party, say Howe & Co., of Albany, cannot draw a check, or if the creditor, E. L. Harris & Co., in Syracuse, will not accept one, a draft (a banker's check) may be bought at a bank, for a small cost of exchange, like an order at a post office. Thus:—

| | |
|---|------------------------|
| \$234--- | Albany, July 12, 1904. |
| Tenth National Bank | |
| Pay to the order of <i>Howe & Co.</i> ----- | |
| Two Hundred thirty-four-----Dollars. | |
| To the | |
| <i>Farmers' National Bank,</i> | |
| <i>Syracuse, N. Y.</i> | |
| <i>Patriek Matthews,</i>
Cashier. | |

1. Of what bank is Matthews cashier?
2. Where is the draft payable?
3. How can Howe & Co. make it payable to other creditors?
4. Make a draft from the National Exchange Bank of Peoria to the Traders' Mutual Bank of Chicago. The Western Machine Co. pays \$400 for it.
5. Why is a bank draft more likely to be "good" than a private check?

NOTE. A bank draft on New York may be cashed almost anywhere in the United States. Drafts on another city will usually be paid before collection in the region of which it is a commercial center.

6. If Howe & Co. are going to mail this to E. L. Harris & Co., why should it be "indorsed in full"?
7. If Howe & Co. prefer, they can have the draft made payable to E. L. Harris & Co. Why is the first plan better?

A great deal of the collecting of debts is done through banks by what are called **commercial drafts**.

Suppose Haines & Co., of Topeka, has bought goods of S. T. Richards & Co., of Chicago, and has not paid the bill when due, say 30 or 60 days after the goods were bought. Richards & Co. may make out a draft as follows and deposit it with their bank in Chicago for collection.

No. 3465.

Chicago, Ill., May 6, 1904.

At sight pay to the order of

The State Bank of Chicago.....\$348--

Three Hundred Forty-eight.....Dollars.

To Haines & Co.,

Topeka, Kas.

S. T. Richards & Co.

The Chicago bank now sends this draft to some bank in Topeka. The Topeka bank sends a messenger to Haines & Co. and presents the draft for payment.

Haines & Co. either write across the face, "accepted," with the date, and firm's signature, or they refuse to pay it.

In case they refuse to pay it, the draft is returned to the State Bank of Chicago and Richards & Co. are notified. They must now take other means of collecting.

The form shown here is that of a **sight draft**. If "Thirty days after date," or some time is written in place of "at sight," the draft becomes a **time draft**.

When a draft is **accepted** it then has the same force as a promissory note owned by the payee and may be discounted at a bank as any promissory note.

1. Name the *maker*, the *payee*, and the *drawee* of the draft above.

1. Suppose the draft on the preceding page to have been a 60-day time draft and to have been accepted May 10. What is the day of maturity and how long has the draft to run before due?

2. If sold on the day of acceptance at 6% discount, what are the proceeds? (Remember that when accepted it is practically a promissory note.)

3. The bank at Topeka will likely charge a small fee for collecting the draft. If they charge $\frac{1}{4}\%$ (of the face of the draft) for collecting (exchange) and discount it on the day it is bought, what shall they remit the State Bank at Chicago?

4. F. Alton of New Orleans draws at sight on R. Fay of Waco, Tex., Aug. 3, 1903, for \$ 500. Make the draft.

5. Suppose the debt not due till Nov. 1. Make a proper time draft dated Aug. 3.

6. If discounted Aug. 18, the proceeds would be \$ *x*.

7. If discounted Aug. 3, less an additional $\frac{1}{2}\%$ for exchange, the proceeds would be \$ *y*.

8. Paine of Macon, Ga., owes Drew of Atlanta \$ 4000, due Jan. 1. Paine accepts a draft Oct. 1 and discounts it himself for \$ *x*.

Sight or time drafts if known to be "good" may be sold to a bank by allowing a percentage for *cost of exchange*.

Thus the State Bank of Chicago might have bought the draft on the preceding page of Richards & Co., and charged them a fee of say $\frac{1}{2}\%$ for collecting.

9. A Charleston bank buys a draft on Richmond for \$ 2100, charging $\frac{1}{2}\%$ for exchange. If it had taken the draft for collection only, the charge would have been 25¢. To the maker what is the difference in money? In which case is payment made more quickly?

10. Write a draft of which you are the maker, your teacher the payee, and a neighboring bank the drawee.

1. $(17\frac{1}{2})^2 - \sqrt{361} = 10\%$ of what?
2. How much pays for
 $3\frac{7}{8}$ yd. silk at \$2.50;
 2 pr. blankets at \$7.37 $\frac{1}{2}$;
 10% off to the trade and 2% for cash?
3. Add horizontally: 3.75, 23.08, 176.97, 0.83 $\frac{1}{4}$, 12.37 $\frac{1}{2}$.
4. \$156.91, \$73.99, \$1439, \$76.84, \$97 $\frac{1}{2}$, \$42.97, \$198 $\frac{1}{2}$.
5. I bought a bill of goods invoiced at \$350. I received two discounts of 10% and 5% each. Find the net cost including \$13.40 freight.

Find cost, but write only the products for adding:—

- | | | |
|-------------------------------|--|---------------------------------------|
| 6. 16 $\frac{2}{3}$ at 80¢ | 8. 901 at 25¢ | 10. $1\frac{1}{6}$ at 20¢ |
| 48 at 66 $\frac{2}{3}$ ¢ | 1200 at 75¢ | $1\frac{5}{11}$ at 37 $\frac{1}{2}$ ¢ |
| 75 at 12 $\frac{1}{2}$ ¢ | 12 at \$1.16 $\frac{2}{3}$ | 4 $\frac{3}{8}$ at 25¢ |
| 24 at 62 $\frac{1}{2}$ ¢ | 39 at \$2.33 $\frac{1}{3}$ | 100 at 2 $\frac{1}{8}$ ¢ |
| 37 $\frac{1}{2}$ at 10¢ | 42 at 83 $\frac{1}{3}$ ¢ | $1\frac{5}{12}$ at \$1.25 |
| 7. 6400 at 87 $\frac{1}{2}$ ¢ | 9. 2 $\frac{2}{3}$ at 21¢ | 11. $1\frac{5}{8}$ at 90¢ |
| 279 at 11 $\frac{1}{5}$ ¢ | 4 $\frac{7}{8}$ at 50¢ | 48 at 18 $\frac{3}{4}$ ¢ |
| 108 at 8 $\frac{1}{8}$ ¢ | $1\frac{7}{12}$ at \$1.00 | 1000 at 2 $\frac{1}{8}$ ¢ |
| 144 at 6 $\frac{1}{4}$ ¢ | 10 $\frac{3}{4}$ at 28¢ | $1\frac{7}{12}$ at \$2.50 |
| 1608 at 33 $\frac{1}{3}$ ¢ | 16 $\frac{1}{2}$ at 33 $\frac{1}{3}$ ¢ | $1\frac{1}{12}$ at 87 $\frac{1}{2}$ ¢ |

Find total interest due on five notes, as follows. Write only results for adding:—

| 12. | 13. | 14. |
|--------------------------------|--------------------|---------------------------------|
| \$500, 2 $\frac{1}{2}$ yr., 4% | \$720, 45 da., 6% | \$700, 4 yr., 4 $\frac{1}{2}$ % |
| \$630, 60 da., 8% | \$376, 30 da., 3% | \$300, 90 da., 6% |
| \$180, 30 da., 12% | \$200, 12 da., 4% | \$400, 63 da., 3% |
| \$900, 4 mo., 9% | \$820, 10 mo., 6% | \$480, 2 $\frac{1}{2}$ yr., 5% |
| \$200, 15 da., 6% | \$1800, 10 da., 4% | \$150, 54 da., 6% |

1. Estimate the commission on a sale of \$5000, at 5%.
2. Insured a mill, valued at a quarter of a million of dollars, for \$200,000, at the following rates for 5 years: \$50,000 in each of three companies, at $\frac{3}{4}\%$; \$20,000 at $\frac{5}{8}\%$; and the remainder at $\frac{7}{8}\%$. What per cent of the value of the property is my annual premium?
3. \$40,000 is to be raised by taxation for building a schoolhouse. The assessed valuation of the town is \$6,400,000. My property is assessed at \$24,000. What shall I pay towards the cost of the schoolhouse?

Find proceeds of notes without grace:—

4. \$ 60, 3 mo., 8%. 6. \$850, 47 da., 4%. 8. \$1900, 63 da., 4%.
5. \$260, 90 da., 6%. 7. \$946, 62 da., 3%. 9. \$1217, 14 da., 8%.
10. Imported 40,000 lb. sugar invoiced at $2\frac{3}{4}\phi$, at 40% *ad valorem*, and $\frac{1}{8}\phi$ per pound, specific. Required the cost of the importation.
11. Bought 19 shares of 7% manufacturing stock, at 128 $\frac{3}{4}$. Received a semiannual dividend, and then sold for 126. No brokerage. My gain or loss?
12. Sold \$50,000 worth of hides at 2% commission, and with the net proceeds bought cotton at the same commission. Required my total commission.
13. Bought 500 tons Franklin coal at \$5.62 $\frac{1}{2}$. Sold at an average advance of 22%, but lost 5% in bad debts. Required net gain. Allow 25 ϕ a ton storage, and \$32.60 for other expenses.
14. A 6-months' note for \$500, bearing 6% interest, is discounted at a bank 40 days before maturity. Required the proceeds.
15. Invested \$5000, in stock paying 5%, at par value. I borrowed the money at 3 $\frac{1}{2}\%$. What was my annual gain?
16. A dealer sold a piano for 25% less than his asking price and yet made \$100, or 20% on the cost. Required the cost.

1. The rate of postage to publishers is 1¢ a pound. If *The Century* averages $1\frac{1}{2}$ pounds per copy, what will be the postage bill for a month if 90,000 copies are mailed at this rate?

2. If wine contains $7\frac{1}{2}\%$ of alcohol, how much alcohol in 6 dozen quart bottles that are 5% short of full?

3. The price of silver Nov. 22, 1902, in London was the lowest on record, $22\frac{1}{2}d.$ per ounce. What was this per troy pound in our money? Take English money at par.

4. \$18,000,000 New York Central $3\frac{1}{8}\%$ bonds were sold by J. P. Morgan & Co. at $106\frac{3}{4}$. What was the total premium? What is the total and annual income? What per cent of income do they yield at this rate?

5. Hired \$50,000 at 5% simple interest for 3 years and 6 months. I loaned it at $4\frac{1}{2}\%$ compound interest. What did I gain or lose by the transaction?

6. In the Battle of Gettysburg the Union army lost 23,000 men, 16,500 of whom were either killed or wounded. What per cent of the loss were taken prisoners?

7. Gained 28% by selling a coach at \$640. What per cent should I have gained, if I sold it for \$728?

8. I gained \$1110, and this was 37% of what I lost on another transaction. Required the net loss or gain.

9. The gross income of the Prudential Insurance Company is \$126,000. If the average premium upon the risks taken during the year is $2\frac{3}{8}\%$, what is the face of the policies which the company has written during the year?

10. The price on a certain quantity of standard furniture, as quoted by J. H. Harper, amounts to \$546.84, 20% and 5% off; and as quoted by H. A. Warner & Co., \$575.30, 25% and 10% off. Which is the better offer and how much?

1. What is the ratio of 3 to 5? This ratio may be written $3 \div 5$; $\frac{3}{5}$; or 3 : 5.

Two equal ratios will make a proportion. Thus:—

$$12 : 3 = 16 : 4$$

$$5 : 20 = 6 : 24$$

$$\text{or } \frac{12}{3} = \frac{16}{4}$$

$$\frac{5}{20} = \frac{6}{24}$$

The four numbers are said to be *in proportion*, and are read "12 is to 3 as 16 is to 4," or "the ratio of 12 to 3 is equal to the ratio of 16 to 4."

Test the following proportions to see whether the ratios are equal:—

2. $6 : 48 = 5 : 40$.

6. $\frac{2}{3} : \frac{3}{4} = 8 : 9$.

3. $32 : 4 = 96 : 12$.

7. $7 : 42 = 9 : 54$.

4. $5 : 3 = 35 : 21$.

8. $2\% : 4\% = 12\frac{1}{2}\% : 25\%$.

5. $2\frac{1}{2} : 5 = 4\frac{1}{2} : 9$.

9. $3 \text{ yd.} : 4 \text{ yd.} = \$0.75 : \$1.00$.

The *first* and *last* terms of a proportion are the **extremes**; the *second* and *third* terms are the **means**. The first term in a ratio is an *antecedent* and the other a *consequent*.

10. Which terms are dividends? Which divisors? Which may be numerators? Which denominators?

11. Arranging the proportion $16 : 8 = 10 : 5$ in a fractional form, we have $\frac{16}{8} = \frac{10}{5}$; multiplying both sides of this equation by 5×8 or 40, we have $16 \times 5 = 10 \times 8$.

What terms are 16 and 5? Which are the means?

12. What conclusion may we draw from Exercise 10?

Principle. *In a proportion, the product of the extremes is equal to the product of the means.*

13. At least one pair of the ratios must be abstract numbers before we may apply this principle. Why is this?

NOTE. All the terms of a proportion may be expressed as abstract numbers for 5 ft. \div 2 ft. $= \frac{5}{2}$ or $5 \div 2$.

14. Write the proportion in Exercise 8 in abstract terms.

15. Test the proportions in Exs. 1-8 by the principal on page 206.
16. Find the values of x (make use of the principle on page 206):—
- $$3 \times 18 = x \times 9 \qquad 3 : x = 9 : 18 \qquad 7 : 16 = x : 18$$
- $$9 \times x = 6 \times 15 \qquad 9 : 6 = 15 : x \qquad x : 24 = 6 : 72$$
17. How is a missing factor found? How is the missing term of a proportion found?

Find the missing term in these proportions:—

- | | |
|--|---|
| 18. $17 : 60 = 85 : x$. | 24. $\$x : \$9 = 60 \text{ lb.} : 27 \text{ lb.}$ |
| 19. $27 : x = 81 : 100$. | 25. $10 \text{ T.} : \frac{1}{2} \text{ T.} = \$x : \$3.75$. |
| 20. $x : 16 = 130 : 26$. | 26. $15 : x = 3 : 12$. |
| 21. $324 : 243 = 12 : x$. | 27. $24 \text{ lb.} : x \text{ lb.} = 2 : 8$. |
| 22. $\frac{1}{12} : x = 8 : 16$. | 28. $9\frac{1}{2} : 16\frac{1}{4} = 38 : x$. |
| 23. $\$90 : \$48 = 15 \text{ yd.} : x \text{ yd.}$ | 29. $7.25 : 0.5 = x : 3$. |
30. Why must the terms of a ratio be like numbers?

Since the principle of proportion may be applied to the solution of problems in which three terms are given to find the fourth, the subject is sometimes called the "**Rule of Three.**"

Written

1. If 16 yards cost \$40, what will 10 yards cost?

PROCESS

$$16 : 10 = \$40 : \$x$$

$$\begin{array}{cc} 5 & 5 \\ \$x = \frac{10 \times \$40}{16} = \$25 \\ & 8 \end{array}$$

EXPLANATION. Since the ratio of their costs must be equal to the ratio of their lengths, or since their costs are proportional to their lengths, we have the proportion. The question then is to find one of two numbers when the product is known. Write the dividend above the divisor, and use cancellation. Since the cost is wanted, we write the abstract ratio of their lengths.

2. When 60 bushels of oats cost \$36, what will 25 bushels cost?
3. What will 18 tons of hay cost when 7 tons cost \$147?

1. If 14 men can do a piece of work in 9 days, how long will it take 3 men ?

$$3 : 14 = 9 \text{ days} : x \text{ days.} \quad 2. \text{ Does the time required to do a}$$

$$x = \frac{14 \times 9 \text{ days}}{3} = 42 \text{ days.} \quad \text{piece of work increase or decrease as}$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{the number of workmen increases or}$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{decreases ?}$$

3. What is the ratio of 14 men to 3 men ?

4. What is the ratio of the time of 14 men, or 9 days, to time of 3, or 42 days ?

Since the time decreases as the number of men increases, a proportion of this sort is called an **inverse proportion**.

5. 27 men build a bridge in 12 days. How long will it take 36 men to build it ? What is the ratio of 27 to 36 ?

6. In the Boer War a town was garrisoned by 2100 men, who had provisions for a 9 months' siege. How long would the provisions last, if they should receive a reinforcement of 600 men ?

7. $2\frac{1}{8}$ dollars is paid for $4\frac{1}{4}$ bu. oats. What will $19\frac{1}{2}$ bu. cost ?

8. When $3\frac{3}{4}$ tons cost \$ 27.50, what will $4\frac{5}{8}$ tons cost ?

9. A certain piece of work was to have been performed by 288 men in 72 days. A number of workmen having been taken sick, 108 days were required. How many men were taken sick ?

10. The ratio is $2\frac{2}{3}$; the first term, $\frac{1}{2}$ of $\frac{5}{8}$. What is the second ?

11. If a loaf weighs 6 oz. when flour is \$ 4.50, what ought it to weigh when flour is \$ 6 per barrel ?

12. 112 men can pave a street in 18 days. The work must be done, however, in 12 days. How many more men must be employed ?

13. If a loaf of bread cost 5¢ when flour is \$ 6 a barrel, what is the probable price when the same loaf can be bought for 4¢ ?

Notice that the problems given under the subject of proportion are not new, but just like those we have solved by other forms of analysis. The only new feature is the way of writing the statement of the relations and the method of solution depending upon the principle on page 206. This form of solution is used very little in practical arithmetic, but is of great value in problems in physics and geometry.

1. If 17 tons of coal are worth \$ 135, what are 85 tons worth ?

SOLUTION. $5 \times \$ 135 = \$ 675$. When one ratio can be easily seen, perform that part of the work mentally and thus save time. Use this method when you can.

2. Compare \$18 with \$198. If I earn the smaller sum in $5\frac{1}{2}$ days, how long ought I to be in earning the larger sum ?

3. The product of the means in a proportion is $17\frac{4}{15}$. The first extreme is $7\frac{1}{2}$. What is the missing extreme ?

4. If 13 sheep cost \$ 91, what will 117 sheep cost ?

5. A clock ticks x times in $1\frac{1}{2}$ hours if it ticks 90 times in a minute.

6. If a post 14 ft. high casts a shadow 17 ft. long, how high is the church whose shadow is 136 ft. long at the same time ?

7. Two men are traveling toward each other. A travels $\frac{5}{8}$ as far as B in a day. If A must go 105 miles before they meet, how far will B have to go, and what is the distance between them ?

8. A farmer raised 320 bu. of potatoes on $3\frac{1}{2}$ acres of land, and the next year he decided to plant $10\frac{1}{2}$ acres with potatoes. How many bushels do you think he might expect if the yield was a good one ?

9. A grocer paid \$ 6.80 for 17 doz. eggs, and found that he must have $8\frac{1}{2}$ doz. more to fill his orders. How much did he remit with his second order ?

10. If 6 bbl. of flour are made from 30 bu. of wheat, how many barrels should be made from 55 bu. of wheat ?

1. Compare $37\frac{1}{2}$ with $7\frac{1}{2}$. If $7\frac{1}{2}$ barrels cost \$12.50, what will $37\frac{1}{2}$ barrels cost? Explain: $5 \times \$12.50 = \62.50 .

2. Compare $4\frac{2}{3}$ with 46. If 27 mi. of track are laid in 46 da., how many miles may be laid in $4\frac{2}{3}$ da.?

3. One mason can lay $\frac{5}{8}$ as many brick in a day as another. If the better man has \$3 a day, what ought the other to be paid?

4. Compare 3 sq. ft. with 2 sq. yd. If napkins containing the smaller quantity sell for \$3 a dozen, what ought I to pay for a dozen tablecloths containing the larger quantity?

5. If 36 yards of carpet must be bought for a floor when the width of a strip is 36 inches, how many yards are required when the strip is 27 inches wide?

6. A cog-wheel having 8 cogs plays into another having 24 cogs. When the small wheel has made 42 revolutions, how many has the larger wheel made?

7. If 18 men can do a piece of work in 30 days, in how many days can they do it with the assistance of 6 more men?

8. A piece of work can be done by 24 men in 30 days. How many men will it take to do $\frac{1}{2}$ of the work in 20 days?

9. If 9 weeks' board costs \$94.50, what will 12 weeks' board cost?

10. If a yacht sails 24 miles in 70 minutes, how long will she be in sailing 108 miles?

11. If 72 men lay 2 miles of water pipe in 15 days, how many days will 48 men require?

12. If a train runs 1000 miles in 28 hours, how many miles can it run in 120 hours?

13. If $2\frac{3}{4}$ yd. of cloth can be bought for \$23.10, what should be paid for $15\frac{1}{4}$ yd. at the same rate?

14. If 1 A. yields 22 bu. 3 pk. of corn, how many acres would yield 546 bu.?

1. Find these powers and roots:—

$$10^2; \sqrt{100}; 40^2; \sqrt{1600}; 70^2; \sqrt{4900}.$$

$$20^2; \sqrt{400}; 50^2; \sqrt{2500}; 80^2; \sqrt{6400}.$$

$$30^2; \sqrt{900}; 60^2; \sqrt{3600}; 90^2; \sqrt{8100}.$$

2. How is a number squared?
3. How are roots related to powers?
4. How many equal factors make a square?
5. What is the square root of 25×49 or 1225 ?
6. Find the square root of $11,025$.

PROCESS

$$\begin{array}{r} 5 \overline{) 11,025} \\ 5 \overline{) 2,205} \\ 3 \overline{) 441} \\ 3 \overline{) 147} \\ 7 \overline{) 49} \\ 7 \end{array}$$

7. Explain the process.

8. In the same way find the square root of 9216 .

9. $\sqrt{396,900}$.

11. $\sqrt{117,649}$.

10. $\sqrt{194,481}$.

12. $\sqrt{176,400}$.

Therefore $5 \times 3 \times 7$, or 105 , is the square root of $11,025$.

Extracting the square root of a number or separating it into two equal factors is the reverse of squaring one of these equal factors. A careful analysis of the process of squaring will enable us to reverse the process and find the square root of a number when it cannot be readily found by factoring.

1. Square 47 .

PROCESS

$$\begin{array}{r} 47 \\ 47 \\ \hline 329 = 7^2 + 7 \times 40 \\ 1880 = 7 \times 40 + 40^2 \\ 2209 = 7^2 + 2 \times 7 \times 40 + 40^2 \end{array}$$

Observe that in the process of multiplication we first find 7×7 or 7^2 ; next, 7×40 ; next, 40×7 ; and finally, 40×40 or 40^2 .

2. 83^2 in the same way is equal to $3^2 + 3 \times 80 + 80 \times 3 + 80^2 = ?$

3. Compare 3×80 with 80×3 . Then $3 \times 80 + 80 \times 3 = 2 \times 3 \times 80$.
4. $36^2 = 30^2 + 2 \times 6 \times 30 + 6^2$, or —.
5. In the same way find 57^2 ; 72^2 ; 28^2 .

1. Square 64 by the method on the preceding page.

WORK

64

64

$16=4^2$

$480=2 \times 4 \times 60$

$3600=60^2$

$4096=64^2$

2. Which of the partial products is the largest?

3. From which digit was it obtained?

4. Which is the smallest of the partial products?

5. From which digit was it obtained?

6. If 3600 were taken from the product, most of what remains is made from what factors?

7. Give the squares of all the numbers from 1 to 9 inclusive.

8. How many figures in each of these squares?

9. Square the numbers 10, 20, 30, and so on to 100.

10. Compare the square of 30 with the square of 3.

11. Compare 80^2 with 8^2 ; 60^2 with 6^2 ; 40^2 with 4^2 .

12. How many figures in the squares of numbers from 10 to 99 inclusive?

13. What is the square of 100? Of 200? Of 999?

14. How many figures in the squares of numbers from 100 to 999 inclusive?

15. If there are four figures in the square, how many in the root? How many in the root when five figures are in the square?

16. Give the number of figures in the square root of:—

9409, 381, 27,225, 182,329, 49,434,961.

17. How does the number of places in a square compare with the number of places in the root?

18. Square 0.2; 0.02; 0.4; 0.12; 0.25; 0.03; 0.005.

19. Compare the number of decimal places in the power with the number in the root.

20. Why can the square of a decimal never contain an odd number of decimal places?

To find the square root, or one of the two equal factors, of 2809.

| PROCESS | |
|--|---|
| 2809 (50+3) | 1. How many figures in this square? |
| 2500=50 ² | 2. Then how many in the root? |
| 309=2 × 50 × x + x ² (why?) | 3. What is the square of 50? Of 60? |
| 300=2 × 50 × 3 | 4. Between what two squares does 2809 come? |
| 9=3 ² | |

- Then its root lies between what two numbers?
- If the root lies between 50 and 60, the largest of the three partial products that make the square is what?
- When 2500 is taken from 2809, what two partial products are contained in the 309 remaining?
- Most of the 309 is made from which of the partial products?
- Then since 309 is more than 2 × 50 times the number yet to find, about what must the number be?
- When 2 × 50 × 3 is taken from 309, what one of the three partial products remains?
- Is 9 equal to 3²? Then the second number must be 3 and the entire root is 50 + 3 or 53. Prove by squaring.
- Give the complete process of finding the root of a square containing three or four figures.

In the same way find the square root of:—

- | | | | | |
|-----------|-----------|-----------|-----------|-----------|
| 13. 528. | 14. 676. | 15. 1156. | 16. 1764. | 17. 2025. |
| 18. 2916. | 19. 3969. | 20. 4624. | 21. 5625. | 22. 7056. |
- Prove each by squaring by the method on pp. 211, 212.
 - Give two factors of 25. Of 25 ft. Of 25 sq. ft. Of \$ 25.
 - In which of these were the factors equal? Of which were you able to extract the square root?

A concrete number cannot have a square root, that is, two equal factors, for one factor must be abstract. Why?

1. Find the square root of 8836.

SHORT PROCESS

$$\begin{array}{r} 88'36(94 \\ 81 \\ 2 \times 90 = 180 \overline{) 736} \\ \underline{4} \\ 184 \overline{) 736} \end{array}$$

We may omit the zeros in the square of 90, also the zero of the 90.

2. Compare $4 \times 180 + 4 \times 4$ with 4×184 .

Thus we see that we may also save work by adding the 4 to 180 before multiplying by 4.

Solve by both processes and show what you save by the shorter:—

- | | | | |
|--------------------|--------------------|---------------------|---------------------|
| 3. $\sqrt{784}$. | 6. $\sqrt{3136}$. | 9. $\sqrt{5329}$. | 12. $\sqrt{7569}$. |
| 4. $\sqrt{3364}$. | 7. $\sqrt{6889}$. | 10. $\sqrt{4489}$. | 13. $\sqrt{2809}$. |
| 5. $\sqrt{8464}$. | 8. $\sqrt{2704}$. | 11. $\sqrt{9801}$. | 14. $\sqrt{9409}$. |

To find the square of 347.

WORK

$$\begin{array}{r} 347 \\ 347 \\ 90000 = 300^2 \\ 12000 \} = 2 \times 300 \times 40 \\ 12000 \} \\ 1600 = 40^2 \\ 2100 \} = 2 \times 300 \times 7 \\ 2100 \} \\ 280 = 2 \times 40 \times 7 \\ 280 \} \\ 49 = 7^2 \\ \hline 120409 \end{array} \quad \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} = 2 \times 340 \times 7$$

1. Of the partial products, which is the greatest? The next in size? The smallest?

2. Give the five partial products in the square of 265, giving the largest first.

3. In the same way square 321; 248; 563.

To find the square root of 120409.

$$\begin{array}{r} \text{Root} \\ 120409 \overline{) 300} \\ 90000 \overline{) 40} \\ 2 \times 300 = 600 \overline{) 30409} \\ 24000 = 600 \times 40 \\ 6409 \\ 2 \times 340 = 680 \overline{) 1600} \\ 4809 \\ 4760 = 680 \times 7 \\ 49 = 7^2 \end{array}$$

1. How did we find the 300 of the root? What use was made of it in finding the 40?

2. How did we use the 300 and the 40 in finding the 7?

3. Show by referring to the problem at the left just what each remainder in this solution contains.

The process on the preceding page may be used to find the square root of any number.

To find the square root of 2137444.

| PROCESS | |
|---------------------|-------------------|
| 1000 ² = | 2'13'74'44 1000 |
| 2000 | 1 00 00 00 400 |
| 400 | 1 13 74 44 60 |
| 2400 | 96 00 00 2 |
| 2800 | 1462 = root |
| 60 | 177444 |
| 2860 | 171600 |
| 2920 | 5844 |
| 2 | 5844 |
| 2922 | |

| SHORT PROCESS | |
|---------------|-----------------|
| | 2'13'74'44(1462 |
| | 1 |
| 2 | 113 |
| 24 | 96 |
| 28 | 1774 |
| 286 | 1716 |
| 292 | 5844 |
| 2922 | 5844 |

Describe the process.

Find the square root of: —

- | | | |
|------------|-------------|--------------|
| 1. 283024. | 6. 6017209. | 11. 769129. |
| 2. 299209. | 7. 529984. | 12. 935089. |
| 3. 404496. | 8. 484416. | 13. 1153476. |
| 4. 556516. | 9. 638401. | 14. 1481089. |
| 5. 755161. | 10. 725904. | 15. 2540836. |

To find the square root of 0.501.

| PROCESS | |
|--------------------|---------------|
| 0.7 ² = | 0.501(0.707 + |
| 1.4 | 0.49 |
| .007 | 0.0110 |
| 1.407 | 0.009849 |
| | 0.001151 |

EXPLANATION. As the square of *tenths* gives *hundredths*, to get the first root figure we take the first two figures at the right of the point or .50, the root of which is nearly 0.7. Each new quotient figure is determined by division, as in the case of integers. Since the square of a decimal cannot give an odd number of figures, this decimal must be an *imperfect* power, and the root

cannot be *exactly* determined. This is indicated by a + or - after the last root figure.

Find the square root of:—

- | | | | |
|------------|------------|-------------|---------------|
| 1. 0.5625. | 5. 0.89. | 9. 0.64. | 13. 225.9009. |
| 2. 0.9216. | 6. 19.467. | 10. 0.064. | 14. 2044900. |
| 3. 42.225. | 7. 824.9. | 11. 1932.4. | 15. 76.3876. |
| 4. 0.783. | 8. 17.035. | 12. 783.95. | 16. 0.8. |

A. $\sqrt{\frac{829}{405}} = \sqrt{\frac{84}{81}} = \frac{8}{9}$.

B. $\sqrt{\frac{8}{9}} = \sqrt{0.375} = 0.612 +$

C. $\sqrt{7\frac{1}{9}} = \sqrt{\frac{64}{9}} = \frac{8}{3} = 2\frac{2}{3}$.

D. $\sqrt{4\frac{4}{9}} = \sqrt{\frac{40}{9}} = \frac{6.3245+}{3} = 2.108 \pm$.

E. $\sqrt{6\frac{8}{9}} = \sqrt{6.375} = 2.52 \pm$.

In finding the root of fractions:—

I. First change them to simplest form, as in A or C.

II. Use the method in A or C when both terms are perfect powers.

III. Use B or E when both terms are imperfect powers.

IV. D may be used when the denominator is a square.

- | | | | |
|--------------------------------|-----------------------------|----------------------------------|-------------------------------|
| 17. $\sqrt{\frac{147}{482}}$. | 19. $\sqrt{6\frac{1}{2}}$. | 21. $\sqrt{82\frac{1}{2}}$. | 23. $\sqrt{151\frac{1}{3}}$. |
| 18. $\sqrt{\frac{5}{16}}$. | 20. $\sqrt{5\frac{2}{3}}$. | 22. $\sqrt{\frac{1205}{8480}}$. | 24. $\sqrt{2\frac{8}{16}}$. |

Extract the square root of:—

Oral

- | | |
|--------------------------|------------------------|
| 1. 14400. | 11. 0.49. |
| 2. $\frac{1}{9}$. | 12. 0.049. |
| 3. $(5\frac{1}{2})^2$. | 13. 0.00490. |
| 4. $(16\frac{1}{2})^2$. | 14. 625. |
| 5. 0.09. | 15. 0.625. |
| 6. 36×49 . | 16. 16 million. |
| 7. 1735. | 17. $7\frac{1}{2}$. |
| 8. $(87\frac{1}{2})^2$. | 18. $30\frac{1}{2}$. |
| 9. 9×81 . | 19. $272\frac{1}{4}$. |
| 10. 0.0625. | 20. $10\frac{9}{16}$. |

Written

- | | |
|--|------------------------|
| 1. 94249. | 11. $95\frac{1}{8}$. |
| 2. 0.729. | 12. 16^3 . |
| 3. $13\frac{7}{8}$. | 13. 2. |
| 4. 1008016. | 14. $\frac{1}{9}$. |
| 5. 9834496. | 15. $\frac{8}{27}$. |
| 6. 62742241. | 16. $127\frac{1}{2}$. |
| 7. 2033.1081. | 17. 785. |
| 8. $\frac{8}{9}$. | 18. 3444736. |
| 9. $\frac{3}{4} + \frac{5}{8} + \frac{8}{9}$. | 19. $17^2 + 25^2$. |
| 10. 998001. | 20. 0.741. |

1. The law governing the three partial products forming a square may be more easily remembered if stated in a formula as follows —

$$(a + b)^2 = a^2 + 2ab + b^2$$

Thus, $23^2 = (20 + 3)^2 = 20^2 + 2 \times 3 \times 20 + 3^2 = 400 + 120 + 9 = 529$.

2. $16^2 = (12 + 4)^2 = 144 + 96 + 16 = 256$; thus we see that a and b need not represent *tens* and *ones* respectively, but *any* two numbers.

3. Knowing that $25^2 = 625$, find 27^2 , as in Example 2.

Square Root

PROCESS

$$\begin{array}{r}
 \begin{array}{cc} a & b \\ a^2 + 2ab + b^2 = 7056. & (80 + 4 \\ a^2 = 6400 \\ 2ab + b^2 \text{ or } (2a + b)b = & 656 \\ 2a = 160 \\ b = & 4 \\ (2a + b) = 164 & \end{array} \\
 \hline
 656 = 2ab + b^2
 \end{array}$$

EXPLANATION. 7056 comes between the square 6400 and 8100, hence its root is between 80 and 90. Then a must be 80, and a^2 , 6400. Taking this out, what remains?

The remainder, 656, then is made up of two factors, $2a + b$, and b . $2a$ of one being known is used as a *trial divisor* to get a clew to the other factor, b , which in this cannot be more than 4. The whole factor, or the *complete divisor*, then is $160 + 4$, or 164. Since this with 4 makes 656, the exact root is 84.

If there had been a remainder, we should have next considered all the root found, or 84, as a and used $2a$, or 168, for our next trial divisor to find the next root figure.

Using the formula, find to two decimal places the square root of: —

- | | | | |
|-----------|-----------|-----------------------|-----------------------|
| 1. 80. | 6. 17.45. | 11. 3.33. | 16. $5\frac{1}{2}$. |
| 2. 10.35. | 7. 15. | 12. 0.7854. | 17. $79\frac{1}{2}$. |
| 3. 6.43. | 8. 8.56. | 13. $39\frac{1}{2}$. | 18. $34\frac{1}{2}$. |
| 4. 9.8. | 9. 9.42. | 14. $7\frac{1}{2}$. | 19. 64.01. |
| 5. 7.2. | 10. 0.7. | 15. 0.360. | 20. $36\frac{2}{5}$. |

1. Draw a right triangle with the sides which form the right angle, 3 inches and 4 inches respectively
2. Measure the length of the other side, or **hypotenuse**.
3. Draw a square on each of the three sides as base.
4. Compare the square on the hypotenuse with the sum of the squares on the other sides.

Pythagoras proved about 500 B.C. that the fact that we find true here is true for *any* right triangle, viz. that

The square on the hypotenuse is equal to the sum of the squares on the other two sides.

5. Carpenters make use of this fact, in laying out the foundation for a building, when they want to form a right angle. A line 8 feet long is taken in one direction along which the foundation is to be made. Another line 6 feet long is fastened to one extremity of the first line and moved until a 10-foot rod will just reach the outer extremity of the two lines. Draw such a figure, and show that this gives a right triangle.

6. Use the test in 5, and find whether the walls of your school-room are perpendicular to the floor.

7. If the square on the hypotenuse is 100 sq. in. and on one of the sides 36 sq. in., what is the length of each side of the triangle?

Denoting the hypotenuse by H , the base by B , and the perpendicular by P , when these are abstract numbers representing the *number* of units in the dimensions, we may state from the above principle the following formulæ:—

$$H = \sqrt{B^2 + P^2}$$

$$B = \sqrt{H^2 - P^2}$$

$$P = \sqrt{H^2 - B^2}$$

8. Explain the formulæ.

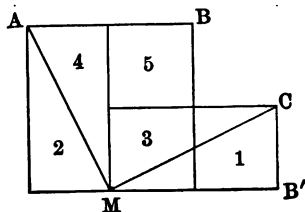
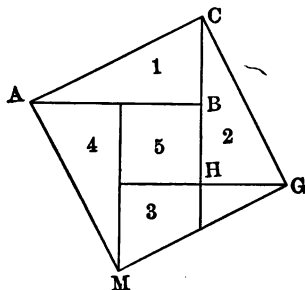
9. If $H = 15$ and $P = 14$, $B = x$.

10. If $B = 15$ and $P = 16$, $H = x$.

11. If $H = 25$ and $B = 20$, $P = x$.

The Right Triangle

The truth of the Pythagorean theorem, stated on the preceding page, may be seen by drawing, or cutting from cardboard, figures like the following:—



Let ABC be the right triangle. The square on the hypotenuse AC is equal to the 4 triangles, 1, 2, 3, and 4, and the small square, 5. Now put 1 and 2 in the position of the figure at the right, and the figure is equal to a square on AB and one on CB' .

1. The base of a right triangle is 48 feet and the perpendicular is 36 feet. What is the hypotenuse?

2. The hypotenuse is 85 feet and the perpendicular is 51 feet. What is the base?

3. The base is 76 feet and the hypotenuse is 95 feet. What is the perpendicular?

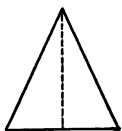
4. What is the diagonal of a rectangle 92 ft. long and 69 ft. wide?

5. What is the diagonal of a 30-ft. square?

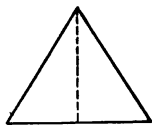
6. What is the longest line that can be drawn on a sheet of paper 16 inches wide and 20 inches long?

7. What is the diameter of the largest wheel that can be got through a doorway measuring 7 feet by 5?

8. What is the distance between the opposite corners of a field 200 rods long and half as wide?



ISOSCELES TRIANGLE



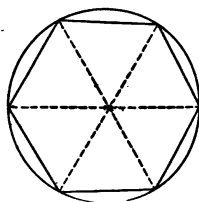
EQUILATERAL TRIANGLE

Prove by cutting or measuring that —

- (1) *The altitude of an isosceles triangle bisects the base.*
- (2) *The perpendicular from any vertex of an equilateral triangle to the opposite side bisects it.*

Since an equilateral triangle is also isosceles whatever side is taken as base, (2) could have been inferred from (1).

1. If the base of an isosceles triangle is 12 and the equal sides 10, what is the altitude?
2. Find the altitude of a triangle whose sides are each 10 inches.
3. A rectangle measures 22 ft. by 10 ft. How long is its diagonal?
4. The foot of a 25-foot ladder is 12 ft. from the side of the house against which it leans. How far from the ground is its top?
5. What is the area of a right triangle whose longest side is 20 ft. and its shortest 8 ft.?
6. Find the altitude of an equilateral triangle whose side measures 24 ft. Find the area.
7. What will it cost to fence a square field containing 5 A. at \$1.25 a rod?
8. A *regular hexagon* is made up of six equilateral triangles. Study the figure and discover how to inscribe one in a circle.
9. Find the area of a regular hexagon whose sides are each 10 inches.



A REGULAR HEXAGON

Remember that we cannot take the square root of a concrete number, such as 25 sq. ft., but of 25. In all the formulæ that follow, we are to consider areas, lengths, etc., as the *number* of units, and hence deal with abstract numbers.

1. Since the area of a circle $= \pi r^2$, or $3.1416 \times$ the square of the radius, $r = \sqrt{\frac{\text{area}}{\pi}}$. What is the radius of a circle whose area is 78.54 sq. in.?

2. What is the diagonal of a floor 25 feet long and 16 feet wide?

3. The diagonal of a rectangle 30 feet long measures 42 feet. What is the width of the rectangle?

4. The perimeter of a rectangle is 36 feet. Its width is half its length. What is its diagonal?

5. What is the diagonal of a square containing 32 square inches?

6. The top of a 30-foot ladder, which is placed 16 feet from the side of a house, reaches a window sill in the third story. How far from the ground to the window sill?

7. Two yachts start together. One sails due north and the other due east, each at the rate of 12 miles an hour. How far apart are they at the end of 4 hours?

8. What is the area of a circle drawn with an 18-inch radius?

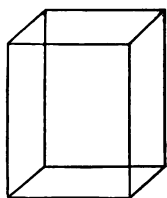
9. The area of a circle is 24.3474 sq. in. What is its diameter?

10. A line reaching from the bank of a stream to the top of a 50-foot pole on the other side is 275 feet long. What is the width of the stream?

11. The base of an isosceles triangle is 20 feet and its altitude 15 feet. What is the length of one of the equal sides?

12. A gable-roof house is 24 feet wide. The distance from the plate to the ridgepole is 12 feet. The rafters project 1 foot over the eaves. How long are they?

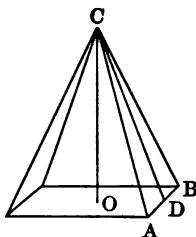
1. What is the length of a square equal in area to a rectangle 24 rd. long and 33 ft. wide?
2. What is the longest straight line that can be drawn on the ceiling of your schoolroom if it measures 32 ft. by 30 ft.?
3. Compare the perimeter of a rectangle 48 in. by 12 in. with that of a square of equal area.
4. How much do I save by crossing along the diagonal of a square that contains 1296 sq. rd. instead of going around its two sides?
5. How long is an acre of land in the form of a square?
6. How long a guy will support a derrick 48 ft. high if fastened 85 ft. from its base?
7. The hypotenuse of a right triangle measures 90 ft. The other sides are equal. How long are they?
8. Two poles are 100 ft. apart. One is 60 ft. high, and the other 80 ft. How long a line will connect their tops?
9. A rectangle 3 times as long as wide contains 3888 sq. ft. What are the dimensions? (*Hint.* Divide it into 3 squares.)
10. What is the altitude of an equilateral triangle whose base is 24 feet?
11. The base of an isosceles triangle is 84 feet, and one of the other sides is 50 feet. What is the altitude?
12. From the corner of a 12-inch square I cut an isosceles triangle one of whose sides is 4 inches. What is the area of that part of the large square which remains?
13. My son's kite is 1500 feet directly above the spot on which I am standing, and my son holds the string 1800 feet away. How much string has he let out? Allow 25 ft. for sagging.
14. How many rods do I save by taking the diagonal of a field 75 rods wide and 200 rods long, instead of going around the corner?



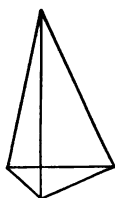
RECTANGULAR
PRISM



TRIANGULAR
PRISM



RECTANGULAR
PYRAMID



TRIANGULAR
PYRAMID

A **Pyramid** is a solid whose base is a polygon and whose sides or faces are triangles meeting at a common point called the **vertex** of the pyramid.

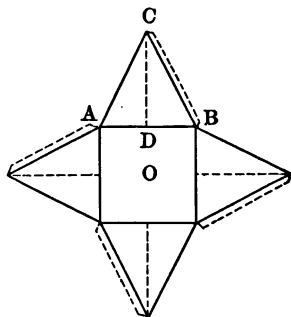
If the base is a regular polygon, as a square, or an equilateral triangle, and the sides are equal isosceles triangles, the pyramid is a **regular pyramid**.

The distance from the vertex to any side of the base of a regular pyramid is the **slant height**.

1. The distance from the vertex to the side is the altitude of the triangle, hence it divides the side into two equal parts. Why?

2. Construct from cardboard a pyramid whose base is a 4-inch square, and whose edges AC , etc., are 6 inches.

Draw a model, and leave lapels for pasting.



Written

3. What is the length CD , or the **slant height** of this pyramid?

4. Having found CD , and knowing OD , observe the figure at the top of the page, and find the height. (Observe that the altitude of a regular pyramid meets the base at its center.)

5. How could you have found AO and then the height from OA and AC ? Find it.

1. Make a prism having exactly the same base and altitude as the pyramid you have made. Test the accuracy of your construction by measuring, as in the figure.



Make an opening in the base of the pyramid, and fill with dry sand, and fill the prism from this as a measure.

2. What do you find true of their volumes ?
3. Make other prisms and pyramids as your teacher may direct, and test the accuracy of the following : —

The volume of a pyramid is $\frac{1}{3}$ of that of a prism having an equal base and an equal altitude.

4. A square pyramid is 12 ft. high and measures 3 ft. along one side of its base. What is its volume ?
5. What part of a square prism is whittled away by a boy who is making the largest pyramid possible out of it ?
6. The volume of a square prism is 36 cubic inches. What is the volume of a square pyramid of the same base and altitude ?
7. The area of the base of a triangular prism is 4 square feet. Its altitude is 5 feet. What is its volume ?
8. The contents of a square prism are 28 cubic feet. Its base covers 4 square feet. What is its altitude ?
9. The contents of an hexagonal prism are 42 cubic inches. The altitude is 6 inches. What is the area of the base ?

1. A granite shaft 10 ft. high and 20 in. square is surmounted by a square pyramid 2 ft. in altitude. The contents of both ?

2. Which is more easily measured, the slant height or the altitude of a pyramid ? Which line of a triangle is the slant height of a regular pyramid ?

3. The slant height of a square pyramid is 15 inches, and the side of the base 10 inches. Find its contents.

4. The area of the base of a pentagonal prism is $62\frac{1}{2}$ sq. in.; its altitude is 24 in.; the contents ?

5. A square prism has a base 2 feet long and an altitude of 10 feet. It is made of granite weighing 165 pounds to the cubic foot. What is the weight of the prism ?

6. How many surfaces has a square prism ? What is the shape of each one ?

7. A hexagonal pyramid is one having a hexagon as base. How many triangles make its visible surface ? Of what kind ?

8. What two lines in an isosceles triangle must be known in order to find its area ?

9. What is meant by the slant height of a regular pyramid ?

The total area of all the faces of a prism is called its **convex surface**.

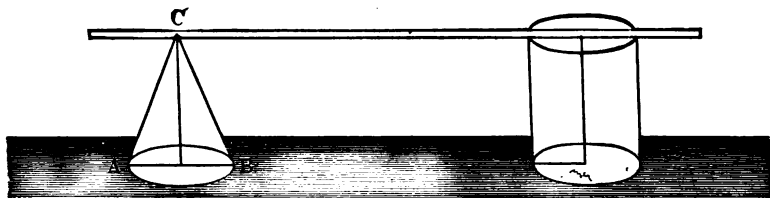
10. A square pyramid has a slant height of 12 inches and a base of 4 inches. What is its convex surface ?

11. An octagonal pyramid's slant is 16 inches, and the perimeter of its base is 48 inches. What is its convex surface ?

12. All four sides of a pyramid are equilateral triangles 6 inches long. Find the slant height and the convex surface.

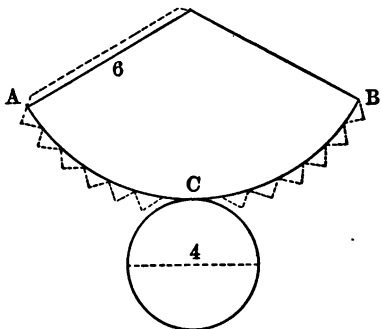
13. The altitude of a square pyramid is 15 in. and the side of its base is 12 in. Required its contents.

14. Find its slant height and convex surface.



A solid having a circle for a base and tapering uniformly to a vertex is a **cone**. AC is the **slant height**. While there are other kinds of cones, we shall consider the kind described above. In this kind the altitude from the vertex passes through the center of the base.

1. How can you find the altitude when the slant height is known?
2. Make a cone whose base is a circle, whose radius is 2 inches, and whose slant height is 6 inches. Make a model as in the margin.
3. How long is the arc CB ? What is its radius?
4. What will be the height of the cone?
5. Make a model for a cylinder of the same dimensions.
6. What will be the size and shape of the convex surface?
7. Test the accuracy of your construction by measuring as shown in the figure at the top of the page.
8. Using the cone as a measure, fill the cylinder with dry sand.
9. How do their volumes compare?
10. Make other sizes and show that—



The volume of a cone is equal to $\frac{1}{3}$ of that of a cylinder having an equal base and the same height.

11. How do we find the volume of a cylinder?

1. If a cylinder weighs 3 pounds, what will be the weight of a cone of the same material having the same base and altitude?
2. The largest possible cone is turned in a lathe out of a cylinder 6 inches long and 3 inches in diameter. What part of the cylinder goes into shavings? How many cubic inches in the cone?
3. The base of a cone is 6 square inches and its altitude is 12 inches. Find its contents.
4. The contents of a cone are 24 cubic inches. The altitude is 12 inches. What is the area of the base?
5. The diameter of the base of a cone is 4 feet. Its altitude is 9 feet. What are its contents?
6. A cylinder of ebony weighs 1 lb. 8 oz. What will an ebony cone of the same base and altitude weigh?
7. What is meant by a sector? How many lines in its boundary?
8. Show that the arc of a sector multiplied by $\frac{1}{2}$ of the radius will give the area of the sector. Compare the method with that of finding the area of a circle.
9. The convex surface of a cone is a sector, the circumference of the base being the arc of the sector, and the slant height of the cone its radius. The circumference of the base is 8 inches, and its slant height 10 inches. Area of its convex surface?
10. What is the convex surface of a cone whose altitude is 4 feet, and the diameter of whose base is 3 feet?
11. The radius of the base of a cone is $3\frac{1}{2}$ feet. The slant height is 5 feet. Find the entire area. (The area of the base + area of the convex surface.)
12. What is the convex surface of a cylinder 7 feet in diameter and 10 feet high?
13. How many yards of cloth will be required to make a conical tent 12 feet in diameter and 15 feet high? Add 5% for seams.

1. Cut away any slice of a sphere, as an apple. What is the form thus exposed?

2. When a sphere is bisected, that is, divided into two **hemispheres**, the plane surfaces thus exposed are **great circles** of the sphere. Would the diameter and circumference of one of these circles be the diameter and circumference of the sphere?

3. Wind the surface of a hemisphere as in figure C with a hard, waxed cord. (Place a small tack in the center, and around this wind the cord.)

4. With the same cord wind a great circle as in figure D.

5. Compare the two lengths, and thus the two surfaces.

6. Using cords of different sizes, and if convenient several sizes of spheres, show that —

The surface of a hemisphere is twice that of a great circle.

7. Compare the surface of a sphere with that of a great circle.

8. What is the area of a great circle of a sphere whose radius is 4 inches? What then is the surface of the sphere?

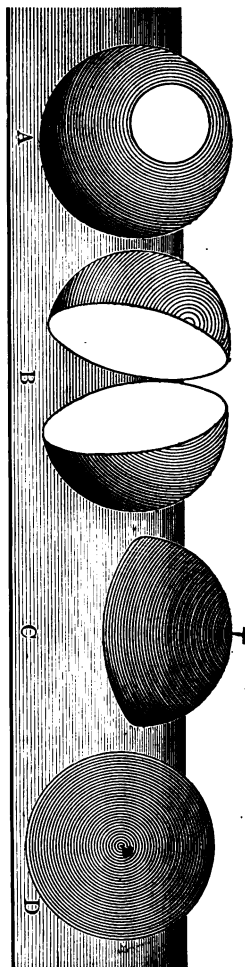
Remember then that —

The surface of a sphere = $4\pi r^2$ or πd^2 .
(r = radius; d = diameter.)

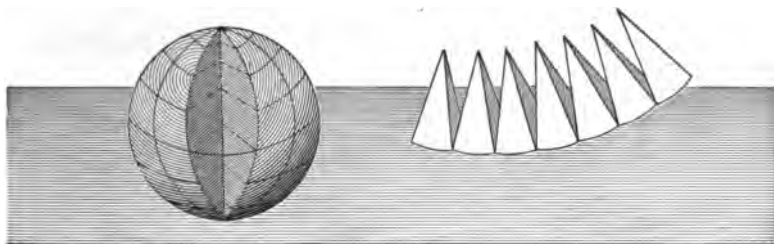
9. Show that $4r^2 = d^2$.

10. What is the surface of a sphere whose diameter is 10 inches?

11. When the radius is 4 inches, what is the surface of a sphere?



1. If a sphere should be dissected as in the accompanying illustration, what solids would its parts most resemble?



2. What line in the sphere forms the altitude of each pyramid-like solid?
3. What forms the base of each?
4. Taken together, what will the bases of all the pyramid-like solids make?
5. If these were perfect pyramids, how would the volume of any one be found?

While these solids are not pyramids, for their bases are not plane figures, yet it is proven in geometry that—

The volume of a sphere is the same as that of a pyramid whose base is the surface of the sphere and whose height is the radius of the sphere.

6. What should we obtain by multiplying the surface of a sphere by $\frac{1}{3}$ of its radius?
7. The surface of a sphere is 113 sq. in., and its radius 3 in. What are its contents?
8. How is the surface of a sphere found? $\frac{1}{2}$ of the radius is what part of the diameter?

9. Read and explain the following:—

$$\frac{R}{3} \times 4 R^2 \times 3.1416 = \frac{4}{3} R^3 \times 3.1416, \text{ or } \frac{4}{3} \pi R^3 = \text{volume of a sphere; or}$$

$$\frac{D}{6} \times D^2 \times 3.1416 = \frac{D^3}{6} \times 3.1416, \text{ or } \frac{\pi D^3}{6} = \text{volume of a sphere.}$$

1. What part of a 2-inch cube is a 2-inch sphere?
2. If a sphere 3 inches in diameter is carefully turned out of a 3-inch cube, what part of the cube will go into shavings, and what part will remain in the sphere?
3. If a sphere is 0.5236 of a cube of the same diameter, what will be the contents of a sphere 4 inches in diameter? Compare 0.5236 with $\frac{\pi}{6}$.
4. If a cubic foot of iron weighs 450 pounds, what is the weight of an iron sphere 12 inches in diameter?
5. A cubic foot of ivory weighs 114 pounds. What is the weight of a set of 4 billiard balls 2 inches in diameter?
6. How many cubic miles in the moon if we call its diameter 2000 miles?
7. If we call the diameter of the earth exactly 8000 miles, how many moons will be equal in volume to the earth? Shorten your work by cancellation.
8. Two 4-inch spheres are dropped into a pail even full of water and holding 864 cubic inches. How many cubic inches of water are displaced?
9. Find the square inches in the surface of a 6-inch sphere.
10. How many square miles in the surface of the moon? Call its diameter 2000 miles.
11. A cubic foot of water weighs 1000 ounces, and gold is about 19 times as heavy. What will a sphere of gold 3 inches in diameter weigh?
12. If it costs \$3 to gild a 3-inch ball, what will it cost to gild a 4-inch ball? Shorten your work by cancellation.
13. If a ball 2 inches in diameter weighs 18 oz., what will a ball of the same material, and 6 inches in diameter, weigh?

1. What is the ratio of a 2-inch square to a 4-inch square?
2. What is the ratio of a 2-inch square to a 5-inch square?
3. What is the ratio of a 2-inch circle to a 4-inch circle? What common factors enter into the areas of any two given circles?
4. Compare a 3-inch circle with a 5-inch circle.
5. Explain the statement: —

A 3-inch circle : a 5-inch circle = $(\frac{3}{5})^2 \times 3.1416 : (\frac{5}{5})^2 \times 3.1416$, or a 3-inch circle : a 5-inch circle = $3^2 : 5^2$.

6. If a rectangle is 2 by 3, what will be the length of another of the same form or shape that is 4 wide? 6 wide? 12 wide?

7. Two rectangles whose corresponding sides have the same ratio are **similar**. All surfaces of the same form or shape are **similar surfaces**. Are all squares similar? All circles? All equilateral triangles?

8. Compare a rectangle 2 by 3 with one 4 by 6.
9. Compare a rectangle 3 by 6 with one 5 by 10.
10. Are the rectangles in Exs. 8 and 9 similar? Why?
11. Observe that in the squares, circles, and rectangles that you have tried, their areas have the same ratio as the squares of their corresponding lines. Try other figures that you know to be similar, and see whether this is true: —

Similar surfaces have the same ratio as the squares of the ratios of their corresponding lines.

12. Compare two equilateral triangles whose sides are respectively 3 inches and 5 inches.
13. Are the surfaces of all spheres similar surfaces? If it costs 50¢ to gild a 3-inch sphere, what will it cost to gild a 9-inch sphere?
14. If a lot 60 ft. square costs \$300, what will one 120 ft. square cost at the same rate?

1. What is the volume of a 2-inch cube? Of a 4-inch cube?
2. Compare a 2-inch cube with a 4-inch cube. What is the ratio of 2 to 4?
3. What is the volume of a 4-inch sphere? Explain: $\frac{4}{3} \times 2^3 \times 3.1416$.
4. What two factors are common to the volume of any two spheres?
5. Compare a 3-inch with a 5-inch sphere. Explain:—
 $\frac{4}{3} \times (\frac{3}{5})^3 \times 3.1416 : \frac{4}{3} \times (\frac{5}{3})^3 \times 3.1416 = 3^3 : 5^3 = 1\frac{2}{3} : 5$.
6. What is the ratio of the diameters in Ex. 5? What was the ratio of their volumes?
7. Compare a 3-inch cube with a 5-inch cube as to their edges and volumes.
8. Solids that have the same shape are similar solids. Name some similar solids.
9. In the preceding problems, what did you notice to be the relation of volumes of similar solids to their *like lines*?
10. Take other solids that you know to be similar, and see whether you find this to be true:—

Similar solids have the same ratio as the cubes of the ratios of their corresponding lines.

11. The ratio of a 4-inch cube to a 12-inch cube is $(\frac{1}{3})^3$, or $\frac{1}{27}$.
12. What is the ratio of a 4-inch sphere to a 12-inch sphere?
13. Of a $6\frac{1}{2}$ -inch cube to a $12\frac{1}{2}$ -inch cube?
14. Of a $6\frac{1}{2}$ -inch sphere to a $12\frac{1}{2}$ -inch sphere?
15. Of a $16\frac{3}{4}$ -inch sphere to a $33\frac{1}{8}$ -inch sphere?
16. If a $2\frac{1}{2}$ -inch sphere weighs 3 lb., what will a 5-inch one of the same material weigh?
17. How many $1\frac{1}{2}$ -inch cubes can you put into a box 6 inches each way?
18. How many $\frac{1}{2}$ -inch balls can be molded from a 6-inch ball?

Similar Surfaces and Volumes

1. The ratio of the flow of water through two pipes depends upon the area of the cross section of the two pipes. Compare the flow through a $\frac{1}{2}$ -inch nozzle and a 2-inch nozzle.

2. If two triangles are similar, their corresponding sides and altitudes all have the same ratio. If the ratio of the altitude of two triangles is as 3 to 9, or $\frac{1}{3}$, what is the ratio of their areas?

3. If a square lot 60 ft. long costs \$300, what will one of the same shape 3 times as long cost?

4. Compare the strength of a rope 2 in. round with that of one 3 in. round.

5. If it costs \$8.25 to gild a sphere 20 inches in circumference, what will it cost to gild one 30 inches?

6. A $\frac{5}{8}$ -inch faucet fills a tank in 20 minutes; a $\frac{3}{4}$ -in. faucet will fill it in x min.

7. The ratio of two similar triangles is 25; the ratio of their altitudes is x .

8. To paint a conical steeple 30 ft. high costs \$35; to paint a similar one 45 ft. high costs, at the same rate, \$ x .

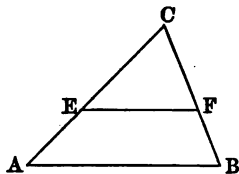
9. A conical tent 16 ft. slant height costs \$13.50; one of the same shape measuring 4 ft. more would cost \$ x .

10. If a 2-inch sphere weighs 1 lb., how much will a 6-inch sphere weigh?

11. If a rectangular bin 5 ft. long contains 75 bu. of oats, how many bushels will a similar bin $12\frac{1}{2}$ ft. long contain?

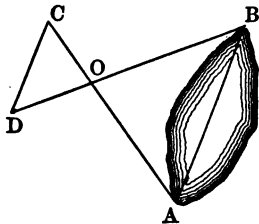
12. It requires 90 min. to fill a cylindrical tank $3\frac{1}{2}$ ft. in diameter. At the same rate, how many minutes will be required to fill a similar tank 14 ft. in diameter?

1. Describe similar surfaces.
2. Make two similar triangles. Are their corresponding angles equal?
3. Draw a triangle, ABC , as in the figure. Draw EF parallel to AB . By the use of a protractor compare the angles FEC and BAC . Also angles CFE and CBA .
4. Are triangles ABC and EFC similar; that is, do they have the same shape?
5. Make a triangle in which AC is 6 inches and CB 3. Mark off CE equal to 4 inches and CF 2 inches. Are the two triangles similar?
6. What is the ratio of CA to CE ? Of CB to CF ?
7. Cut similar triangles from cardboard. Measure their sides and discover that—



In similar triangles the ratios of the corresponding sides are equal, and the ratio of any two sides of one is equal to the ratio of the corresponding sides of the other.

8. Inaccessible distances may be found by the principle of similar triangles. Suppose we are to find the distance AB across a small lake. By measuring from A to C , and from B through O to D , making the ratio of OC to OA the same as of OD to OB , we have similar triangles.

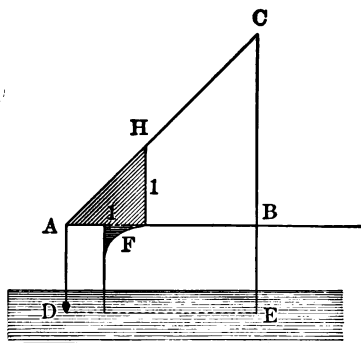


If $OC = \frac{1}{2} OA$, and CD measures 20 rods, what is AB ?

NOTE. Make OC any convenient part of OA and then OD the same part of OB . In the figure $OC = \frac{1}{2} OA$, and $OD = \frac{1}{2} OB$.

9. When a vertical rod 6 feet high casts a shadow 9 feet long, a tree casts a shadow 150 feet long. How high is the tree?

1. A boy, wishing to find the height of a pole CE , made a piece of apparatus which he called his "surveying instrument." It consisted of a right triangle whose two legs were equal. It stood 3 feet from the ground. He moved it along until the point C could just be seen along the hypotenuse of the triangle when the base of the triangle AF was parallel with the ground. A line with a weight (a plumb line) hung from A . If DE was 27 feet, how high was the pole? (Triangles AFH and ABC are similar. Why?)



2. If HF had been twice AF , and DE had been 40 feet, what would CE have been?

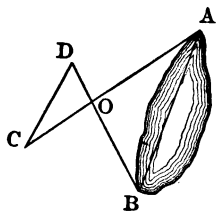
3. Make such an instrument, and find the height of trees, telegraph poles, etc.

4. Make one with the triangle having one leg twice the other, and measure the same heights. Do your results check?

5. Two triangles are similar. One has sides 4, 5, and 7 inches respectively. The long side of the other is 21 inches. What are the other sides? What if the short side of the latter were 2 inches?

6. I wish to find the distance AB . AC is 15 rods and OC is 5. I measure from B through O to D . If BO is 8 rods, what shall I make OD ? Why?

I measure DC , and find it to be $7\frac{1}{2}$ rods. How far from A to B ?



7. In this way measure distances on your school lot.

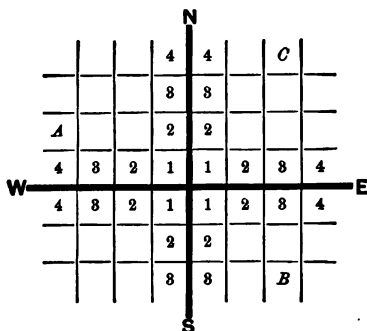
1. Government lands are divided by parallels and meridians into townships six miles square, containing 36 sections or square miles. Each section is divided into *half sections* and *quarter sections*. How many acres in a section? In a quarter section?

2. A township is designated by its number north or south of a **base line** running east and west, and east or west of a **principal meridian** running north and south.

Thus, *C* is *Township 4 N., Range 3 E.* What is *A*? What is *B*?

3. The 36 sections into which a township is divided are numbered as in the accompanying figure. Point out section 15.

4. Half and quarter sections are designated as W. or N. half sections, etc.; and S.W. or N.E. quarter sections, etc. How many acres in S. $\frac{1}{4}$ S.W. $\frac{1}{4}$ Sec. 15?



| | | | | | |
|----|----|----|----|----|----|
| 6 | 5 | 4 | 3 | 2 | 1 |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 18 | 17 | 16 | 15 | 14 | 13 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 30 | 29 | 28 | 27 | 26 | 25 |
| 31 | 32 | 33 | 34 | 35 | 36 |

Township.

| | |
|------------------------|-------|
| N. $\frac{1}{4}$ Sect. | |
| S. W. | S. E. |

Section 15.

Surveyors generally use, in measuring land, a steel chain 100 ft. long, divided into foot links, or a steel tape line of the same length graduated to feet and tenths. Sometimes a *Gunter's Chain* is used. It contains 100 links, each 7.92 in. long. The chain is 4 rods, or 66 ft., or 792 in. in length. 80 chains, or 320 rods, measure a mile.

5. How many square rods in a square chain?

6. How many square chains make 160 sq. rds, or 1 acre?

7. How many acres in S.W. $\frac{1}{4}$ N.E. $\frac{1}{4}$ Sec. 27? Draw diagram and locate it.

1. Find the cost of plastering the walls and ceiling of a room 18 ft. long, 15 ft. wide, and 9 ft. high, allowing $\frac{1}{3}$ of the walls for openings and wood-covered portions. Price $12\frac{1}{2}$ ¢ per square yard.
2. Find the area of a right triangle, base 25 ft., hypotenuse 60 ft.
3. A rhomboidal field contains 5 acres, and measures 50 rods along a straight road. How wide is it?
4. Name the six quadrilaterals and the four parallelograms. Draw a trapezoid and its equivalent rhomboid and rectangle.
5. Required the area of the entire surface of a stick of timber 18 ft. long, 4 in. thick, 8 in. wide at one end, 12 in. at the other.
6. The diagonal of a trapezium is $22\frac{1}{2}$ ft., and the perpendiculars drawn from the vertices of the angles opposite it are 16 ft. and 12 ft. respectively. What is the area of the trapezium? Draw it.
7. Find the cost of 28 "six-by-four" joists, averaging 18 ft. in length at \$32 per M.
8. What is the area of a walk 3 ft. wide, around a semicircular flower bed, the straight edge of which measures 12 ft.?
9. There is a difference of 6 in. in the diameter of the wheels of a carriage. The fore wheel turns 1000 times in going a certain distance. The hind wheel turns x times, and is 4 ft. in diameter.
10. What is the axis of a sphere 4 ft. in circumference?
11. From a sheet of zinc, weighing 16 lb., and measuring 8 ft. by 4 ft., a square was cut, reducing its weight to $11\frac{1}{2}$ lb. How long was the square?
12. How much ground is covered by 12 cords of 4-ft. wood piled 2 ft. high?
13. A granite sphere barely clears a gateway 2 feet wide. Find its contents.

1. A span of horses draws a load of brick weighing two tons. The brick are of the ordinary size, $8 \times 4 \times 2$, and weigh 100 lb. to the cubic foot. How many bricks in the load?

2. Find the contents of a cone 6 in. in altitude and 2 in. in diameter at the base.

3. Find the slant height of a square pyramid 12 in. in altitude, base 8 in. on a side.

4. Find the entire area of a hemisphere 15 in. in diameter.

5. How long is the equator, the equatorial semidiameter of the earth being 3963.296 miles?

6. Compare the perimeters of a rectangular field 60 rd. by 30 rd., of an equivalent square field, and of a circular field of the same area.

7. The inside dimensions of a cellar are 16 ft., 12 ft., and 8 ft. The wall is to be two feet thick. How many cubic yards of earth will need to be removed?

8. A cylindrical 8-in. driven well is 76 ft. deep. How many cubic feet of earth, etc., have been taken out?

9. One fire-engine throws a 2-inch stream, another a $1\frac{1}{4}$ -inch stream. Compare the quantities of water thrown.

10. One piece of shafting 2 in. in diameter weighs 300 lb. What is the weight of a similar piece $3\frac{1}{2}$ in. in diameter?

11. How deep shall a 12-foot square bin be made to hold 1728 bushels?

12. How many cords of wood in a section of a giant pine 18 ft. long and 16 ft. in diameter?

13. If a 2-in. rope breaks with a weight of 8000 lb., what weight might break a similar 3 in. rope?

14. Give area of basin required for a fountain that throws its spray out 15 feet.

The **English system of weights and measures**, the system in common use in the United States, lacks a uniform scale of relation. For example, 12 in. = 1 ft.; 3 ft. = 1 yd.; $5\frac{1}{2}$ yd. = 1 rd., etc., and 2 pt. = 1 qt.; 4 qt. = 1 gal. The United States system of money has a uniform decimal scale, and thus any part of a dollar can be expressed as a decimal fraction. Thus 3 dollars, 5 dimes, and 7 cents can be expressed \$3.57.

The **metric system** grew out of an attempt by the French government to supply a system of weights and measures that would have a uniform decimal scale of relation that would thus facilitate computation by enabling one to change a unit to any other unit by simply moving the decimal point.

The committee appointed to report upon a *standard unit* thought to make this unit, which was to be the **unit of length**, some part of some well-defined portion of the earth's circumference. A careful computation of the length of earth's quadrant through Paris was made, and 0.0000001 of this distance was taken and called the **meter**. The meter is about 39.37 inches in length.

NOTE. While a slight error was made in fixing the standard unit, this does not affect the usefulness of the system.

From the meter, all units of length, surface, volume, capacity, and weight are derived.

The **unit of weight** is the weight of a cube of water 0.01 of a meter on an edge. This is the **gram**.

The **unit of capacity** is a cubical vessel 0.1 of a meter on an edge. This is a **liter** (lê'ter).

NOTE. The Metric system is in general use by nearly all civilized nations except Great Britain and the United States. It is used by some departments of the United States government, and in the sciences. It was legalized in France in the early part of the nineteenth century.

The Metric system is a *decimal system*, *ten* units of one denomination making *one unit* of the next higher.

Decimal parts of the standard or principal unit are denoted by Latin prefixes; multiples of the same, by Greek prefixes.

| FROM THE LATIN | FROM THE GREEK |
|-------------------|--------------------|
| Milli means 0.001 | Myria means 10,000 |
| Centi means 0.01 | Kilo means 1000 |
| Deci means 0.1 | Hekto means 100 |
| | Deka means 10 |

In the tables units in common use are in **bold-faced type**.

There is no uniformity as to the abbreviations used. The ones given here are in general use.

UNITS OF LENGTH

Standard unit, the **Meter**

| TABLE | EQUIVALENTS |
|---|-----------------------------------|
| 10 millimeters (mm.) = 1 centimeter (cm.) | = 0.3937079 inch |
| 10 centimeters = 1 decimeter (dm.) | |
| 10 decimeters = 1 meter (m.) | = 39.37079 inches |
| 10 meters = 1 dekameter (Dm.) | |
| 10 dekameters = 1 hektometer (Hm.) | |
| 10 hektometers = 1 kilometer (Km.) | = { 3280.9 feet
0.621382 miles |
| 10 kilometers = 1 myriameter (Mm.) | |

UNITS OF SURFACE

Principal unit, the **Square Meter**

As the units of surfaces are squares whose dimensions are the corresponding linear units, it takes 10² or 100 units of one denomination to make one of the next.

| TABLE | EQUIVALENTS |
|--|------------------|
| 100 sq. millimeters (sq. mm.) = 1 sq. centimeter (sq. cm.) | = 0.155 sq. in. |
| 100 sq. centimeters = 1 sq. decimeter (sq. dm.) | |
| 100 sq. decimeters = 1 sq. meter (sq. m.) | = 10.764 sq. ft. |
| 100 sq. meters = 1 sq. dekameter (sq. Dm.) | |
| 100 sq. dekameters = 1 sq. hektometer (sq. Hm.) | |
| 100 sq. hektometers = 1 sq. kilometer (sq. Km.) | = 247.114 acres |

When used in measuring land the square meter is called a *centare* (ca.), the square dekameter an *are* (a.), and the square hektometer a *hektare* (Ha.).

UNITS OF VOLUME

Principal unit, the **Cubic Meter**

NOTE. As the units of volume are cubes whose edges are the corresponding linear units, it takes 10^3 or 1000 units of one denomination to make one of the next higher.

| TABLE | EQUIVALENTS |
|--------------------------------|--|
| 1000 cu. millimeters (cu. mm.) | = 1 cu. centimeter (cu. cm.) = 0.06102 cu. in. |
| 1000 cu. centimeters | = 1 cu. decimeter (cu. dm.) |
| 1000 cu. decimeters | = 1 cu. meter (cu. m.) = 35.314 cu. ft. |

In measuring wood the cubic meter is called a **stere** (1 st. 0.2759 cd.); a **decister** (1 dst.) is one tenth of a stere.

UNITS OF CAPACITY

Principal unit, the **Liter** = a cu. decimeter

| TABLE | EQUIVALENTS |
|----------------------|---|
| 10 milliliters (ml.) | = 1 centiliters (cl.) = 0.6102 cu. inch |
| 10 centiliters | = 1 deciliter (dl.) |
| 10 deciliters | = 1 liter (l.) = { 1.0567 liquid quarts
0.908 dry quarts |
| 10 liters | = 1 dekaliter (Dl.) |
| 10 dekaliters | = 1 hektoliter (Hl.) = { 26.417 gallons
2.8375 bushels |
| 10 hektoliters | = 1 kiloliter (Kl.) |

The **liter** is used in measuring liquids and small fruits, the **hektoliter** in measuring grain, vegetables, and liquids in larger quantities.

1. With a meter stick find the dimensions of your crayon box. How many liters will it hold?

2. Observe that a liter (a cubic decimeter) is about the usual size of a quart berry-box. This will give a way of changing approximately from one table to the other. In a peck of nuts about how many liters?

3. Is a barrel ($2\frac{1}{2}$ bu.) more or less than a hektoliter? At \$1 per bushel, how much a hektoliter?

4. Try to picture a box 1 m. long, 1 m. wide, and 10 cm. deep. How many hektoliters will it hold? How many bushels?

UNITS OF WEIGHT

Principal unit, the gram

| TABLE | | EQUIVALENTS |
|---------------------|---------------------|-------------------|
| 10 milligrams (mg.) | = 1 centigram (cg.) | = 0.15432 grain |
| 10 centigrams | = 1 decigram (dg.) | |
| 10 decigrams | = 1 gram (g.) | = 15.432 grains |
| 10 grams | = 1 dekagram (Dg.) | |
| 10 dekagrams | = 1 hektogram (Hg.) | |
| 10 hektograms | = 1 kilogram (Kg.) | = 2.20462 pounds |
| 10 kilograms | = 1 myrogram (Mg.) | |
| 10 myrograms | = 1 quintal (Q.) | |
| 10 quintals | = 1 metric ton (T.) | = 2204.621 pounds |

The **gram** is the weight of a cubic centimeter, the **kilogram** of a cubic decimeter, and the **metric ton** of a cubic meter of distilled water at its greatest density.

The *gram* is used in mixing medicines, and in weighing jewels, precious metals, letters, etc. Ordinary articles are weighed by the *kilogram* (commonly called *kilo*) and heavy articles by the *metric ton*.

TABLE OF EQUIVALENTS

(FOR REFERENCE)

| Common | Metric | Common | Metric |
|------------|--------------------|----------------|---------------------|
| 1 inch | = 2.54 (cm.) | 1 cu. foot | = 28.317 (cu. dm.) |
| 1 foot | = 30.48 (cm.) | 1 cu. yard | = 0.7645 (cu. m.) |
| 1 yard | = 0.9144 (m.) | 1 cord | = 3.624 (st.) |
| 1 rod | = 5.029 (m.) | 1 liquid quart | = 0.9463 (l.) |
| 1 mile | = 1.6093 (Km.) | 1 gallon | = 3.785 (l.) |
| 1 sq. inch | = 6.452 (sq. cm.) | 1 dry quart | = 1.101 (l.) |
| 1 sq. foot | = 9.2903 (sq. dm.) | 1 bushel | = 0.3524 (Hl.) |
| 1 sq. yard | = 0.8361 (sq. m.) | 1 grain | = 0.0648 (g.) |
| 1 sq. rod | = 0.2529 (a.) | 1 ton | = 0.9072 (met. ton) |
| 1 sq. mile | = 2.59 (sq. Km.) | 1 troy ounce | = 31.1035 (g.) |
| 1 Acre | = 0.4047 (Ha.) | 1 av. ounce | = 28.35 (g.) |
| 1 cu. inch | = 16.387 (cu. cm.) | 1 av. pound | = 0.4536 (Kg.) |

The metric system is not used in this country except in scientific work, hence there is little need of actually reducing from one system to another. That we may be able to form approximate estimates of lengths, weights, etc., in the English system from lengths known in the metric system learn the following table:—

APPROXIMATE EQUIVALENTS

| | | | |
|-------------|-----------------------------|--------------|--|
| 1 meter | = 3 ft. 3 $\frac{3}{4}$ in. | 1 liter | = 1.06 liq. qt. or $\frac{1}{4}$ dry qt. |
| 1 kilometer | = $\frac{5}{8}$ miles | 1 hektoliter | = 2 $\frac{1}{2}$ bushels |
| 1 are | = 4 sq. rd. | 1 gram | = 15 $\frac{1}{2}$ grains |
| 1 hectare | = 2 $\frac{1}{2}$ acres | 1 kilogram | = 2 $\frac{1}{2}$ av. pounds |
| 1 stere | = $\frac{1}{4}$ cord | 1 metric ton | = 2200 pounds |

Since the metric system is a decimal system, units may be changed to those of a higher or lower denomination by moving the decimal point.

Explain the following changes or reductions:—

1. 3247.28 m. = 324728 cm. = 32.4728 Hm. = 3.24728 Km. = 3247280 mm.

2. 67317.96 sq. cm. = 673.1796 sq. dm. = 6.731796 sq. m. = 0.06731796 sq. Dm.

3. 8.3724 Ha. = 837.24 a. = 83724 ca. = 83724 sq. m.

4. 47.234 cu. m. = 47.234 cl. = 47234 cu. dm. = 47234000 cu. cm.

5. 247.831 l. = 2.47831 Hl. = 24783.1 cl.

6. 1346.982 g. = 1.346982 Kg. = 134698.2 cg.

7. How many mm. in a Km.? In a Hm.? In a cm.?

8. 25 Kg. are equal to how many pounds to the nearest unit?

9. Estimate the length of the room in meters.

10. Estimate the weight of familiar objects in Kg.

11. Estimate in cm. lengths of lines drawn on the blackboard.

12. Give the dimensions of your books, slates, etc., in cm.

1. In 847.2 Kg., how many grams? How many pounds?
2. Change 75 bushels to hektoliters.
3. How many square meters in a rectangle 18 ft. by 10 ft.?
4. An importer pays duty on 1200 meters of cloth. How many yards?
5. How many square rods in a square hektometer?
6. How many liters in a cubic meter?
7. An importer buys 250 l. of liquor at \$0.75 a liter. He sells it for \$3 a gallon. What does he gain or lose?
8. A rectangular stone is 1 m. long, 5 dm. wide, and 24 cm. thick. How many kilograms does it weigh, being eight times as heavy as water?
9. How many kilograms of flour in a barrel?
10. Add 18.32 Km., 648 m., 94.8 Hm., 38.4 dm.
11. What will a stere of stone cost at \$12 a cord?
12. How many hectares in a field 14 Hm. long and 40 Dm. wide? How many acres?
13. How many gallons in a cubic meter of water?
14. How many times is 16 dm. contained in 1.28 Km.?
15. If goods are bought at \$2.35 per yard, at what price per meter must they be sold to gain 25%? (1 meter = 39.37 inches.)
16. A hektoliter of fruit weighs 63 kilograms, and 32 liters of sirup can be obtained from it. How many kilograms of fruit will it take to make a hektoliter of sirup?
17. The distance between two places on a map is 12.5 centimeters. What is the actual distance between the places if the scale of the map is 1 to 60,000?
18. If a certain stone is 2.83 times as heavy as water, what is the weight of a piece of this stone which is 5.39 m. long, 17.36 dm. wide, and 52.6 cm. thick?
19. A stone measuring 5.2 cm. by 17.3 cm. by 0.43 m. weighs 7.25 kgs. How many times as heavy as water is the stone?

1. What causes day and night ?
2. Through what kind of path does any point on the surface of the earth pass when the earth makes one revolution on its axis ?
3. What unit is used in measuring circles ? How many degrees in a circle ?
4. Through how many degrees does any point on the earth's surface pass from noon to-day until noon to-morrow ?
5. How many hours from noon to-day until noon to-morrow ?
6. Since 360° pass under the sun every 24 hours, how many degrees pass under the sun every hour ?

Distance (measured in degrees) on circles running east and west around the earth is **longitude**.

Since the earth turns from west to east, 15° degrees of longitude pass under the sun every hour, and we have the following : —

TABLE

360° of longitude correspond to 24 hrs. of time
 15° of longitude correspond to 1 hr. of time
 1° of longitude corresponds to 4 min. of time

Also $15'$ of longitude correspond to 1 min. of time
 $15''$ of longitude correspond to 1 sec. of time

7. Suppose your town or city is just passing under the sun, i.e. that it is just noon ; has a point 15° west of you yet passed under the sun ? Why ? How long before it will do so ? Why ?
8. When you have noon, what time is it 15° west of you ?
9. What time 30° west ? What time 15° east ?
10. What time $7\frac{1}{2}^\circ$ west of you when your time is 10 o'clock A.M. ?
11. Where is it 3 o'clock P.M. when it is noon where you live ?
12. A boat-race occurs on the Thames at 5 P.M. The result is known in Philadelphia at 1 P.M. Account for this.

1. When you have noon, what points have 11 A.M. ? 1 P.M. ?
2. The difference in longitude between two places is 17° . What is the difference in time ?

Longitude is generally reckoned from the meridian passing through Greenwich, England. Thus longitude 23° west means that the meridian through that place is 23° west of the meridian through Greenwich.

The longitude to the nearest minute of ten cities is here given.

| | | | |
|---------------------------|-----------------------------|-------------------------|------------------------------|
| Berlin, Germany | $13^{\circ} 24' \text{ E.}$ | Detroit | $83^{\circ} 43' \text{ W.}$ |
| Boston | $71^{\circ} 3' \text{ W.}$ | London | $6' \text{ W.}$ |
| Calcutta | $88^{\circ} 20' \text{ E.}$ | Madras, India | $80^{\circ} 15' \text{ E.}$ |
| Chicago | $87^{\circ} 37' \text{ W.}$ | Paris, France | $2^{\circ} 20' \text{ E.}$ |
| Cleveland | $81^{\circ} 40' \text{ W.}$ | San Francisco | $122^{\circ} 26' \text{ W.}$ |

- 3-12. When it is noon at Greenwich, Eng., find the time at each of the places named above.

Standard Time

The time considered above is called **local time**. In 1883 the railroads of the United States adopted for their own convenience what is known as **standard time**. At any particular place the time is reckoned as the local time of some specified meridian rather than the meridian through that place. The meridians used are 75° , 90° , 105° , and 120° west of Greenwich. Thus places within about $7\frac{1}{2}^{\circ}$ of any of these meridians use the local time of that meridian.

1. From what meridian do you reckon time in your town or city ? Has the meridian (a north and south line) through your place passed under the sun before or after this meridian does ?

2. Do you live east or west of the meridian from which you reckon time ?

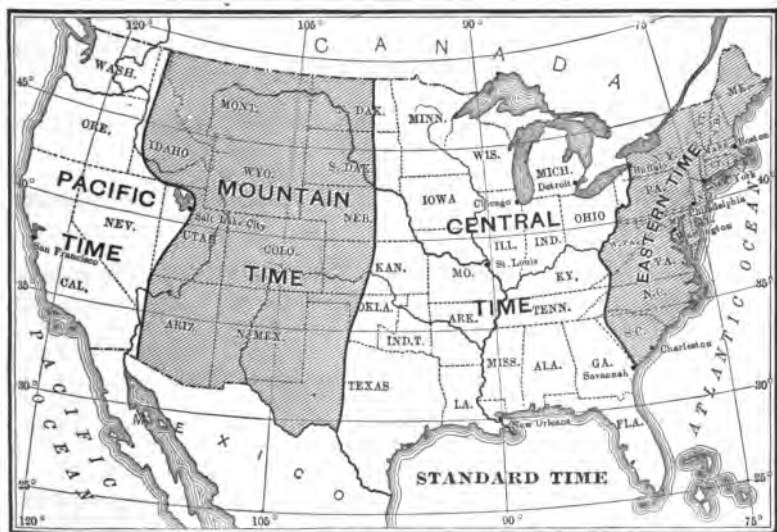
3. How many degrees is your meridian from this meridian ? Compare your local time with the standard time.

4. Suppose one lives 7° east of the 90th meridian. Is standard time fast or slow ? How much ?

The local time of the 75th meridian is **Eastern Time**, of the 90th, **Central Time**, of the 105th, **Mountain Time**, of the 120th, **Pacific Time**

5. Which time do you use?
6. When it is 8 A.M. Eastern Time, what is it Pacific Time?
7. When it is 4 P.M. Mountain Time, what is it Eastern Time?
8. At noon by Central Time, what is it by each of the others?
9. If one is traveling from New York to Chicago, how will he change his watch as he changes into Central Time?

The line of division between standard meridians is not a straight line midway between them, but depends upon important railroad terminals. It was fixed by the roads. This is shown in the following map.



10. When it is noon in Chicago, what time is it in New York? Charleston? New Orleans?
11. When 3 P.M. in San Francisco, what time is it in St. Louis? Detroit? Boston?

1. $\frac{3}{4}\%$ of a number is 50. What is the number?
2. The area of a trapezoid is 350 square feet. The parallel sides are 30 feet and 40 feet. What is the distance between them?
3. Find the missing term: $6:8=16:x$.
4. The area of a triangle is 25 square feet. Its altitude is 5 feet. What is its base?
5. The cost of 12 oranges at the rate of 3 for 5¢ is what per cent of 80¢?
6. A boy has 50¢. He told his friend that this was 40% of what his books cost him. What was the cost of his books?
7. One boy rides his wheel 30 miles an hour. Another boy rides at the rate of $\frac{1}{4}$ mile in 30 seconds. How far does each ride in a minute?
8. If I sell 68 yards of cloth to-day, and this is $6\frac{1}{4}\%$ more than I sold yesterday, how many yards did I sell yesterday?
9. 10 added to 35 is the same as the product of what 2 factors less than 10?
10. An engine ran 25 rods from the engine house, then back 18 rods, then forward 60 rods. How far from the engine house did it stop? Draw a line.
11. It is $1\frac{1}{4}$ in. on a map between two cities. How far apart are they if the map is drawn 200 mi. to the inch?
12. Interest of \$ 234 for 60 da. at 3 per cent is \$ x .
13. What per cent is made by buying fish at \$6 per hundred-weight and selling it at 10¢ a pound?
14. How much 3% stock must be purchased to yield an annual income of \$ 450?
15. A commission merchant charges 5% for selling potatoes at 60¢ a bushel. His bill is \$ 30. How many bushels did he sell?

1. If a quart of cream will serve 8 people, how many gallons shall I order for 300?

2. A horse harnessed is worth \$150. If the harness increases his value 20 per cent, what is the harness worth?

3. If the working week is reduced from 66 hr. to 60 hr. and wages in proportion, what does the \$2 man get after the cut?

4. Flour goes up from \$5 to \$7. What ought a loaf to sell for after the rise if it had previously sold for 10¢?

5. A bankrupt's liabilities are \$14,000 and his assets \$8000. What does the creditor lose who receives \$400?

6. 15 ounces is $\frac{3}{4}\%$ of how many pounds?

7. A man can do a job in two days. A boy can do it in 6 days. What shall I pay the boy if the man has \$2 a day?

8. Bought stock at 112 and sold at 98. Loss on 30 shares?

9. A mason works 8 hours for \$3, and a carpenter 10 hours for \$3.50. How much does one earn more than the other in 100 hours?

10. 90,000 copies printed in an hour are x copies per second.

11. What are the proceeds of a 60-day note for \$200 discounted at date at 9%? No grace.

12. Of a flock of 1000 sheep 400 die. What must each of the remainder be sold for to cover the loss? The sheep cost \$8 each.

13. I carry \$36,000 insurance at $\frac{3}{4}\%$. Find the premium.

14. What is the square root of $2\frac{1}{4}$?

15. What is the length of the longest line that can be drawn in a rectangle 6 by 8 inches?

16. A building lot contains 3382 sq. ft. and is 100 ft. long. How wide is it?

1. Read as per cents: $\frac{1}{16}$, $\frac{1}{12}$, $\frac{3}{16}$, $\frac{5}{12}$, $\frac{7}{8}$, $\frac{5}{16}$, $\frac{7}{12}$, $\frac{1}{24}$, $\frac{7}{16}$.
2. Read as fractions in smallest terms: $12\frac{1}{2}\%$, $18\frac{3}{4}\%$, $31\frac{1}{4}\%$, $37\frac{1}{2}\%$, $43\frac{3}{4}\%$, $56\frac{1}{4}\%$, $83\frac{3}{4}\%$, $68\frac{3}{4}\%$.
3. Of a mixture 15 gal. of alcohol make $31\frac{1}{4}\%$ of it. How many gallons in the mixture?
4. Give the cost of: —

| | |
|-------------------------|---------------------|
| 8 at \$ 2 per dozen | 9 at \$ 4 a dozen. |
| 7 at \$ 2.50 per dozen | 2 at \$ 5 a dozen. |
| 5 at \$ 1.50 per dozen. | 16 at \$ 3 a dozen. |
5. Required the semi-annual dividend from \$ 18,000 in $3\frac{1}{2}\%$ bonds.
6. What is the value of a ton of soap in 4 oz. cakes at 48 ¢ a dozen?
7. How many envelopes can I buy for \$ 12 at 75 ¢ per M?
8. Bought pencils at \$ 3 per gross and sold them at 50 cents a dozen. Gain on 5 gross?
9. Paid \$ x for a pair of \$ 8 shoes less two 10% discounts.
10. Of 300,000 immigrants 40% are illiterate. How many of them can read?
11. If a half million out of a gift of 3 millions for a hospital is spent for the building, what is the annual income from the remainder at $3\frac{1}{8}\%$?
12. Each angle of a triangle measures 60° . What kind of triangle is it?
13. x minutes will take a horse 6 times around a $\frac{1}{2}$ mile track if he trots at a 2:10 gait.
14. It is 160 rods round a square field containing x acres.
15. What will pay a note of \$ 800 that has been running 8 months at 6%?
16. $12\frac{1}{2}\%$ of 64 is $33\frac{3}{4}\%$ of what?

1. A broker received \$75 for purchasing bonds at $\frac{1}{8}\%$ commission. What was the face value of the bonds?
2. A man offered to sell his horse for 25% more than it cost him. He afterwards sold it for \$190 which was 5% less than he first asked for it. What did the horse cost him?
3. A house cost \$3000, and rents for \$25 a month. If the taxes and other expenses amount to \$50 annually, what per cent does it pay on the investment?
4. A man desires to settle an annual income of \$700 on his son. How much must he invest in U.S. bonds, paying $3\frac{1}{2}\%$ at 105 to yield that income?
5. A city map is drawn to a scale of $\frac{1}{12}$ mile to the inch. What is the length of a line which represents a street $\frac{1}{2}$ mile long?
6. Mr. Jones kept 75 bushels of cranberries through the winter, but found $\frac{1}{5}$ of them worthless. He sold the good ones for \$3.50 a bushel, receiving \$15 more than he would had he sold them in the fall. What was the price per bushel in the fall?
7. If a gas jet burns 4 cubic feet of gas in an hour, and 4 jets are lighted each evening from 6.30 to 10, what will be the gas bill for February at 10¢ for 100 cubic feet?
8. On a certain locomotive the driving wheel 7 feet in diameter turns 24,000 times in going from Boston to Springfield. How many times will a car wheel 30 inches in diameter turn in going the same distance?
9. $\frac{5}{7}$ of an article sold for the cost of $\frac{7}{8}$ of it. What was the gain per cent?
10. I send \$4935 to a broker in Chicago for the purchase of flour at a commission of 5%. If he paid \$5 per bbl., how many barrels did I receive?
11. Sold 1500 lbs. of coal for what a ton cost. Gain = $x\%$.

1. When 5% bonds are quoted at 104, what sum must be invested to have an annual income of \$1600? Brokerage $\frac{1}{4}\%$.

2. Borrowed \$500 at 6% on June 10, 1902. When it was paid it amounted to \$546. On what date was it paid?

3. If the holder of a \$150 note running 4 mo. had it discounted at a bank July 15, 1903, at 5%, what did he receive? Date, June 5, 1903.

4. If the note was unpaid when due, and drew interest at 5% from the time it became due, what would settle it March 27, 1904?

5. A young man whose salary is \$24.00 a week pays \$7.50 a week for board and \$8.75 a week for other expenses. In how many weeks can he save enough to pay a debt of \$279?

6. Received a consignment of 2000 barrels of flour which sold at \$5.50 per barrel. I paid \$74 for storage and \$27 for carting. How much ought I to remit, after deducting a commission of $\frac{1}{2}\%$?

7. A sewing machine was sold at a discount of $12\frac{1}{2}\%$ on the asking price and at a gain of 40% on the cost. If the cost was \$50, what was the selling price? The asking price?

8. How many feet of wire will be required to fence a square field containing 3136 square feet, if there are three rows of wire in the fence?

9. A hall 20 feet high has a square floor containing 6561 square feet. A merchant agreed to furnish burlap one yard wide for the walls at 15¢ a yard, if the bill was paid on or before Feb. 4, 1903. It was not paid until Dec. 22, 1903, when interest was required at 5%. What amount was required to pay the bill?

10. A 90-days' note for \$350, dated Nov. 25, was discounted Dec. 19. What did the payee receive?

11. How many gallons will a standpipe 15 feet in diameter and 80 feet high contain? One gallon equals 231 cubic inches.

1. A physician received \$76 from a collector whose commission was 5%. What was the amount collected?
2. I pay \$13.50 for insuring my furniture at $\frac{1}{4}\%$. For how much is it insured?
3. \$16.50 will settle a bill on which the discount is 10% for cash. What difference will it make if I delay the payment?
4. What per cent is made by buying coal at \$4.65 a long ton and selling it at \$8 a short ton?
5. After deducting his commission of 4%, how many barrels of apples at \$1.50 can an agent buy with a remittance of \$2000?
6. How much will a broker charge, whose commission is $\frac{1}{4}$ for selling 85 shares of stock, — par value \$5?
7. Shall I increase or diminish my income by selling 80 shares of 5% stock at 72 and investing in 8% stock at 108 $\frac{1}{2}$?
8. What is the tax on property assessed for \$4780 at \$16.30 a thousand?
9. \$360 was paid an agent for buying cotton at a commission of $\frac{3}{4}\%$. He afterwards sold the cotton at a profit of \$4500, deducting his commission of $1\frac{1}{4}\%$. What was his commission on the selling price, and what were the proceeds?
10. An agent receives \$2184 with which to buy meat at 16 $\frac{3}{4}$ ¢ per pound. How many pounds can he buy at a commission of 5%?
11. In a city \$275,000 was raised from a $1\frac{1}{4}\%$ tax. What was the assessed valuation of the property?
12. What is the net price of an article listed at \$50 and sold at a discount of 25, 10, and 4%?
13. The surface of a cube is 576 square inches. Find its volume.
14. Find the exact interest of \$250 from July 18, '03, to Jan. 5, '04.

1. Find the cost of \$300 in U.S. bonds bought at $126\frac{1}{2}$, brokerage $\frac{1}{8}\%$.
2. What is the per cent of profit if I buy oranges at \$1.50 a hundred, lose 10% of them by decay, and sell the remainder at the rate of 3 for 10 cents?
3. The proceeds of a 90-day note, discounted at a bank at date, were \$492.50. What was the face of the note?
4. What premium must be paid on a building valued at \$7500 and insured for $\frac{3}{4}$ of its value at 4%?
5. A man can do a piece of work in 3 days. His brother can do it in 4 days. What part of the work does each do in a day?
6. What part would both do in a day working together? If they work together, how long will it take them to do the work?
7. A farmer can mow a field in 10 hours. His hired man requires 12 hours to mow the same field. Suppose they work together, how long will it take?
8. A can do a piece of work in 6 days, B can do it in 8 days, and C can do it in 12 days. How long will it take the 3 men working together to do the work?
9. Three men can paint a boat in 4 days. Two of them can do it in 6 days. How long would it take the third man working alone?
10. A lot is 200 feet by 90 feet. A house, the main part of which is 24 feet wide and 42 feet long, with an ell 14 by 20 feet, is built upon the lot. A cellar 8 feet in depth is dug under the whole house, and the soil used in filling the rest of the lot. How deep was the filling?
11. I have \$15,000 to invest. Which had I better do, buy 8% stock at a premium of 50, or loan the money at $5\frac{1}{2}\%$ interest?
12. Required the amount of a note for \$628 on interest from the 4th of July, 1903, to May 30, 1904, at $3\frac{1}{2}\%$.

1. Find the sum of:— 2. A cubical bin is 100 inches long. How many cubic feet does it contain?

\$467.95

784.87

591.23

467.89

321.10

456.78

432.19

567.89

987.65

439.87

420.65

398.76

987.65

849.77

976.93

842.67

649.34

897.66

307.95

3. A lot of land rectangular in shape is 186 feet long and 151 feet wide. What is its value at 16½ cents a square foot?

4. An automobile was sold for \$600 at a loss of 20%. What would have been the gain or loss per cent if it had been sold for \$800?

5. Find the interest of a half million dollars at 2½% for 111 days.

6. What can I secure at a bank for a ninety-day note for \$2200, the rate of discount being 5 per cent? No grace.

7. Draw an equilateral triangle. Draw a line representing its altitude. Calling the side of the triangle 50 feet, what is its altitude to the nearest hundredth of a foot?

8. I am obliged to pay three months' interest at 7 per cent on my tax bill. The rate is \$16.70 per thousand, and my property is assessed for \$18,600. What

shall I be obliged to pay in all?

9. A man bought a house for \$2500 and sold it for \$1875. What per cent of the cost did he lose?

10. What is the interest of \$320, at 6 per cent per annum, from Jan. 2, 1903, to Nov. 20, 1905?

11. A merchant sold goods for \$240, thereby losing 20 per cent of the cost. For what amount should he have sold them to gain 15%?

12. A note for \$800, dated May 1, 1900, falls due Aug. 17, 1903. What will settle it if it draws 4 per cent interest?

1. I am offered a \$40 suit for \$35, or a \$35 suit for \$30. Which is the better offer and why?

2. A note for \$750 dated October 19, 1903, drawing 4 per cent interest, falls due May 17, 1904. What will settle it?

3. It costs \$120 to fence a square lot at \$2 a rod. What is the land worth at \$1600 an acre?

4. I buy tea, pay an *ad valorem* duty at 25 per cent, and double my money by selling at \$1.00 a pound. Required the cost without duty.

5. A train running 50 miles an hour is 36 minutes in going from one station to another. How far apart are the stations?

6. A note for \$800, drawing 5% interest for 18 months, is discounted by a bank 90 days before maturity, at 7%. Required proceeds.

7. By selling a farm for \$4800, the owner lost $\frac{1}{4}$ of what he paid for it. Find the per cent of loss.

8. The discount on goods at 30%, 10%, and 5% off is \$23.94. What is the list price?

9. What sum of money at 5% simple interest will produce in one year and three months the same interest that \$2940 will produce at 4% in two years and six months? (Solve by proportion.)

10. The square on the diagonal of a square room is 648 sq. ft. What will it cost to carpet the room with carpet $\frac{3}{4}$ yd. wide, at 90¢ a yard?

11. What must be paid for stock paying an annual dividend of 3% to secure an annual income of 7% from the investment?

12. The proceeds of a note for \$500 discounted at 6% are \$492.50. Find the term of discount.

13. In what time will a sum of money double itself at 8%?

14. Draw a line 0.0001 of mile long.

1. After spending $\frac{2}{3}$ of his money in travel and $\frac{1}{4}$ of the remainder, Mr. Emerson finds he has \$440. He had x dollars at first.
2. When \$1 $\frac{1}{2}$ will buy a gallon of oil, how much will \$ $\frac{2}{11}$ buy?
3. A tank containing 480 gallons is emptied by two faucets in 5 and 7 minutes, respectively. How much water will pass through each, if both are open at once?
4. A rectangular cistern contains 294 $\frac{1}{2}$ cu. ft. of water and measures 13 $\frac{2}{3}$ ft. \times 6 ft. \times x ft. Value of x ?
5. A garden roller is 4 $\frac{1}{2}$ ft. long and 6 $\frac{1}{2}$ ft. in circumference. How much surface will it pass over in 48 revolutions?
6. At \$40 per M, what will be paid for 6 boards 16 ft. long and in width tapering from 20 in. to 15 in.?
7. Invested \$26,250 in N.P. bonds at 87 $\frac{1}{2}$ and sold them the next day at 91. I gained x dollars.
8. How many times larger is a 100-ft. circle than a 50-ft. circle?
9. If a man is satisfied with 3 $\frac{1}{2}$ % of his investment, what can he afford to pay for U.S. 4's?
10. The *Boston Journal* is printed from rolls of paper 64 in. wide. Each page is 16 \times 22. If 160,000 copies of 8 pages each are printed daily, how many miles of 64 in. paper are required?
11. Two trappers have provisions for 9 months. If A had been away, they would have lasted B 12 months. They would have lasted A alone x months.
12. If 24 men can mow 66 acres in 2 days, how many acres can 14 men mow in 7 days?
13. What is the interest of \$248 for 6000 days at 7 $\frac{1}{2}$ %?
14. Mr. Green purchased a \$40 suit at 10% discount and yet the dealer made 25%, for it cost him only x dollars.

1. James B. Chase buys of John R. Knight, Nov. 30, 1903, goods to the amount of \$600, for which he gives his note payable in 3 months, without interest. On Dec. 30 Mr. Knight sells the note to a bank at a discount of 5%. What sum does he receive?
2. Write the above note as it will appear after the bank gets it, making your town or city the place of the transaction.
3. If $\frac{1}{4}$ of a grocer's cash sales are profit, what is his profit per cent?
4. The extremes of a proportion are 49 and 196, and the means are equal to each other. What is the proportion? Explain the operation.
5. Two kinds of silk are sold, each at \$3.60 a yard. On one kind the merchant makes a profit of $33\frac{1}{4}\%$, and on the other $33\frac{1}{4}\%$ of the selling price is gain. On which does he make the greater gain, and how much per yard?
6. A dealer buys oats at 42¢ a bushel. In selling he makes a profit of $16\frac{2}{3}\%$, which was a reduction of $12\frac{1}{2}\%$ from his asking price. His asking price was x ¢.
7. What income will be derived semiannually by investing \$8214 in 4% R.R. stock at 26% below par?
8. What must be the depth of water in a cylindrical cistern 2 ft. 6 in. in diameter to contain 153 gallons?
9. Received a remittance of \$25,375 for the purchase of cotton at $12\frac{1}{2}$ ¢ a pound; this included my commission of $1\frac{1}{2}\%$. I bought x pounds.
10. May 14, I get a 90-day note for \$1292 discounted at 6%. The note was dated April 10, 1903. Proceeds?
11. If it costs \$320 to fence a rectangular lot 120 rods \times 80 rods, what will it cost to fence an equal square lot at the same rate?

1. In 1860 there were 40 high schools in the United States. In 1900 there were 6005. What was the per cent of increase?

2. Pure water weighs a thousand ounces to the cubic foot. Dead Sea water weighs 77 pounds to the cubic foot. What per cent of impurities in the latter?

3. The tire of a wheel on a watering-cart is 6 inches wide. If the wheel is 5 feet in diameter, what surface of ground is covered at each revolution?

4. Merchandise may be sent 3000 miles by mail for 1¢ an ounce; by express the charge is \$1.25 a hundred. What will it cost to send 75 pounds in the cheaper way?

5. A tax of \$6971.60 is to be assessed upon a town containing 430 polls, taxed at \$1.25 each. Its real estate is valued at \$1,354,000 and its personal property at \$75,000. Find the tax of a man whose property is assessed at \$3640 and who pays for one poll.

6. A person borrows \$100, and at the end of each year he pays \$25 which goes to reduce the principal, after paying interest at 4% on what he has owed during the year. How much does he owe at the end of 3 years?

7. A man having \$2655 invests it in $3\frac{1}{2}\%$ bonds at 88 $\frac{1}{4}$. Afterward, when they are 93, he sells out and invests his money in a $5\frac{1}{4}\%$ mortgage. What difference has he made in his income?

8. The total expense of maintaining the schools in a certain city is \$287,000. If the school keeps 40 weeks, 5 days in a week, and there are 10,500 school children in the city, what is the cost of each day's schooling for each child?

9. The inside diameter of a hollow sphere is 7 in. The thickness of the shell is $3\frac{1}{2}$ in. Find the volume of the shell.

10. Required the proceeds of a 4-mo. note for \$5000, discounted at $5\frac{1}{2}\%$ 37 da. after date.

1. What per cent of discount lets a \$225 horse go for \$185 $\frac{1}{2}$?
2. Which produces the greater per cent of income and how much, 5% bonds at 120 or 4% bonds at 105?
3. After my bill has been reduced by successive discounts of 20% and 10%, I can pay it for \$1016.64. What was the gross amount charged?
4. What is the rate of duty on jewelry if \$402.50 is charged on an invoice of \$1150?
5. A savings bank pays 4% interest compounded semiannually. A man makes a deposit of \$50 every six months beginning July 1, 1901. What amount stands to his credit Jan. 1, 1903?
6. The successful candidate received 6750 votes. His opponent had 3825. How were every 100 votes divided?
7. What is the exact interest of \$81 from Dec. 31, 1902, to March 12, 1903, at 6%?
8. What is the tax rate of \$1 where \$73,000 is to be raised on a valuation of \$5,309,090?
9. The last reading of my gas meter was 67,300 cu. ft. The previous reading was 64,900. At \$1.50 per thousand, with a discount of 15¢ per thousand cu. ft., my gas bill was x dollars.
10. Find the square root of $\frac{2}{3}$, correct to thousandths.
11. A rectangular field is 64.8 rods long and 36.05 rods wide, and a square field is of equal area. At \$1.10 a rod, how much more will it cost to fence one than the other?
12. I bought 1185 lb. of hay at \$19.50 per ton, for x dollars.
13. Sold a 7% mortgage for \$3000 at 25% premium, and bought 6% railroad stock at par with the proceeds. Did I increase or lessen my income?
14. One third of the sum of two numbers is 384, the difference is 64. What are the numbers?

1. I bought a house for \$2500, and sold it so that 20% of the selling price was profit. What did I receive for it?

2. $\frac{3}{4}$ yd. carpeting is used for a room 20 ft. square. The waste in matching is 6 in. to a strip. The cost at \$1.75 per yard is \$ x .

3. The wheels of a bicycle are 30 in. in diameter; the gearing is such that each wheel makes two revolutions to every turn of the pedals. How many times will each pedal turn in a ride of one mile?

4. Mr. Brown is taxed on x dollars. His tax bill is \$110.00, including \$2 for poll tax, the rate being 9 mills on the dollar.

5. A horse and wagon were sold for \$120 each; the horse was sold at a loss of 25%, the wagon was sold at a gain of 25%. Find how much was lost or gained on the whole transaction.

6. Bought railroad stock at $114\frac{3}{4}$, and sold at $117\frac{1}{4}$. In each case I paid $\frac{1}{8}\%$ brokerage. What was my profit on 200 shares?

7. My broker bought for me 26 shares of stock at 107 and sold them for $118\frac{3}{4}$, brokerage in each case $\frac{1}{4}\%$; find my gain.

8. Which is the better investment, 5% stock at 120, or 6% at 150?

9. If $\frac{3}{4}$ of a day's wages are \$1.40, what shall be paid for $\frac{7}{16}$ of a day?

10. A man fails and pays 67 cents on a dollar; after paying his lawyer's fee of 4%, how much would a creditor receive on a claim of \$625?

11. I bought cloth by the meter (39.37 in.) and sold at the same price per yard. What per cent did I gain?

12. A garden 145 feet long and 120 feet wide is inclosed by a tight board fence 6 feet high; find the cost at 8¢ a square yard, of painting both sides of the fence.

13. Find the surface of a sphere 25 inches in diameter.

14. How many square inches are left of a sheet of paper 14 in. by 21 in. after the largest possible circle is cut out of it?

1. Simplify $\frac{2 \times (\frac{2}{3} + \frac{2}{23} + 2\frac{2}{7} - \frac{5}{12} \times 1\frac{1}{10})}{1 + (9 - 2 \times 2)}$.
2. How far is a man from the starting-point, who travels west 48 miles then due north 62 miles, and then east 14 miles?
3. Sold my carriage at $\frac{3}{4}\%$ gain, and with the money bought another which I sold for \$182 and lost $12\frac{1}{2}\%$. What did each carriage cost?
4. A 90-day note for \$500 without interest, dated December 10, 1904, will yield what proceeds if discounted at 6% Jan. 10?
5. A man borrowed \$2700 November 11, 1903, with interest at 5%; find the amount of his debt Aug. 5, 1905.
6. Find the difference in a bill of \$825 between a discount of 25% and a discount of 10%, 10%, and 5%.
7. The distance around a rectangular field whose width is $\frac{3}{4}$ its length, is 98 rods; find the area of the field.
8. A dealer obtained \$480 for a piano, the list price of which he had discounted 50%. He still made a profit of 20%. At what was it listed? What did it cost him?
9. Bought \$15,000 worth of goods on 4 months' time, and sold immediately for \$14,900 cash. Money being worth 5%, what did I gain?
10. Find the cost at \$20 a ton of 12 bales of hay averaging 218 pounds each.
11. What sum will cancel a 5% note for \$763, dated April 19, 1901, and maturing Aug. 11, 1904?
12. An article sells for \$1.29; if the profit is 50%, what was the cost?
13. 4% government bonds yield an annual income of \$1000. What is their face?
14. \$120 yields \$8 annually. What rate per cent is this?

1. Simplify the following: $\frac{(\frac{3}{8} + \frac{7}{8}) \times (\frac{3}{8} - \frac{1}{8})}{2\frac{1}{2} \times \frac{7}{8} + 4\frac{1}{8} - 2}$.
2. How many gallons in a cylindrical can 2 feet in diameter and 3 feet deep?
3. William Snow bought this day of John West for cash the following: 4 lb. tea at 45 cents, 2 lb. coffee at 40 cents, 2 bushels potatoes at 50 cents, 25 lb. sugar at 5 cents. Make a receipted bill.
4. Each side of a triangle measures 30 feet. Its area is x .
5. A bookseller buys books from the publishers at 40% off the list price. He sells a set of Thackeray's works which list at \$30 at a discount of 20%. What per cent does he make?
6. Hats bought at \$15 a dozen are sold at \$2 apiece; find the gain per cent.
7. Find the amount of \$585 at simple interest for 1 year, 5 months, 17 days at $5\frac{1}{2}\%$.
8. Bought hats at \$27 a dozen and sold them at \$3.75 each; find the gain per cent.
9. At \$6 a thousand, find the cost of shingles laid 4 inches to the weather to cover a roof of 1750 square feet, the shingles averaging 4 inches wide.
10. At what price must stock paying 4% be bought in order that 5% may be realized on the investment?
11. Find the proceeds of a bank-note for \$650 discounted for 90 days at 6%.
12. A house and lot cost \$5000; the insurance is \$25, taxes are \$50, and repairs \$75 annually; what rent must be received in order to realize 6% on the investment?
13. Find the cost of the following lot of lumber: —
 - 20 pieces 13 ft. \times 6 in. \times 10 in. at \$14 per M.
 - 10 pieces 16 ft. \times 2 in. \times 4 in. at \$16 per M.
 - 6 pieces 24 ft. \times 8 in. \times 10 in. at \$15 per M.

1. Reduce the following to its simplest form : $\frac{363.25}{482\frac{1}{2}} + \frac{5}{8}$.
2. Find the amount of \$486.50 for 1 year, 5 months, and 17 days at $4\frac{1}{2}\%$ simple interest.
3. A man 6 feet high casts a shadow 42 inches long. Find the height of a flagstaff which at the same time casts a shadow 28 feet long.
4. Find the proceeds of a note for \$560 discounted for 90 days at $3\frac{1}{2}\%$.
5. How many times will a wheel 4 ft. in diameter revolve in going one mile ?
6. Find the cost of 6 pieces of timber each of which is 32 ft. long, 10 in. wide, 8 in. thick, at \$14 a thousand feet, board measure.
7. On a bill of goods amounting to \$485.50 I received commercial discounts of 15%, 10%, and 5%. Find the net cost of the goods.
8. What is the market value of 25 shares of New York Central stock at $46\frac{1}{4}$ premium ?
9. 4% government bonds yield an annual income of \$1400. What is their face ?
10. The cost of 50 gallons of molasses is \$25. If $\frac{1}{5}$ is lost by leakage, and 20 gallons are sold at $62\frac{1}{2}\text{¢}$ a gallon, at what price per gallon must the remainder be sold to gain \$5 ?
11. At $2\frac{1}{2}\%$ discount, what shall be paid for 4% stock so that the annual income shall be \$2400 ?
12. Find the proceeds of a note for \$3000 at 69 days when discounted at 5%.
13. The diagonal of a square field is 20 rods. What is its perimeter ?
14. A field is 42 rods long and 35 rods wide. Find its value at \$37.50 an acre.

1. Reduce $\frac{4\frac{1}{2} - 1\frac{1}{2}}{4\frac{1}{2} \times 5\frac{1}{2}}$ to a simple fraction in its lowest terms.

2. I sell an article at an advance of 25% on the cost and then discount the bill 5% for cash payment. My net gain is \$63.75. Find the cost.

3. Find the cost of 8 sticks of timber each 42 ft. long, 10 in. wide, 8 in. thick, at \$18 per M, board measure.

4. An agent received \$32 as his commission at 4% for buying flour at \$5 a barrel. How many barrels were bought?

5. Find the cost, at \$28 per M, of lumber for a floor 21 feet long by 16 wide, allowing $\frac{1}{8}$ of the lumber for matching.

6. Find the cost of the following bill of goods: 1840 lb. hay at \$14 a ton; 2460 lb. coal at \$5 a ton of 2240 lb.; 5120 lb. oats at 24 cents a bushel of 32 lb. Make and receipt the bill.

7. A plot of ground in the form of a triangle contains 2 acres of land; the base of the triangle is 40 rods. Find the altitude.

8. An agent remits to me \$247.38 after retaining a commission of 5% for collection. What sum did he collect? What was the amount of his commission?

9. If a square field contains 10 acres, what is the length of the diagonal?

10. Find the cost of papering the walls of a hall 36 feet long, 24 feet wide, and 18 feet high, with paper $1\frac{1}{2}$ feet wide at \$0.25 a roll of 16 yards, allowing 64 square yards for doors and windows.

11. A house was sold for \$1850 at an advance of 15% on the cost. What would it have brought at a gain of 20%?

12. Sold $\frac{3}{4}$ of an article for what $\frac{3}{4}$ of it cost. What was the gain per cent?

13. What per cent would I receive on my investment if I should buy at 10% discount stock which pays an annual dividend of $4\frac{1}{2}\%$?

1. The owner of $\frac{1}{11}$ of a mine sold $\frac{1}{6}$ of his share for \$40,500. What should he who owns $\frac{2}{3}$ of the mine get for $\frac{1}{3}$ of his share?

2. A bookseller buys a book, whose catalogue price is \$3.50, at a discount of 20% and 5%, and sells it at 10% above the catalogue price. What per cent profit does he make?

3. A farm costing \$3500 sold for \$5400. What was the per cent of gain?

4. A coal-bed 10 feet thick covers a square mile. How many 15-T. carloads, allowing 45 cu. ft. to a ton?

5. How many acres in a street $1\frac{1}{2}$ miles long and 4 rods wide?

6. I buy apples at \$2 a barrel and lose 20% of them. At what price per barrel must I sell the remainder to gain 10% on the transaction?

7. At what price must I buy stock that pays annual dividends of 8% in order to realize $4\frac{1}{2}\%$ on my investment?

8. I sell goods at 15% below the marked price and still make a profit of 10%. What per cent above cost was the marked price?

9. A owns $\frac{2}{3}$ of a manufacturing plant. The plant is valued at \$48,870. A sells a part of his interest for \$10,860. What part of his interest does he sell?

10. A certain stock pays 10%. At what rate must it be bought to yield 6% on the investment?

11. How many bushels of oats will be contained in a bin 30 feet long, 15 feet wide, and 10 feet deep?

12. Sold a horse so that $\frac{1}{3}$ of the gain equalled $\frac{2}{5}$ of the cost. What was the gain per cent?

13. A square field containing $27\frac{1}{2}$ A. has a diagonal path across it x rods in length.

14. Find the cost of 8246 lb. of coal at \$5.50 a ton of 2000 lb

15. When \$3460 is a loss of 20%, \$ x would be a gain of 20%.

1. Simplify the following: $\frac{3\frac{1}{2} + 2\frac{1}{8} - 1\frac{1}{4}}{\frac{3}{8} \times \frac{1}{8}} + 1.375$.
2. I paid \$25 for linoleum, at \$1.25 per square yard. The length of my floor was 15 ft. What was its width?
3. From a field containing 50 acres I sold a corner 100 rods long and 40 rods wide. What per cent remained?
4. By selling stock at 84 there is a gain of 5% on the investment. At what price was the stock purchased?
5. Find the contents in bushels of a bin 8 feet long, 4 feet wide, and 6 feet deep.
6. A merchant buys goods for \$1125. He sold $\frac{1}{2}$ at an advance of 25% on the cost, $\frac{2}{3}$ at an advance of $12\frac{1}{2}\%$, and the remainder at one half their cost. What was his profit?
7. Find the cost of 20 boards, each 14 ft. long, 8 in. wide, and $1\frac{1}{4}$ in. thick, at \$24 per M.
8. What will it cost to cement a cellar bottom 36 ft. long, 23 ft. 7 in. wide, at 96¢ a square yard?
9. A merchant bought 3 yards for \$2 and sold 2 yards for \$3. What was his gain per cent?
10. Bought 240 barrels of apples at \$1.75 a barrel; lost 40 barrels through frost. At what price a barrel must I sell the remainder to gain 25% on the money invested?
11. A coat cost \$8. How shall it be marked that the dealer may lower the price 20% and still gain 20%?
12. Find the face of a 60-day note which, when discounted at a bank, will yield \$250.
13. A tree 100 feet high casts a shadow on level ground 75 feet long. How far from the end of the shadow to the top of the tree?
14. Find the cost of a stone walk 4 rd. long and 5 ft. wide at 60 cents a square foot.

1. Make a receipted bill for the following:—

Bought of Dyer & Co. for cash: $3\frac{1}{2}$ lb. of tea at 45 ¢, 20 lb. sugar at $5\frac{1}{8}$ ¢, 2 lb. coffee at 50 ¢, 10 yd. of muslin at $7\frac{3}{4}$ ¢.

2. I buy oranges at the rate of 15 cents a dozen, and sell them at the rate of 3 for 10 cents; find the gain per cent.

3. At 8 cents a foot, what will be the cost of a board 12 ft. long, 10 inches wide at one end, and tapering to a point?

4. If 14 quarts of grass seed are required for an acre of ground, what will be the cost of the seed for a field 36 rods by 24 rods, the seed being worth \$ $3\frac{1}{2}$ a bushel?

5. School bonds bearing $4\frac{1}{2}\%$ interest sell at 10% premium; what rate per cent does the buyer get on his investment?

6. Leaving $\frac{2}{3}$ of my money at home, I spend 5% of the rest for eggs that cost me 29 cents a dozen. I bought eggs enough to fill 8 baskets, 5 dozen to the basket. How much money had I at first?

7. Find the cost at 25 cents a rod, of building a fence around a square field of 10 acres.

8. I sold two cows at \$45 apiece. On one I gained 20%, and on the other I lost \$17.50. For what should I have sold the two to have gained $5\frac{1}{2}\%$?

9. Find the amount of \$375 for 11 months 17 days, at $4\frac{1}{2}\%$ simple interest.

10. A field of 18 acres produces 26 bushels of wheat per acre; each bushel of wheat makes 52 lb. of flour; if 196 lb. of flour are worth \$5, what is the value of the crop?

11. The list price of a carriage is \$260. I am allowed 20% and 10% discounts. What is the net price?

12. The net price of a mowing machine is \$158.40, and the trade discounts are 20% and 10%. Find the list price.

1. Simplify $\frac{1}{3}$ of $\frac{31}{4} + 2\frac{1}{2} \times 14$.
2. I buy stocks at 4% discount and sell at 4% premium; what per cent profit do I make on the investment?
3. A house rents for \$40 a month, the annual expenses on it are: taxes \$92.50, water rate \$20, and repairs \$60. The landlord has 5% clear profit. What did he pay for the house?
4. What must be the length of a field 88 feet wide containing one third of an acre?
5. After buying some goods, a merchant lost 20% of them by fire. He sold the remainder at a gain of $33\frac{1}{3}\%$, receiving \$250.75 more than he paid for the whole. What did the goods cost?
6. I buy a lot of land at \$250 an acre. I divide it into building lots 66 ft. \times 99 ft., and sell these lots at \$400 each. Find the gain per cent.
7. What will it cost to paint the walls and ceiling of a hall 48 feet long, 27 feet wide, 18 feet high, at 95 cents a square yard?
8. At \$ $3\frac{1}{4}$ a cord, a pile of 4-ft. wood 32 ft. long cost \$17 $\frac{1}{2}$. How high was the pile?
9. On an article listed at \$8, a trade discount of 20%, 10%, and 5% is made; find the selling price.
10. Multiply 3 and 15 thousandths by one and five thousandths. Divide the product by five million, and express the result in words.
11. A grocer bought 75 lb. of soap at $6\frac{1}{4}$ cents a pound. While on hand it dried away $\frac{1}{4}$ in weight. He sold it at $8\frac{1}{2}$ cents a pound. What was his gain or loss per cent?
12. I retail oranges at 3 cents each, gaining 150% on the purchase price. What did the oranges cost a dozen?
13. A man bought a pair of horses for \$400, which was 20% less than their real value, and sold them for 25% above their real value. What was the selling price?

1. Simplify and express decimally $\frac{18\frac{1}{2} \times 7\frac{3}{4}}{8\frac{3}{4} - 6\frac{1}{4}}$.
2. Find the cost at \$7 per 100 square feet, of slating a trapezoid of which the parallel sides are 64 feet and 32 feet and the perpendicular distance between them is 20 feet.
3. Find the cost of the shingles required to cover a roof 40 feet long, 20 feet wide, at \$5 a thousand, if it requires 36 shingles to cover 5 square feet.
4. A box 6 ft. long, 4 ft. wide, and 3 ft. deep is full of oats. What is the value of the oats at 30 cents a bushel?
5. Find the cost of paving and curbing one mile of street, the paving being 30 feet wide and costing \$2.75 a square yard and each line of curbing costing 30 cents a linear foot.
6. Find the result of $1.76 \times 49.647 \div 0.0088$.
7. The interior of a rectangular tank is $2\frac{1}{2}$ feet by 3 feet by 5 feet; in how many minutes will this tank be filled by a pipe that admits 18 quarts of water a minute? (1 gallon = 231 cubic inches.)
8. How many gallons of water in a circular cistern 4 ft. 4 in. in diameter, the water 16 feet deep?
9. Find the ratio of lighting surface to floor surface in a room 30 by 35 ft., with 4 windows, each 3 ft. by 8 ft. 9 in.
10. A merchant sold a case of goods which cost \$14.40 at 10% below the marked price, thus gaining 25% on the cost; find the marked price.
11. Find the cost of the ties and rails for 1 mile of single track railway, the ties being placed 2 feet apart from center to center and each rail weighing 90 lb. a yard, if the ties cost 40 cents each and the rails cost \$29 a ton of 2240 lb.
12. A man bought Pacific R.R. bonds at 107, sufficient to give an annual income of \$252 at 6%. What did he pay for them, brokerage $\frac{1}{8}\%$?

1. Change $\frac{1}{8}$ and $\frac{3}{4}$ to decimals, and divide the first decimal by the second.
2. Find the cost of carpeting a room 15 feet long, 12 feet wide with carpet 27 inches wide, at 75 cents a yard.
3. A bookseller buys a book whose catalogue price is \$3.50, at a discount of 20% and 5%, and sells it at 10% above the catalogue price. What per cent profit does he make?
4. Find the square root of 6,115,729.
5. A dealer bought 100 bushels of potatoes at 40 cents a bushel. If he lost 30% of them, at what price per bushel must he sell the remainder to gain 20% on his investment?
6. What is the value at \$5 a cord, of a pile of wood 4 feet wide, 10 feet high, and 20 yards long?
7. A man endowed a professorship with a salary of \$2000 per annum. What sum must he invest at 6% to provide this salary?
8. A grocer pays \$12 for 5 bushels of cranberries, and sells them so as to gain $33\frac{1}{3}\%$; find the selling price per quart.
9. A pupil who attends school 68 days during a term was marked 85% for attendance. How many days was he absent?
10. John Hartford borrows this day of Charles Smith \$280, giving his note for 3 months at 5%. Write the promissory note in proper form and find its amount at maturity.
11. If a cubic foot of iron weighs 500 lb., what will a cannon ball 6 in. in diameter weigh?
12. Find the exact contents in cubic yards of a solid wall 8 feet high and 18 inches thick around a rectangular court 20 yards by 32 yards.
13. A's farm is 240 rods wide; he sells 18 acres off one end. How much shorter is his farm than it was before?

1. Simplify $[(14\frac{7}{8} + \frac{1}{8}) - (6\frac{3}{8} \times \frac{7}{8})] \times .0625$.
2. The owner of $\frac{7}{8}$ of a mill sold $\frac{7}{8}$ of his share for \$4,060. What should he who owns $\frac{3}{8}$ of the mill get for $\frac{3}{8}$ of his share?
3. On a note of \$400, at 6%, dated Jan. 12, 1901, the following payments were made: May 22, 1901, \$200; Oct. 2, 1903, \$150. Find the amount due Dec. 10, 1904.
4. Find the cost at 75 cents a square yard of paving a circular court whose radius is 40 feet.
5. A capitalist buys U.S. 4% bonds to the amount of \$50,000, par value, at $112\frac{3}{8}$, brokerage $\frac{1}{8}\%$. Find the cost of the bonds and the rate of income on the investment.
6. Find the cost of the following lot of lumber:—
3 pieces 8 in. \times 6 in. \times 12 ft. at \$17 per M.
30 pieces 12 in. \times 2 in. \times 14 ft. at \$20 per M.
20 pieces 10 in. \times $\frac{7}{8}$ in. \times 16 ft. at \$25 per M.
7. A certain house was built by 40 workmen in 48 days, but being burned, it is required to rebuild it in 30 days. How many men must be employed?
8. A man pays \$75 for insuring his house for $\frac{3}{4}$ its value at $1\frac{1}{4}\%$. Find the value of the house
9. When chairs are sold for \$4.80 a dozen, with a discount of 5% for cash, what is the cash value of 200 chairs?
10. A note for \$350, at 5% simple interest, was given Nov. 23, 1902. Find the amount of this note June 15, 1904.
11. A speculator buys bonds whose par value is \$10,000 at $113\frac{3}{8}$, and sells them at $115\frac{1}{8}$. How much does he gain if brokerage is $\frac{1}{8}\%$ in each transaction?
12. Find the proceeds of a note for \$425 at 90 days, when discounted at $3\frac{1}{4}\%$.

1. Simplify $(\frac{3}{8} + \frac{1}{2} \times \frac{2}{3}) + (\frac{1}{8} + \frac{5}{7} \times \frac{7}{8}) - (0.59 + \frac{4}{5})$.
2. The ice on a circular pond is 2 feet thick. If the pond is 1000 feet in circumference, how many cubic feet of ice does it contain?
3. A yard is 84 feet long and 80 feet wide. What is the length of a clothes line that will reach from one corner to the other corner diagonally opposite?
4. A man sells 2 horses for \$200 each; on one he gains 25%, and on the other he loses 20%. Does he gain or lose on both, and how much?
5. Make a receipted bill for the following: William Stone buys this day of Flagg Bros., 2 barrels of flour at \$5.50, 20 lb. sugar at 5½ cents, 4 lb. coffee at 35 cents, 5 lb. butter at 28 cents, 2 bushels potatoes at 45 cents.
6. Change $\$4\frac{2}{3}$ to its lowest terms.
7. Find the cost of paving a circular court 42 feet in diameter at 62½ cents a square yard.
8. The gross amount of a bill of goods is \$750.35, and the rates of discount are 10%, 10%, and 5%. What is the net cost to the purchaser?
9. When the duty on a quantity of lace at 30% *ad valorem* was \$115.80, what was the cost of the lace, and the duty in francs at \$0.193 per franc?
10. A tax of \$6750 is levied on a certain village whose assessed valuation is \$4,500,000. What is the tax on a house assessed at \$8500?
11. Change $\frac{1}{3}$ to the form of a decimal, and multiply it by 0.035.
12. A man gave away $\frac{1}{3}$ of the books in his library, lent $\frac{1}{4}$ of the remainder, and sold $\frac{1}{5}$ of those left; he then had in his possession 360 books. How many books had he at first?
13. A house rents for \$30 a month, and the owner pays \$75 a year for taxes and repairs. What is the value of the house if his net profit is 5% per annum?

1. Simplify $\frac{(\frac{3}{4} \times 2\frac{1}{2}) + (\frac{3}{8} \times 7\frac{1}{2}) + \frac{3}{4} \times \frac{3}{8}}{\frac{1}{2} \text{ of } (\frac{3}{4} - \frac{1}{16}) \times (1 - \frac{3}{8})}$.

2. How many tons of coal can be put into a bin 12 feet square and 6 feet high, allowing 55 lb. of coal to a cubic foot and 2240 lb. to the ton?

3. Multiply three and fifteen ten thousandths by one and one hundredth, and divide the product by four and five hundredths. Express the result in words.

4. Find the cost of carpeting a floor $13\frac{1}{2}$ feet by 18 feet, the carpet being $\frac{3}{4}$ of a yard wide and costing \$1.20 a yard.

5. The list price of a bill of goods is \$120; find the net cost when the successive commercial discounts are 20%, 10%, and 5%.

6. A physician whose charges are \$2 a visit, made on an average 5 visits per day in a year of 365 days. He collected 55% of his charges and saved \$2 out of every \$5 collected. At this rate how much did he save in 2 years and 6 months?

7. At what price must I buy 5% bonds in order to get 4% on my investment?

8. Owing to a deficiency in the appropriation bill, the salaries of the clerks in a bureau were reduced 18% for the last quarter of the fiscal year. How much did a clerk who was paid \$287 for the last quarter receive during the whole fiscal year?

9. At $3\frac{1}{2}$ bushels an acre how many bushels of seed oats will be required for a field 660 feet long and 462 feet wide?

10. A merchant asks for successive discounts of 15% and 5% on a bill of \$850, but he is offered instead discounts of 10% and 10%; find the difference between the two net amounts.

11. What is the loss on 40 shares of stock bought at $109\frac{1}{4}$ and sold at $106\frac{3}{8}$, brokerage being $\frac{1}{8}\%$ in each case?

1. Simplify $\frac{17\frac{5}{2} - 9\frac{1}{2} + 4\frac{1}{2}}{\frac{5}{8} \times 9\frac{1}{2}}$.

2. A house was sold for \$7050 at a loss of 6%; for what price should it have been sold to gain 15%?

3. Find the net proceeds on the sale of 576 barrels of flour at \$7.50 a barrel, the commission being $3\frac{1}{2}\%$ and the freight and storage being 33¢ a barrel.

4. At what price must 5% bonds be bought to realize $7\frac{1}{2}\%$ on the investment?

5. I buy oranges at 8 cents a dozen and retail them at the rate of 2 for 3 cents; find the per cent profit.

6. Find the cost at 45 cents a roll, of papering the walls of a room $16\frac{1}{2}$ feet long, 15 feet wide, and 12 feet high, making no allowance for openings. (A roll of paper is 8 yards long and 18 inches wide.)

7. Received 6% dividend on stock bought at 25% below par; what rate of interest did the investment pay?

8. Find the cost of the following bill of lumber: 20 scantlings 14 ft. long, 4 in. wide, and 3 in. thick at \$30, per M; 16 planks 10 ft. long, 14 in. wide, and 2 in. thick, at \$36 per M.

9. If $5\frac{1}{2}$ bushels of wheat cost \$4.75, how much will $8\frac{3}{4}$ bushels cost?

10. A merchant buys cloth at \$1.20 a yard, and marks it so as to sell it at a discount of 20% from the list price and still gain 20%; find the list price of the goods.

11. It costs \$36.18 to insure a store at $\frac{3}{4}\%$; find the face value of the policy.

12. A schoolhouse costing \$9500 is to be built in a district whose property is valued at \$1,920,000; find (a) the rate of taxation, (b) the amount of tax to be paid by a man whose property is valued at \$6500. (No allowance for collection.)

1. Simplify $\frac{26.7 - 11.80 + 6.45}{\frac{5}{8} \times 3\frac{1}{8} \times 0.72}$.
2. The diameter of a bicycle wheel is 28 inches. Find the number of revolutions it makes in going a mile.
3. Find the number of square yards in the entire surface of the four walls and ceiling of a room 18 feet 6 inches long, 12 feet 4 inches wide, and 9 feet high.
4. If 4% bonds to the amount of \$8000 face value are bought at 92½, find the cost of the bonds and the rate of income on the investment.
5. Find the interest of \$465, at 5%, from May 1, 1904, to Jan. 15, 1906.
6. Bought U.S. 4% bonds at 115½ (brokerage ½%) to the amount of \$5000 face value. Find the annual income and the rate of interest on the investment.
7. Find the cost of four sticks of timber, each 8 inches by 10 inches and 30 feet long, at \$15 per M, board measure.
8. Find the cost of the following: —
78 boards 13 ft. × 16 in. × ¾ in., at \$16.50 per M.
18 joists 10 ft. × 4 in. × 3 in., at \$13.75 per M.
9. At what price must 4% stock be bought so that the investment may yield 5%?
10. A dealer buys 6 cords of wood at \$4 a cord, and 8 tons of coal at \$4.50 a ton; he sells the wood at 80 cents a cord foot and the coal at 30 cents a hundredweight. Find his entire gain.
11. Find the cost at \$17.50 per M of 35 3-in. planks each 22 feet long and 16 inches wide.
12. What will it cost to carpet in the most economical way a room 36 feet by 20 feet, with matting 27 inches wide, at 45 cents a yard?
13. Two successive discounts of 15% and 10% reduced a bill to \$489.60; what was the original bill?

1. Simplify $1 + \frac{3\frac{1}{2} - 2 \times \frac{2}{3} + 1}{0.125 + 0.005 - 12\frac{1}{4}}$.

2. If 240 lb. of sugar are sold for \$19.20 at a gain of 28%, what was the cost per pound?

3. How long must a ladder be to reach a window 15 feet high, if the foot of the ladder is 8 feet from the house?

4. A grocer buys 20 bushels of potatoes at 75 cents a bushel, and sells them at 30 cents a peck; find his entire gain and his gain per cent.

5. On January 1, 1904, Edward White of New York sold to Charles Holt for cash 1600 yards of flannel at $37\frac{1}{2}$ cents a yard, 240 yards silk at \$1.62 $\frac{1}{2}$ a yard, and 1500 yards cotton at $8\frac{1}{4}$ cents a yard. Make out the receipted bill in proper form.

6. If cranberries are bought at \$4 a bushel, at what price per quart must they be sold in order to gain 20%?

7. How many rods of fence will be required to inclose a square field containing 2 acres?

8. A man bought a farm of 196 acres for \$9800, and after spending \$980 for improvements, sold the farm at \$66 an acre. What was his per cent of gain?

9. What income will be derived from investing \$14,060 in $3\frac{1}{2}\%$ bonds purchased at $87\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

10. How many cubical blocks, each edge of which is $\frac{1}{3}$ of a foot, are equivalent to a block of wood 8 feet long, 4 feet wide, and 2 feet thick?

11. A commission merchant sold 744 bushels of wheat and sent his employer \$527.31, retaining a commission of \$30.69. Find the rate of commission and the selling price of a bushel of the wheat.

12. Find the net proceeds of a note of \$500, payable in 90 days without interest, if discounted at a bank at 6%, 40 days after date.

1. Find the cost at 16¢ a square yard, of plastering the walls and ceiling of a room 18 ft. by 16 ft. and 12 ft. high, allowing 75 square feet for openings.
2. Find the cost at \$15 per M of 75 pieces of lumber each 14 ft. by 16 in. by $1\frac{1}{4}$ in.
3. The perimeter of one square is 12 inches and that of another is 16 inches; find the perimeter of a third square whose area equals the area of both.
4. A merchant bought 351 bushels of wheat for \$234; he sold half of the wheat at a gain of 15% and the rest at cost. Find the average gain on one bushel.
5. A 4-months note for \$735 dated May 16, 1904, drawing interest at 5 per cent, is discounted at a bank at 6 per cent two months before maturity without grace. Required the proceeds.
6. How many cords of wood can be stored in a shed 16 feet long, 12 feet wide, and 6 feet high?
7. A man sold a carriage for \$207 thereby gaining $12\frac{1}{2}\%$; how much did he gain?
8. Find the cost of plastering the walls and ceiling of a room 16 feet by 9 feet and 12 feet high, at 38 cents a square yard, making an allowance of $\frac{1}{8}$ for openings.
9. A merchant marks an article \$6, but sells it at a discount of 10% for cash and gains 20%; find the cost of the article.
10. Bought 18,970 lb. of hay at \$9 a ton, and 12,580 lb. of straw at \$7 a ton; sold the hay at 75 cents a hundred pounds and the straw at 60 cents a hundred pounds; find the entire gain.
11. Find the cost at \$50 an acre of a rectangular field 1650 feet long and 825 feet wide.
12. Find the cost at 35 cents per cubic yard of excavating a trench 6 rods long, $1\frac{1}{2}$ yards wide, and 1 foot 6 inches deep.

1. Simplify $\frac{3\frac{1}{2} - \frac{2}{7} \times 4.2}{.5 + \frac{2}{7}}$.

2. Find the amount of \$1357.63 at $5\frac{1}{2}\%$ simple interest from June 1, 1902, to Dec. 13, 1904.

3. A speculator buys bonds whose par value is \$10,000 at $113\frac{3}{8}$ and sells them at $115\frac{1}{8}$; how much does he gain if brokerage is $\frac{1}{8}$ on each transaction?

4. Find the square root of 0.729 to *three* decimal places.

5. A person failing in business owes \$10,800 and has property worth \$7200; what will a creditor receive whose claim is \$180?

6. Divide the sum of four thousandths and four millionths by their difference, extending the result to four places of decimals.

7. How many shares of stock at 4% discount can be bought for \$3076 if the broker charges $\frac{1}{8}\%$?

8. A person borrows \$100, and at the end of each year he pays \$25 to reduce the principal, and pays interest at 4% on what he has owed during that year. How much does he owe at the end of 3 years?

9. A man sold two horses for \$124 each, on one he gained 20% , on the other he lost 20% ; find the whole gain or loss.

10. Change $\frac{1}{8}\frac{2}{3}\frac{1}{4}$ to its lowest terms.

11. The sides of a rectangle are 8 ft. and 10 ft.; find the diagonal to three places of decimals.

12. George Dent gives you to-day a note for \$480 for 3 months without interest; write the note and find the proceeds if it is discounted to-day at a bank at 6% .

13. What is the middle minute of a calendar year?

14. A merchant gained $12\frac{1}{2}\%$ by selling 48 yards of silk for \$4.50 more than cost; find the cost of a yard of the silk.

15. Sold $\frac{3}{4}$ of a ton for $\frac{3}{4}$ the cost. Gain or loss per cent?

APPENDIX

EXTRACTION OF THE CUBE ROOT

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1. The third power, or cube, of $3 = 3 \times 3 \times 3 = 27$.
The third, or cube, root of $27 = \sqrt[3]{27} = 3$.
What is one of the three equal factors of 27 ?
What is the cube root of 27 ? Of 64 ?
2. Learn the cube of each of the nine digits. How many figures in each cube ?
3. What is the value of
 $\sqrt[3]{8}$; $\sqrt[3]{27}$; $\sqrt[3]{64}$; $\sqrt[3]{125}$; $\sqrt[3]{216}$; $\sqrt[3]{343}$; $\sqrt[3]{512}$; $\sqrt[3]{729}$
4. Cube 10; cube 25; cube 99.
5. How does the number of figures in the cube compare with the number of figures in the root ?
6. Find the value of
 3^3 ; $\sqrt[3]{27}$; 5^3 ; $\sqrt[3]{125}$
 30^3 ; $\sqrt[3]{27'000}$; 50^3 ; $\sqrt[3]{125'000}$
 300^3 ; $\sqrt[3]{27'000'000}$; 500^3 ; $\sqrt[3]{125'000'000}$
7. If the cube of a number contains three times as many figures as the number, or one or two less, how many figures in the cube of 253 ? 67 ? 8000 ?
8. How many figures in the cube root of 729 ? 512,000 ? 8,000,000 ?
9. Cube 0.08. How many decimal places in the cube ? Why ?
10. How many decimal places in the cube of 0.716 ? Of any number ?
11. Why must the number of decimal places in the cube of a decimal be a multiple of 3 ?

Extracting the cube root of a number or separating it into three equal factors is the reverse of cubing one of these equal factors. A careful analysis of the process of cubing will enable us to reverse the process and find the cube root of a number.

1. To find the cube of 64.

PROCESS

NOTE. *t* and *o* stand for *tens* and *ones*.

$$64^3 = 64 \times 64 \times 64; \text{ or, as } 64 = 60 + 4, \\ 64^3 = (60 + 4) \times (60 + 4) \times (60 + 4) = (t + o)(t + o)(t + o).$$

$$\begin{array}{r} 60 + 4 = t + o \\ 60 + 4 = t + o \\ \hline 60 \times 4 + 4^2 = to + o^2 \\ 60^2 + 60 \times 4 = t^2 + to \\ 60^2 + 2(60 \times 4) + 4^2 = t^2 + 2to + o^2 \\ 60 + 4 = t + o \\ \hline 60^2 \times 4 + 2(60 \times 4^2) + 4^3 = t^2o + 2to^2 + o^3 \\ 60^3 + 2(60^2 \times 4) + 60 \times 4^2 = t^3 + 2t^2o + to^2 \\ 60^3 + 3(60^2 \times 4) + 3(60 \times 4^2) + 4^3 = t^3 + 3t^2o + 3to^2 + o^3 \\ 216,000 + 43,200 + 2880 + 64 = 262,144 \end{array}$$

The four partial products of which the cube of 64 is composed, are :

- I. *The cube of the tens* = 216,000.
- II. *3 times the square of the tens times the ones* = 43,200.
- III. *3 times the tens times the square of the ones* = 2880.
- IV. *The cube of the ones* = 64.

2. Observing this law of forming the four partial products that make a third power, or cube, let us cube 75.

PROCESS

$$\begin{array}{r} 70^3 = 343,000 \\ 3 \times 70^2 \times 5 = 73,500 \\ 3 \times 70 \times 5^2 = 5,250 \\ 5^3 = 125 \\ \hline 75^3 = 421,875 \end{array}$$

3. Of these four partial products, which is the largest? The smallest? The second in size?

4. Applying this law, find the cube of 53; 86; 94.

5. Is the cube root of 200,000 more or less than 50? More or less than 60? Why do you think so?

Let us try to reverse this process of cubing a number by finding the factors of the four partial products that make the cube 262,144. In other words —

6. Extract the cube root of 262,144.

PROCESS

$$\begin{array}{rcl}
 t^3 + 3t^2o + 3to^2 + o^3 & = & 262'144(60 + 4 \\
 t^3 & = & 216\,000 \\
 3t^2 = 10,800 & \overline{) 46\,144} & = 3t^2o + 3to^2 + o^3 = (3t^2 + 3to + o^2)o \\
 3to = 720 & & \\
 o^2 = 16 & & \\
 3t^2 + 3to + o^2 = 11,536 & \overline{) 46\,144} & = (3t^2 + 3to + o^2)o
 \end{array}$$

EXPLANATION. 262,144 comes between the cubes 216,000 and 343,000; its cube root then comes between 60 and 70, and the *first* partial product, the t^3 , must be 216,000. Taking this out of the cube, what remains?

The *second* partial product, or $3t^2o$, is much the largest of the three remaining parts of the cube. It is made up of two factors, $3t^2$ and o . The remainder, 46,144, is approximately the product of these two factors. $3t^2 = 3 \times 60^2 = 10,800$. If we divide the approximate product, 46,144, by one of its factors, 10,800, we get the other factor, 4 or o .

The *third* partial product, or $3to^2 = 3to \times o$; $3to = 3 \times 60 \times 4 = 720$

The *fourth* partial product, or $o^3 = o^2 \times o$; $o^2 = 16$; adding $3t^2 + 3to + o^2$, or $10,800 + 720 + 16$, we have 11,536; multiplying this number by o or 4, we have $3t^2o + 3to^2 + o^3 = 46,144$, the sum of the three remaining partial products. Hence we conclude that 64 is the cube root required.

Find the cube root of the following numbers :

1. 12,167.
2. 32,768.
3. 46,656.
4. 74,088.
5. 103,823.
6. 157,464.
7. 175,616.
8. 238,328.
9. 373,248.
10. 474,552.
11. 614,125.
12. 884,736.
13. What is the edge of a cubic block containing 421,875 cu. in.?
14. Extract the cube root of 320.013504.

PROCESS

| | | |
|-------------------------|----------------|-------------------|
| | | 320.'013'504(6.84 |
| | | 216 |
| 3×60^2 | = 10,800 | 104.013 |
| $3 \times 60 \times 8$ | = 1440 | |
| 8^2 | = 64 | |
| | <u>12304</u> | 98.432 |
| 3×680^2 | = 1387200 | 5.581504 |
| $3 \times 680 \times 4$ | = 8160 | |
| 4^2 | = 16 | |
| | <u>1395376</u> | 5.581504 |

EXPLANATION. 1. We begin at the decimal point to separate the power into groups of three figures. How many figures will then be in the root? How many integrals?

2. Having found two figures of the root, we consider the tens to be 68, and proceed as before.

3. If the number is an imperfect power, we may annex decimal ciphers and approximate the root.

15. $\sqrt[3]{3796416}$.
16. $\sqrt[3]{12977875}$.
17. $\sqrt[3]{37.259704}$.
18. $\sqrt[3]{0.1001728}$.
19. $\sqrt[3]{0.087640}$.
20. $\sqrt[3]{25}$.

In extracting the cube root of fractions or mixed numbers, follow the directions given under square root (page 216).

21. $\sqrt[3]{\frac{1728}{125}}$.
22. $\sqrt[3]{\frac{19683}{24888}}$.
23. $\sqrt[3]{3\frac{1}{8}}$.
24. $\sqrt[3]{4\frac{12}{25}}$.
25. $\sqrt[3]{72\frac{1}{2}}$.
26. $\sqrt[3]{405\frac{28}{125}}$.

1. A carpenter wishes to make a cubical cistern that will contain 5832 cu. ft. of water; what must be the length of each of its edges?

2. If the solid contents of a globe are 85,184 in., what are the edges of a cube the cubical contents of which are the same?

3. A man has a pile of wood containing 256 cd. Suppose the wood to be piled in the shape of a cube, what would be the length of each of its edges?

4. What is the depth of a cubical cistern which shall contain 200 gal. of water? (1 gal. = 231 cu. in.)

5. A farmer has a cubical bin which contains 50 bu. of grain; what is its depth? (2150.4 cu. in. = 1 bu.)

6. What would it cost to cement the bottom and sides of a cubical reservoir which contains 1728 cu. ft., at 6 cents a square foot?

7. Find the surface of a cube of granite containing 91,125 cu. in.

8. How large a cube will weigh the same as a ball containing 1500 cu. in.?

9. If a 4-in. ball weighs 12 lb., what will an 8-in. ball weigh?

10. What is the weight of a cube of cork 2 in. long if a cubic foot weighs 15 lb.?

11. What is the length in inches of a half cord of wood in the shape of a cube?

12. What are the dimensions of a cube that has the same volume as a bin 12 ft. 6 in. long, 10 ft. wide, and 5 ft. high?

13. A ball weighs 40 lb. What is the weight of one of the same kind having $\frac{1}{4}$ its diameter?

DEFINITIONS

These definitions, arranged alphabetically, are given here mainly for reference. They may be used, if teachers desire, for review purposes.

Acceptance. The formal agreement by signature of a drawee to pay a draft according to its terms.

Acute Angle. An angle less than a right angle.

Addend. A number to be added to another.

Addition. The process of combining numbers, two by two, into one sum.

Agent or Correspondent. One employed to transact business for another.

Aliquot Part. The quotient of any number divided by an integer.

Altitude. Height. Measured by a straight line perpendicular to the line of the base, and extending from it to the highest point.

Amount. The result of addition; in computing interest, interest and principal added.

Angle. The divergence from a common point of two lines having different directions.

Antecedent. The first term of a ratio; the dividend.

Arabic System of Notation. So called because it came into Europe from Arabia, and was brought by Arabs from India.

Arc. Any portion of a circumference.

Area. The size or total contents of a surface.

Assessment. Money collected from shareholders in stock companies to meet losses or expenses.

Assessors. Officers who estimate the value of taxable property and apportion the tax to be raised.

Bank. A corporation formed to trade in money and securities, or for the custody and loaning of money.

Bank Discount. The allowance made to a bank by the holder of a note for having it paid to him before maturity; bank interest.

Base. The line or surface on which a figure is supposed to stand.

The number of which a percentage is taken.

Bill. An itemized statement showing to whom and by whom goods have been sold, or services rendered, and giving dates, quantity, price, and amount.

Bill of Exchange. A general term for foreign or domestic drafts, especially for the former.

Bonds. A series of interest-bearing notes of a government or corporation.

Broker. An agent who buys and sells securities or other property.

Brokerage. A broker's fee or commission.

Capital. Money or other property invested in business.

Charter. A special act of a legislature setting forth the rights and duties of a corporation.

Check. A depositor's order for the payment of money by his bank.

Chord. A straight line joining the ends of an arc.

Circle. A plane surface bounded by a curve every point of which is equally distant from a point within called the center.

Circumference. The perimeter or boundary of a circle.

Commission. A percentage paid to an agent for transacting business for another.

Common Denominator of two or more fractions. One showing the size of some fractional unit in which all may be expressed.

Common Factor of two or more numbers. A number that is a factor of each of them.

Complex Decimals have a common fraction in the numerator, as $0.27\frac{1}{2}$.

Complex Fractions contain a fraction in the numerator, in the denominator, or in both.

Composite Number. The product of integral factors, 1 not included.

Compound Number. Two or more denominate numbers used to express one quantity; a denominate number having two or more integral units of the same kind of measure, as $3^{\circ} 5'$.

Compound Ratio. The indicated product of two or more simple ratios, as $3:2 \times 4:8$, or $\frac{3:2}{4:8}$.

Cone. A solid having a circle for its base, and tapering uniformly to a point, the vertex of the cone.

Consequent. The second term of a ratio; the divisor.

Consignor. One who sends merchandise (a *consignment*) to an agent (the *consignee*) to be sold.

Convex Surface. The surface of a solid excluding that of its bases.

Corporation. A company authorized by charter to transact business as a single individual.

Couplet. The two terms of a ratio.

Coupons. Interest certificates attached to bonds.

Cube. A solid with six square faces.

Cube (Number). The product of three equal numbers; the third power of a number.

Cube Root. One of the three equal factors forming a third power. $3 \times 3 \times 3 = 3^3$ or 27.

Curvilinear surfaces are those bounded by curves.

Cylinder. A solid having for its

bases equal parallel circles, and having a uniform diameter.

Days of Grace. Three days, in addition to the time named in a note, allowed by law in many states for the payment of a note by its maker.

Decimal Fraction. One or more tenths, hundredths, thousandths, etc., of an integral unit.

Decimal Fractions, or Decimals. Any number of 10ths, 100ths, 1000ths, etc.; commonly expressed at the right of the decimal point without written denominator.

Decimal Point. A period used after ones and before tenths.

Decimals. Decimal fractions written after the decimal point, without a denominator.

Decimal System of Numbers. A system in which *ten* units of any order make one unit of the next higher order.

Degree. A 360th part of a circumference; or, in measuring angles, a 360th part of a revolution.

Denominate Number. One in which the unit is a measure, as 3 lb.

Denominator. The lower term of a fraction. It *names* the fractional units according to their size and shows into how many equal parts the integral unit is divided.

Diagonal of a Polygon. A straight line connecting the vertices of two angles not adjacent.

Diameter. A line measuring the

shortest distance across a circle or square through the center.

Difference. What must be added to the smaller of two numbers to make the larger.

Digits. The numbers for which the nine Arabic figures stand.

Dimensions. Measurements needed to find contents.

Discount. An allowance deducted.

True Discount. The difference between the face and the present worth of a debt due at a future time without interest. The interest on the present worth.

Dividend. Profits of business divided among stockholders in proportion to their shares.

Dividend. A number to be divided.

Division. The process of separating a number into equal parts, or of finding how many times one number is contained in another.

Divisor. A number to divide by; it shows *how large* or *how many* the equal parts of the dividend are to be.

Draft. An order sent by one party to another, requesting him to pay a specified sum to the order of some one named.

Sight Draft. One payable when presented to the drawee.

Time Draft. One payable at a specified time after sight or after date.

Drawee. The party ordered to pay a draft.

Drawer. The maker of a draft.

Duties or Customs. Taxes laid by the government on imported goods.

Duty, *ad valorem*. A tax of a certain per cent of the cost of imports in the country where they are bought.

Duty, *specific*. A fixed tax levied on imports according to weight, number, or measure.

Equation. Two quantities expressed as being equal.

Equiangular. Having equal angles.

Equilateral. Having equal sides.

Exact Divisor. One that gives an integral quotient, without a remainder.

Exchange. A method of making payments or collections in distant places, by means of orders or drafts, without the actual sending of money.

Exponent or Index. One or more figures written above and at the right of a number to show how many times the number is taken as a factor.

Extremes. The first and fourth terms of a proportion.

Face of Note, Check, or Draft. The sum for which it is written.

Factors. Numbers multiplied together in making a product; commonly used as meaning integral factors.

Figure. A surface bounded by lines or a space bounded by surfaces.

Fraction. One or more of the equal parts of an integral unit.

Greatest Common Factor, Divisor, or Measure. The largest factor found in each of two or more numbers.

Gross Weight includes the material used in packing.

Horizontal. Parallel to the plane of the horizon.

Hypotenuse. The longest side of a right triangle.

Imports. Merchandise brought from a foreign country.

Improper Fraction. A number not less than 1 expressed in the form of a fraction.

Inclined. Neither horizontal nor vertical.

Indorsement. A signature on the back of negotiable paper. A record of payment on the back of a note.

Indorser. One who puts his signature on the back of a note, check, draft, etc.

Insurance. Compensation for loss by fire or other disaster.

Integer. A whole number of which the lowest unit is *one*, not any part of one.

Interest. An allowance to the owner for the use of his money.

Interest, Annual. Simple interest on the principal, and simple interest upon any overdue interest.

Interest, Compound. Interest reckoned on both the principal and the overdue interest added to the principal as often as due.

Interest, Exact. Interest computed for parts of a year by taking the exact number of days and reckoning 365 to a year.

Invoice. A bill of goods sold.

Isosceles triangles have two sides equal.

Leakage and Breakage. A discount for liquors lost from casks or bottles during importation.

Least Common Denominator of two or more fractions. One showing the size of the largest fractional unit in which all can be expressed.

Least Common Multiple of two or more numbers. The smallest number of which each is a factor.

Like Fractions have fractional units of the same size and kind.

Like Numbers have units of the same size and kind.

Line. The limit of a surface. The path of a point.

Maker of a Note. The one who makes the promise and signs it. The promisor.

Market Value. Present value in open market.

Maturity. The time when a note, draft, or bond falls due and is legally payable.

Means. The second and third terms of a proportion.

Minuend. A number to be lessened.

Mixed Decimal. A number consisting of an integer and a decimal fraction.

Mixed Number. An integer and a fraction taken together.

Multiple of a number. A number of which it is a factor.

Multiplicand. One of the equal numbers to be combined by multiplication; the factor to be repeated in making a product.

Multiplication. The process of combining equal numbers, by repetition, into one product. It repeats one number "many fold."

Multiplier. The factor that shows how many equal numbers are to be combined in the product.

Negotiable Paper. Notes, drafts, or other written obligations that may be bought and sold.

Net Price or Cost. The price or cost after all discounts or charges have been deducted.

Net Weight. Weight exclusive of packing material.

Notation. A system of writing numbers in figures or letters.

Note, Demand. One payable at the demand of the holder.

Note, Time. One payable at a specified time.

Note, Interest-bearing. One containing the words "with interest."

Number. That which answers the question "How many?"; one or more units.

Numeration. A system of reading numbers expressed in figures.

Numerator. The upper term of a fraction. It *numbers* the fractional units contained.

Oblique lines are neither horizontal nor vertical. Oblique angles are greater or less than right angles.

Oblong. A rectangle whose length exceeds its breadth.

Obtuse angles are greater than right angles.

Parallel. Extending in the same direction, and in all parts equally distant.

Parallelogram. A quadrilateral whose opposite sides are parallel.

Partial Payments. Payments in part of a note or debt.

Par Value. Face value.

Payee. The one to whom or to whose order a note, check, or draft is payable.

Per Cent. Number of hundredths; units out of a hundred.

Percentage. The process of computing by hundredths. The part of the base indicated by the rate per cent.

Perimeter. The circumference of a surface or the sum of its bounding lines.

Period. One of the groups, of three figures each, counting from the ones' place.

Perpendicular. At right angles to another line or surface.

Personal Estate. Property exclusive of land and buildings.

Plane. A plane surface is a flat or level surface.

Point. That which has position,

but no length, breadth, or thickness. The end of a line.

Policy. The written agreement given to the insured by the underwriters.

Poll-tax. A uniform tax on persons of a certain class.

Polygon. A plane surface having straight sides, commonly more than four.

Port of Entry. A city or town containing a custom-house, where U.S. duties are paid.

Power. The product of two or more equal numbers as factors.

Premium. The sum paid for insurance. Excess of market value above par value.

Present Worth. The sum that, at the present time, will pay a non-interest-bearing debt due in the future, without loss to either debtor or creditor.

Prime Number. A number with no other factor than itself and 1.

Principal. A sum upon which interest may be allowed.

Prism. A solid whose sides are parallelograms, and whose bases are equal parallel polygons. Prisms are named from the form of their bases, as *square* prisms, *rectangular* prisms, *triangular* prisms, *hexagonal* prisms, etc.

Proceeds or Avails of a Note. The sum for which the note is sold. Its maturity value less the bank discount. *Net Proceeds.* What is

left after all charges have been deducted.

Product. The result of multiplication.

Promissory Note. A written promise to pay a specified sum of money.

Proper Fraction. A number less than 1; a true fraction.

Proportion. An expression of the equality of two ratios.

Pyramid. A solid, whose base is a regular polygon, and whose sides are triangles meeting in a common point, the vertex of the pyramid.

Quadrant. A fourth part of a circle or of a circumference.

Quadrilateral. A plane surface having four straight sides.

Quotient. The result of division.

Radius. A straight line extending from center to circumference of a circle.

Rate of interest. Per cent of the principal allowed for a year's use of it.

Rate per cent. The number of hundredths used in finding a percentage.

Ratio. The relative size of two numbers expressed by their quotient.

Real Estate. Land and buildings.

Reciprocal of a Fraction. $1 \div$ the fraction, or the fraction inverted.

Reciprocal of a Number. $1 \div$ the number; the fractional unit expressed by that number as denominator, as 3, $\frac{1}{3}$.

Rectangle. A parallelogram having four right angles.

Rectilinear. Bounded by straight lines.

Reduction. Changing the unit of a number without changing its value.

Remainder. What is left when part of a number is taken away.

Remittance. Money or negotiable paper sent to another.

Rhomboid. A parallelogram with oblique angles.

Rhombus. An equilateral rhomboid.

Right Angle. An angle of 90° .

Roman System of Notation. So called because invented and used by the Romans.

Root. One of the equal factors forming a power.

Scalene triangles have their sides unequal.

Secant. A straight line that cuts a curve at two points.

Sector. The part of a circle bounded by an arc and two radii.

Segment. The part of a circle between an arc and its chord.

Semicircle. Half of a circle.

Sextant. One-sixth of a circle.

Share. One of the equal parts into which corporation capital is divided.

Simple Fraction. One having only integral terms.

Slant Height. The shortest distance from the vertex of a cone or pyramid along the outside to the base.

Solid. A form having three dimensions,—length, breadth, and thickness.

Sphere. A solid having a curved surface equally distant from the center at every point.

Square. An equilateral rectangle. A plane surface with four equal sides and angles.

Square (Number). The second power or the product of a number multiplied by itself.

Square Root. One of the two equal factors of a square, or second power.

Stock Certificate. A statement given by a corporation, showing the par value and the number of shares owned by a stockholder.

Stockholders. Owners of the capital or stock of corporations.

Stocks. Shares in the capital of corporations. Government or corporation bonds.

Subtraction. The process of taking part of a number out of it to find the remainder; finding the difference between two numbers.

Subtrahend. A number to be subtracted from another.

Surface. That which has only two dimensions,—length and breadth. The outside of a solid.

Tangent. A line touching a curve at a single point without crossing.

Tare. An allowance for the weight of boxes, bags, etc., used in packing goods.

Tariff. The list of dutiable articles with the rate assessed on each.

Taxes. Money raised by government for public uses.

Term of Discount. The time between the day of discount and the day of maturity.

Terms of a Fraction. The numerator and denominator.

Terms of a Ratio. The antecedent and consequent.

Trapezium. A quadrilateral no two of whose sides are parallel.

Trapezoid. A quadrilateral only two of whose sides are parallel.

Triangle. A plane surface having three straight sides. A *right triangle* has one right angle; an *obtuse triangle* has one obtuse angle; an *acute triangle* has three acute angles. The angles of a triangle are together equal to two right angles, 180°.

Underwriters. Insurance companies.

Unit. One; any thing used as a basis or standard of measurement or comparison.

Vertex. The point in an angle where the sides meet.

Vertical. Relating to the vertex.

Vertical lines point toward the zenith and the earth's center.

+ Plus; and; the sign of addition.

– Minus; less; the sign of subtraction.

- × Times ; multiplied by ; the sign of multiplication.
- + or : Divided by ; signs of division.
-) In ; a sign of division.
- , /, (as in $\frac{1}{3}$, $\frac{2}{3}$). Signs of division.
- = Equals, or equal ; the sign of equality.
- \$ Dollar or dollars.
- % Hundredths ; per cent.
- Ct., c., or ¢ Cent or cents.
- @ At (the rate of).
- ∴ Therefore.
- () as in $(3 + 4) \times 5 = 35$ } *Curves* or
- as in $3 + 4 \times 5 = 35$ } *Vinculum*.
- shows that the numbers inclosed or
- beneath are to be treated as one number.
- $\sqrt{\quad}$, the square root of.
- $\sqrt[3]{\quad}$, the cube root of.
- 6%, $\frac{6}{100}$, .06, or 6 per cent.
- ², ³, as in $5^2 =$ the square of 5 or 25 ;
- $4^3 =$ the cube of 4, or 64.
- Dr., debtor.
- Cr., creditor.
- G.C.D., greatest common divisor.
- L.C.M., least common multiple.
- π. A Greek letter pronounced pī. It stands for 3.1416 —, the ratio of the circumference of a circle to its diameter.

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ANSWERS

THIRD BOOK—PART I

Page 9

1. \$45.40.
2. \$300.49.
3. \$3815.24.
4. \$35,751.37.
5. \$48,590.80.
6. \$11,465.13.
7. \$13,729.55.
8. \$15,850.62.
9. \$16,197.45.
10. \$14,581.69.
11. \$16,678.86.
12. \$88,503.30.
13. \$88,503.30.
15. \$3889.44.
16. \$4480.80.
17. \$4609.18.
18. \$4438.55.
19. \$4435.58.

Page 12

1. \$16,654.13.
2. \$4881.11.
3. \$4194.20.
4. \$7198.54.
5. \$12,728.65.
6. \$15,138.25.
7. \$6668.01.
8. \$3960.21.
9. \$9668.11.
10. \$14,113.23.
11. \$2845.02.

12. \$5536.50.

13. \$4245.66.
14. \$7090.68.
15. \$1554.18.
16. \$1290.84.

Page 14

1. \$473.23.
2. \$140.53.
3. \$309.52.
4. \$127.73 loss.
5. 256,506.
6. 860,758.
7. 850 ft.
8. \$620,581,818.
9. \$476,500,561.
10. \$657,805,399.
11. \$524,955,950.
12. \$664,426,346.

Page 16

1. \$13.44.
2. \$500.78.
3. \$8458.25.
4. \$15,999.60.
5. \$7156.52.
6. \$561.91.
7. \$44.21.
8. \$6368.13.
9. \$778.71.
10. \$541.90.
11. \$4441.18.

Page 21

3. 453,456.
4. 2,373,672.
5. 15,984¢.
6. 35,441 lbs.
7. 114,163.
8. 62,464 oz.
9. \$583,737.
10. 16,308 doz.
11. \$38,880.
12. \$3096.
13. 8064 qts.
14. 6574¢.
15. 729,022.
16. 4214 oz.
17. 29,488 posts.
18. \$15,584.73.
19. \$226,645.44.
20. \$62,652.55.
21. \$1484.28.
22. \$8741.25.
23. \$44,846.50.
24. \$1985.75.
25. \$4984.00.
26. \$3431.25.
27. \$3831.36.
28. \$2277.45.
29. \$8936.28.
30. \$6208.61.
31. \$9660.
32. \$14,456.25.
33. \$2259.84.

34. \$34,320.75.
35. \$12,652.12.
36. \$1769.93.
37. \$141.00.
38. \$230.55.
39. \$4676.25.
40. \$832.48.
41. \$174.06.
42. 17,280,000 rds.
43. 134,928 sq. in.
44. 1,555,200 sec.

Page 22

7. \$6471.36.
8. \$19,849.83.
9. 973,000.
4. \$316.80.
5. \$335.
6. \$1850.

Page 26

9. \$67.00.
10. \$90,817.
11. \$82.
12. \$825.50.
13. \$74 $\frac{1}{8}$.
14. 843 $\frac{4}{7}$.
15. 80 $\frac{1}{4}$.
16. 247 $\frac{1}{8}$.
17. \$2.57 $\frac{1}{4}$.
18. 9736 $\frac{1}{4}$.

19. 13,016 $\frac{1}{2}$.
20. 3199 $\frac{1}{7}$.
21. 993 $\frac{1}{3}$.
22. 2872 $\frac{1}{3}$.
23. 3071 $\frac{1}{7}$.
24. 2763 $\frac{1}{3}$.
25. 2286 $\frac{1}{3}$.
26. 2904 $\frac{1}{3}$.
27. 2023 $\frac{1}{7}$.
28. 1899 $\frac{1}{7}$.
29. 4026 $\frac{1}{3}$.
30. 5751 $\frac{1}{3}$.
31. 2915 $\frac{1}{3}$.
32. 1957 $\frac{1}{3}$.
33. 3015 $\frac{1}{3}$.
34. 1337 $\frac{1}{7}$.
35. 1934 $\frac{1}{3}$.
36. 3004 $\frac{1}{3}$.
37. 3207 $\frac{1}{3}$.
38. 2955 $\frac{1}{3}$.

- a. 1483 ; 244.
- b. 1232 ; 244.
- c. 1411 ; 70.
- d. 580 ; 209.
- e. 991 ; 350.
- f. 719 ; 32.
- g. 810 ; 754.
- h. 563 ; 50.
- i. 2148 ; 258.
- j. 508 ; 868.
- k. 434 ; 424.
- l. 873 ; 872.
- m. 701 ; 900.
- n. 1346 ; 190.
- o. 1130 ; 125.
- p. 1210 ; 288.
- q. 1435 ; 238.
- r. 2258 ; 368.
- s. 1358 ; 354.
- t. 417 ; 32.

- u. 2304 ; 159.
- v. 360 ; 924.
- w. 2477 ; 356.
- x. 307 ; 262.
- y. 939 ; 802.
- z. 286 ; 840.

Page 31

2. 56 $\frac{1}{2}$.
3. \$350 gain.
4. \$525.
5. 20 bbl.
6. \$0.75.
7. 2.10.
8. \$402.

Page 32

1. \$130 gain.
2. \$5750.
3. 8 hrs.
4. 225 da.
5. \$52.50.
6. 28 mi.
7. \$15,000 ; \$2500.
8. \$15.75 gain.
9. Post. ; 3 ϕ .
10. 22 qt.
11. 39.

Page 33

16. 2640.
17. 17,741.
18. 15,924.
19. 15,522.
20. 1,206,060.
21. 32,616.
22. 3690.
23. 11,466 ft.
24. 10,710 lb.

Page 34

1. 63.88 + mi. per hr.
2. 68 T.
3. 18 ϕ .
4. \$2.25.
5. \$2.25.
6. 3 $\frac{1}{4}$ ϕ .
7. 160,000 g.
8. \$63.39.
9. \$996 gain.
10. \$2.50.

Page 36

2. \$22.50.
4. \$93.50.
6. \$432.
7. $\frac{1}{2}$ as long.
8. 6 da.
9. \$1225.
10. 1625.
11. 2052.
12. \$1.50.
13. Hans.
14. \$4.
15. 336 mi. ; 84 mi.
16. \$56.25.

Page 37

1. 78 hr.
2. 1123 min.
3. 305 in.
4. 2699 sq. in.
5. 232 cu. ft.
6. 43 pt.
7. 1200 rd.
8. 224 qt.
9. 467 hr.
10. 1560 sq. rd.
11. 147 oz.

12. 27,600 lb.
13. 296 qt.
14. 420 mi.
15. 588 min.
16. 15,387 cu. in.
17. 89 gal.
18. 9 $\frac{1}{2}$ lb.
19. 650 $\frac{1}{2}$ cu. yd.
20. 9 $\frac{5}{8}$ hr.
21. 6 $\frac{2}{3}$ yr.
22. 61 $\frac{1}{2}$ bu.
23. 128 $\frac{1}{2}$ cu. yd.
24. 108 gal. 6 gr.
25. 83 $\frac{1}{2}$ da.
26. 26,000.
27. 91.
28. 60 $\frac{2}{3}$ cu. ft.
29. 40 lots.
30. 7040.
31. 2666 $\frac{1}{2}$.
32. 2400.

Page 40

1. \$15.40.
2. \$2.31.
3. 366 ; 52 wk. ; 2 da.
4. No profit.
5. \$28.67.
6. 100 bbl.
7. \$153.48.
8. 4 wk.
9. \$426.40.
10. \$10,590.
11. 7920.
12. 416 $\frac{1}{2}$.
13. 92,795,826.

Page 42

1. 2 $\frac{1}{2}$.
2. 37 $\frac{1}{2}$.

3. $427\frac{1}{2}$.
4. $45\frac{1}{2}$.
5. $102\frac{1}{2}$.
6. $416\frac{1}{2}$.
7. $40\frac{1}{2}$.
8. $101\frac{1}{2}$.
9. $101\frac{1}{2}$.
10. $20\frac{1}{2}$.
11. $86\frac{1}{2}$ gal.
12. 2363 eggs.
13. 5182.
14. $142\frac{1}{2}$.
15. 4.

Page 48

3. 4620.
4. 1800.
5. 315.
6. 432.
7. 400.

Page 49

2. $3\frac{1}{2}$.
3. $1\frac{1}{2}$.
4. $1\frac{1}{2}$.
5. $1\frac{1}{2}$.
6. $1\frac{1}{2}$.
7. $1\frac{1}{2}$.
8. $2\frac{1}{2}$.
9. $1\frac{1}{2}$.
10. $1\frac{1}{2}$.
11. $1\frac{1}{2}$.
12. $154\frac{1}{2}$.
13. $133\frac{1}{2}$.
15. $58\frac{1}{2}$.
16. $75\frac{1}{2}$.
17. $56\frac{1}{2}$.
18. $13\frac{1}{2}$.
19. $7\frac{1}{2}$.
20. $56\frac{1}{2}$.

21. $57\frac{1}{2}$.
22. $68\frac{1}{2}$.
23. $79\frac{1}{2}$.
24. $34\frac{1}{2}$.
25. 166.

Page 51

19. \$160.78.
20. \$2.81.
21. \$173.96.
22. \$201.
23. \$33.91.
24. \$32.73.
25. \$66.67.
26. \$43.59.

Page 52

2. $1\frac{1}{2}$.
3. \$20.25.
4. 68.
5. $1\frac{1}{2}$ cd.
6. $1\frac{1}{2}$.
7. $1\frac{1}{2}$; $1\frac{1}{2}$ A.
8. $1\frac{1}{2}$.
9. \$15.
10. 6 yd.; \$0.52.
11. \$2.67.
12. \$149.50.
13. $49\frac{1}{2}$.

Page 53

14. $\frac{1}{2}$.
15. $\frac{1}{2}$.
16. $\frac{1}{2}$.
17. $\frac{1}{2}$.
18. $\frac{1}{2}$.
19. $\frac{1}{2}$.
20. $\frac{1}{2}$.
21. $\frac{1}{2}$.
22. $\frac{1}{2}$.
23. $56\frac{1}{2}$.
24. 5.
25. $19\frac{1}{2}$.
26. 75.
27. $1629\frac{1}{2}$.
28. $\frac{1}{2}$.
29. $78\frac{1}{2}$.

30. $\frac{1}{2}$.
31. $7\frac{1}{2}$.
32. $\frac{1}{2}$.
33. $3\frac{1}{2}$.
34. $2\frac{1}{2}$.
35. $\frac{1}{2}$.

Page 54

1. \$29.39.
2. \$16.76.
3. \$69.65.
4. \$3.63.
5. $\frac{1}{2}$.
6. \$0.43.
7. \$0.13.
8. \$38.99.
9. \$0.0864.

Page 57

1. $3\frac{1}{2}$.
2. $1\frac{1}{2}$.
3. $1\frac{1}{2}$.
4. $1\frac{1}{2}$.
5. $1\frac{1}{2}$.
6. $1\frac{1}{2}$.
7. $1\frac{1}{2}$.
8. $2\frac{1}{2}$.
9. 15.
10. $1\frac{1}{2}$.
11. $\frac{1}{2}$.
12. $\frac{1}{2}$.
13. $\frac{1}{2}$.
14. $\frac{1}{2}$.
15. $2\frac{1}{2}$.
2. $434\frac{1}{2}$.
3. $610\frac{1}{2}$.
4. $1023\frac{1}{2}$.
5. $79\frac{1}{2}$.
6. $294\frac{1}{2}$.
7. $66\frac{1}{2}$.
8. $11\frac{1}{2}$.
9. $15\frac{1}{2}$.
10. $21\frac{1}{2}$.

11. $44\frac{1}{2}$.
12. $32\frac{1}{2}$.
13. $11\frac{1}{2}$ lb.
14. $16\frac{1}{2}$ ft.

Page 58

1. (1) $1\frac{1}{2}$.
- (2) $1\frac{1}{2}$.
- (3) $1\frac{1}{2}$.
- (4) $1\frac{1}{2}$.
- (5) $1\frac{1}{2}$.
- (6) $1\frac{1}{2}$.
- (7) $1\frac{1}{2}$.
- (8) $1\frac{1}{2}$.
- (9) $1\frac{1}{2}$.
- (10) $1\frac{1}{2}$.
2. (1) $1\frac{1}{2}$.
- (2) $1\frac{1}{2}$.
- (3) $1\frac{1}{2}$.
- (4) $1\frac{1}{2}$.
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- (7) $1\frac{1}{2}$.
- (8) $1\frac{1}{2}$.
- (9) $1\frac{1}{2}$.
- (10) $1\frac{1}{2}$.
3. (1) $1\frac{1}{2}$.
- (2) $1\frac{1}{2}$.
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- (4) $1\frac{1}{2}$.
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- (6) $1\frac{1}{2}$.
- (7) $1\frac{1}{2}$.
- (8) $1\frac{1}{2}$.
- (9) $1\frac{1}{2}$.
- (10) $1\frac{1}{2}$.
4. (1) $1\frac{1}{2}$.
- (2) $1\frac{1}{2}$.

- (3) $1\frac{1}{2}$.
 (4) $1\frac{1}{10}$.
 (5) $\frac{3}{5}$.
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 (7) $\frac{3}{10}$.
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 (9) $\frac{3}{10}$.
 (10) $\frac{3}{10}$.
6. (1) $2\frac{5}{10}$.
 (2) $1\frac{3}{10}$.
 (3) $1\frac{3}{10}$.
 (4) $1\frac{3}{10}$.
 (5) $1\frac{3}{10}$.
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 (10) $1\frac{3}{10}$.
7. (1) $1\frac{1}{2}$.
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 (8) $1\frac{1}{2}$.
 (9) $1\frac{1}{2}$.
 (10) $1\frac{1}{2}$.
8. (1) $22\frac{3}{4}$.
 (2) $46\frac{3}{4}$.
 (3) $30\frac{1}{4}$.
 (4) $74\frac{3}{4}$.
 (5) $101\frac{3}{4}$.
 (6) $112\frac{3}{4}$.
 (7) $26\frac{3}{4}$.
 (8) $84\frac{3}{4}$.
 (9) $55\frac{3}{4}$.
 (10) $46\frac{3}{4}$.
9. (1) $24\frac{3}{4}$.
 (2) $54\frac{3}{4}$.
 (3) $35\frac{3}{4}$.
 (4) $81\frac{3}{4}$.
 (5) $111\frac{3}{4}$.
 (6) $121\frac{3}{4}$.
 (7) $31\frac{3}{4}$.
 (8) $94\frac{3}{4}$.
 (9) $65\frac{3}{4}$.
 (10) $50\frac{3}{4}$.
10. (1) $341\frac{3}{4}$.
 (2) $547\frac{3}{4}$.
 (3) $677\frac{3}{4}$.
 (4) $909\frac{3}{4}$.
 (5) $1049\frac{3}{4}$.
 (6) $1585\frac{3}{4}$.
 (7) $2556\frac{3}{4}$.
 (8) $4863\frac{3}{4}$.
 (9) $852\frac{3}{4}$.
 (10) $7601\frac{3}{4}$.
11. (1) $1\frac{1}{2}$.
 (2) $1\frac{1}{2}$.
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 (8) $1\frac{1}{2}$.
- (9) $9\frac{1}{2}$.
 (10) $16\frac{1}{2}$.
12. (1) $\frac{3}{5}$.
 (2) $12\frac{3}{5}$.
 (3) $3\frac{3}{5}$.
 (4) $\frac{3}{5}$.
 (5) $9\frac{3}{5}$.
 (6) $16\frac{3}{5}$.
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- (7) $5\frac{1}{10}$.
 (8) $2\frac{1}{10}$.
 (9) $8\frac{1}{10}$.
 (10) $16\frac{1}{10}$.
16. (1) $9\frac{1}{10}$.
 (2) $10\frac{1}{10}$.
 (3) $12\frac{1}{10}$.
 (4) $53\frac{1}{10}$.
 (5) $70\frac{1}{10}$.
 (6) $69\frac{1}{10}$.
 (7) $6\frac{1}{10}$.
 (8) $59\frac{1}{10}$.
 (9) $17\frac{1}{10}$.
 (10) $41\frac{1}{10}$.
17. (1) $79\frac{1}{10}$.
 (2) $66\frac{1}{10}$.
 (3) $73\frac{1}{10}$.
 (4) $30\frac{1}{10}$.
 (5) $9\frac{1}{10}$.
 (6) $31\frac{1}{10}$.
 (7) $78\frac{1}{10}$.
 (8) $22\frac{1}{10}$.
 (9) $58\frac{1}{10}$.
 (10) $69\frac{1}{10}$.
18. (1) $30\frac{1}{10}$.
 (2) $46\frac{1}{10}$.
 (3) $62\frac{1}{10}$.
 (4) $78\frac{1}{10}$.
 (5) $85\frac{1}{10}$.
 (6) $13\frac{1}{10}$.
 (7) $250\frac{1}{10}$.
 (8) $46\frac{1}{10}$.
 (9) $54\frac{1}{10}$.
 (10) $62\frac{1}{10}$.
19. (1) $1\frac{1}{10}$.
 (2) $2\frac{1}{10}$.
 (3) $3\frac{1}{10}$.
 (4) $4\frac{1}{10}$.
 (5) $5\frac{1}{10}$.

ANSWERS

5

- | | | | |
|----------------------------|--------------------------|-------------------------------|---------------------------|
| (6) $90\frac{3}{15}$. | (3) $64\frac{1}{2}$. | 27. (1) $1\frac{3}{10}$. | (9) 35245 $\frac{1}{2}$. |
| (7) $16\frac{7}{15}$. | (4) $256\frac{3}{8}$. | (2) $\frac{1}{2}$. | (10) 25800. |
| (8) $71\frac{1}{3}$. | (5) $428\frac{1}{2}$. | (3) $1\frac{1}{2}$. | 31. (1) $\frac{1}{2}$. |
| (9) $36\frac{8}{15}$. | (6) $545\frac{1}{2}$. | (4) $1\frac{3}{10}$. | (2) $\frac{1}{2}$. |
| (10) $25\frac{1}{15}$. | (7) $115\frac{1}{2}$. | (5) $1\frac{1}{15}$. | (3) $1\frac{1}{15}$. |
| | (8) 578. | (6) $1\frac{1}{10}$. | (4) $1\frac{1}{10}$. |
| 20. (1) $61\frac{9}{15}$. | (9) $330\frac{1}{2}$. | (7) $1\frac{1}{10}$. | (5) $1\frac{1}{10}$. |
| (2) $2\frac{1}{15}$. | (10) 258. | (8) $1\frac{1}{10}$. | (6) $1\frac{1}{10}$. |
| (3) $61\frac{9}{15}$. | 24. (1) $1\frac{1}{2}$. | (9) $1\frac{1}{10}$. | (7) $1\frac{1}{10}$. |
| (4) $47\frac{1}{15}$. | (2) $1\frac{1}{2}$. | (10) $1\frac{1}{10}$. | (8) $1\frac{1}{10}$. |
| (5) $60\frac{1}{15}$. | (3) $1\frac{1}{2}$. | 28. (1) $100\frac{1}{15}$. | (9) $1\frac{1}{10}$. |
| (6) $60\frac{1}{15}$. | (4) $1\frac{1}{2}$. | (2) $515\frac{1}{2}$. | (10) $1\frac{1}{10}$. |
| (7) $1\frac{9}{15}$. | (5) $1\frac{1}{2}$. | (3) $194\frac{1}{15}$. | 32. (1) $1\frac{1}{2}$. |
| (8) $50\frac{1}{15}$. | (6) $1\frac{1}{2}$. | (4) $661\frac{1}{15}$. | (2) 7. |
| (9) $7\frac{1}{15}$. | (7) $1\frac{1}{2}$. | (5) $1343\frac{1}{15}$. | (3) $4\frac{1}{2}$. |
| (10) $24\frac{1}{15}$. | (8) $1\frac{1}{2}$. | (6) $1984\frac{1}{15}$. | (4) 30. |
| 21. (1) $3\frac{1}{2}$. | (9) $1\frac{1}{2}$. | (7) $162\frac{1}{15}$. | (5) $29\frac{1}{2}$. |
| (2) $3\frac{1}{2}$. | (10) $1\frac{1}{2}$. | (8) $886\frac{1}{15}$. | (6) 20. |
| (3) $4\frac{1}{2}$. | 25. (1) $1\frac{1}{2}$. | (9) $688\frac{1}{15}$. | (7) $19\frac{1}{2}$. |
| (4) $4\frac{1}{2}$. | (2) $1\frac{1}{2}$. | (10) $540\frac{1}{15}$. | (8) $42\frac{1}{2}$. |
| (5) 4. | (3) $1\frac{1}{2}$. | 29. (1) $21205\frac{1}{2}$. | (9) 27. |
| (6) $4\frac{1}{2}$. | (4) $1\frac{1}{2}$. | (2) $33075\frac{1}{2}$. | (10) $26\frac{1}{15}$. |
| (7) $3\frac{1}{2}$. | (5) $1\frac{1}{2}$. | (3) $42997\frac{1}{2}$. | 33. (1) $1\frac{1}{2}$. |
| (8) $4\frac{1}{15}$. | (6) $1\frac{1}{15}$. | (4) $55462\frac{1}{2}$. | (2) $2\frac{1}{2}$. |
| (9) $3\frac{1}{2}$. | (7) $1\frac{1}{15}$. | (5) $62770\frac{1}{2}$. | (3) $1\frac{1}{2}$. |
| (10) $2\frac{1}{2}$. | (8) $1\frac{1}{15}$. | (6) $98099\frac{1}{2}$. | (4) $6\frac{1}{15}$. |
| 22. (1) $6\frac{1}{2}$. | (9) $1\frac{1}{15}$. | (7) $169,181\frac{1}{15}$. | (5) $4\frac{1}{2}$. |
| (2) $3\frac{1}{2}$. | (10) $1\frac{1}{15}$. | (8) $319,548\frac{1}{2}$. | (6) $2\frac{1}{2}$. |
| (3) $7\frac{1}{2}$. | 26. (1) $1\frac{1}{2}$. | (9) $566,915\frac{1}{2}$. | (7) $1\frac{1}{2}$. |
| (4) $1\frac{1}{2}$. | (2) $1\frac{1}{2}$. | (10) $505,896\frac{1}{15}$. | (8) $4\frac{1}{2}$. |
| (5) $2\frac{1}{15}$. | (3) $1\frac{1}{2}$. | 30. (1) $\$1583\frac{1}{2}$. | (9) 21. |
| (6) $3\frac{1}{2}$. | (4) $1\frac{1}{2}$. | (2) 7083 $\frac{1}{2}$. | (10) $1\frac{1}{2}$. |
| (7) $4\frac{1}{15}$. | (5) $1\frac{1}{2}$. | (3) $10687\frac{1}{2}$. | 34. (1) $2\frac{1}{2}$. |
| (8) $2\frac{1}{2}$. | (6) $1\frac{1}{2}$. | (4) 36992 . | (2) $20\frac{1}{2}$. |
| (9) 4. | (7) $1\frac{1}{2}$. | (5) 51450 . | (3) $11\frac{1}{2}$. |
| (10) $4\frac{1}{2}$. | (8) $1\frac{1}{2}$. | (6) 58176 . | (4) $23\frac{1}{2}$. |
| 23. (1) $15\frac{1}{2}$. | (9) $1\frac{1}{2}$. | (7) 11880 . | (5) 46. |
| (2) $56\frac{1}{2}$. | (10) $1\frac{1}{2}$. | (8) 57800 . | (6) $24\frac{1}{2}$. |
| | | | (7) $14\frac{1}{2}$. |

- (8) $35\frac{1}{10}$.
 (9) $152\frac{1}{4}$.
 (10) $107\frac{1}{2}$.
35. (1) $24\frac{1}{10}$.
 (2) $24\frac{1}{10}$.
 (3) $13\frac{1}{10}$.
 (4) $13\frac{1}{10}$.
 (5) $13\frac{1}{10}$.
 (6) $24\frac{1}{10}$.
 (7) $24\frac{1}{10}$.
 (8) $12\frac{1}{10}$.
 (9) $13\frac{1}{10}$.
 (10) $57\frac{1}{10}$.
36. (1) $27\frac{1}{10}$.
 (2) $11\frac{1}{10}$.
 (3) $27\frac{1}{10}$.
 (4) $62\frac{1}{10}$.
 (5) $58\frac{1}{10}$.
 (6) $41\frac{1}{10}$.
 (7) $19\frac{1}{10}$.
 (8) $58\frac{1}{10}$.
 (9) $13\frac{1}{10}$.
 (10) $17\frac{1}{10}$.
37. (1) $35\frac{1}{10}$.
 (2) $54\frac{1}{10}$.
 (3) $71\frac{1}{10}$.
 (4) $91\frac{1}{10}$.
 (5) $104\frac{1}{10}$.
 (6) $162\frac{1}{10}$.
 (7) $280\frac{1}{10}$.
 (8) $529\frac{1}{10}$.
 (9) $940\frac{1}{10}$.
 (10) $838\frac{1}{10}$.
38. (1) $41\frac{1}{10}$.
 (2) $61\frac{1}{10}$.
 (3) $81\frac{1}{10}$.
 (4) $11\frac{1}{10}$.
 (5) $12\frac{1}{10}$.
- (6) $19\frac{1}{10}$.
 (7) $33\frac{1}{10}$.
 (8) $63\frac{1}{10}$.
 (9) $112\frac{1}{10}$.
 (10) $100\frac{1}{10}$.
39. (1) $\$38\frac{1}{10}$.
 (2) $\$32\frac{1}{10}$.
 (3) $\$94\frac{1}{10}$.
 (4) $\$84\frac{1}{10}$.
 (5) $\$60\frac{1}{10}$.
 (6) $\$72\frac{1}{10}$.
 (7) $\$152\frac{1}{10}$.
 (8) $\$80\frac{1}{10}$.
 (9) $\$94\frac{1}{10}$.
 (10) $\$253\frac{1}{10}$.
40. (1) $\$6\frac{1}{10}$.
 (2) $\$8\frac{1}{10}$.
 (3) $\$23\frac{1}{10}$.
 (4) $\$82\frac{1}{10}$.
 (5) $\$61\frac{1}{10}$.
 (6) $\$73\frac{1}{10}$.
 (7) $\$43\frac{1}{10}$.
 (8) $\$11\frac{1}{10}$.
 (9) $\$26\frac{1}{10}$.
 (10) $\$38\frac{1}{10}$.
41. (1) $5\frac{1}{10}$.
 (2) $11\frac{1}{10}$.
 (3) $17\frac{1}{10}$.
 (4) $23\frac{1}{10}$.
 (5) $11\frac{1}{10}$.
 (6) $11\frac{1}{10}$.
 (7) $11\frac{1}{10}$.
 (8) $11\frac{1}{10}$.
 (9) $17\frac{1}{10}$.
 (10) $11\frac{1}{10}$.
42. (1) $61\frac{1}{10}$.
 (2) $21\frac{1}{10}$.
- (3) $61\frac{1}{10}$.
 (4) $47\frac{1}{10}$.
 (5) $60\frac{1}{10}$.
 (6) $60\frac{1}{10}$.
 (7) $11\frac{1}{10}$.
 (8) $50\frac{1}{10}$.
 (9) $7\frac{1}{10}$.
 (10) $11\frac{1}{10}$.
43. (1) $11\frac{1}{10}$.
 (2) $11\frac{1}{10}$.
 (3) $11\frac{1}{10}$.
 (4) $10\frac{1}{10}$.
 (5) $11\frac{1}{10}$.
 (6) $11\frac{1}{10}$.
 (7) $11\frac{1}{10}$.
 (8) $11\frac{1}{10}$.
 (9) $11\frac{1}{10}$.
 (10) $25\frac{1}{10}$.
44. (1) $38\frac{1}{10}$.
 (2) $67\frac{1}{10}$.
 (3) $36\frac{1}{10}$.
 (4) $96\frac{1}{10}$.
 (5) $136\frac{1}{10}$.
 (6) $223\frac{1}{10}$.
 (7) $34\frac{1}{10}$.
 (8) $88\frac{1}{10}$.
 (9) $67\frac{1}{10}$.
 (10) $136\frac{1}{10}$.
45. (1) $21\frac{1}{10}$.
 (2) $11\frac{1}{10}$.
 (3) $11\frac{1}{10}$.
 (4) $41\frac{1}{10}$.
 (5) $24\frac{1}{10}$.
 (6) $11\frac{1}{10}$.
 (7) $11\frac{1}{10}$.
 (8) $31\frac{1}{10}$.
 (9) $11\frac{1}{10}$.
 (10) $11\frac{1}{10}$.

Page 59

1. $1\frac{1}{10}$.
2. $\$52.31$.
3. $\$391$.
4. $\$445$.
5. $\$0.165+$.
6. $\$4.50$.
7. $\$375$.
8. $\$6.00$.
9. $\$19.01$.
10. $3\frac{1}{10}$ mi.
11. 160 mi.
12. $127\frac{1}{10}$.
13. $14\frac{1}{10}$ mi.

Page 60

1. $1\frac{1}{10}$.
2. 59.
3. $95\frac{1}{10}$ A.
4. 10 bbl.
5. 3190.
6. $1\frac{1}{10}$.
7. 9.32.
8. 135 ; $\$19,125$.
9. $\$32.56$.
10. 50 yd.
11. $61\frac{1}{10}$.

Page 66

1. $\$9364.88$.
2. $1\frac{1}{10}$.
3. 10,235 steps.
4. 1440.
5. $3\frac{1}{10}$.
6. $1\frac{1}{10}$.
8. Mar. 14.
May 1st.
9. $1\frac{1}{10}$.
10. 179.
11. $\$1166\frac{1}{10}$.
12. $37\frac{1}{10}$; 26 ; 29.

Page 68

1. \$632.76.
2. \$533.80.
3. \$21.46.
4. \$25.66.
5. \$39.36.
6. \$30,700.

Page 69

1. \$116.17.
2. \$38.40.
3. \$11,424.89.
4. \$6198.43.
5. \$5255.57.
6. \$811.
7. \$51.60.
8. Sec. \$15.03.
9. \$22,076.28.

Page 70

1. \$3.99.
2. \$6.50.
3. \$4.35.
4. 800 boxes.
5. \$2.67.
6. $\frac{1}{2}$ T;
1,890 $\frac{1}{2}$ lb.
7. $\frac{1}{2}$.
8. \$2.81.
9. $\frac{1}{16}$; \$100.
10. \$4105.
11. $4\frac{1}{2}$ da.
12. \$201.92;
\$276.92.
13. 80; 18,750.
14. $1\frac{1}{4}$ in.
15. 20 bu.

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I

1. 206.668.

2. 235.722.
3. 21.3725.
4. 22.32589.
5. 700.108.
6. 680.40285.
7. 234.695.
8. 25.1429.
9. 222.585.
10. 123.27114.

II

1. 1.74.
2. 1.5086.
3. 31.13.
4. 8.774.
5. 3.749.
6. 0.3976.
7. 1.767.
8. 1.5306.
9. 27.926.
10. 5.57.
11. 5.893.
12. 2.5416.
13. 23.8352.
14. 8.557.

III

1. 258.797.
2. 47.252.
3. 203.3074.
4. 63.045.
5. 185.41.
6. 44.5387.
7. 226.472.
8. 396.3544.
9. 152.807.
10. 3.192.
11. 23.5237.

IV

1. 182.62.
2. 1.2541 $\frac{1}{2}$.
3. 2.886.
4. 7959.2.

5. 6.
6. 0.235.
7. 0.16 $\frac{1}{2}$.
8. 0.016 $\frac{1}{2}$.
9. 1.6 $\frac{1}{2}$.

Page 79

1. 3600.
2. 0.289.
3. 78.4.
4. 0.39.
5. 70.5.
6. 113.5.
7. \$8.9125.
8. 640.0287.
9. 55.1286.
10. 3560.
11. 0.583 $\frac{1}{2}$.
12. 0.5625.
13. 0.266 $\frac{1}{2}$.
14. 0.428 $\frac{1}{2}$.
15. 0.555 $\frac{1}{2}$.
16. 0.233 $\frac{1}{2}$.
17. 1.0625.
18. 1.277 $\frac{1}{2}$.
19. 0.938 $\frac{1}{2}$.
20. 0.976 $\frac{1}{2}$.
21. 0.260 $\frac{1}{2}$.
22. 2.529 $\frac{1}{2}$.
23. 1.622 $\frac{1}{2}$.
24. 1.371 $\frac{1}{2}$.
25. 0.175 $\frac{1}{2}$.
26. 0.903 $\frac{1}{2}$.
27. 0.916 $\frac{1}{2}$.
28. 0.555 $\frac{1}{2}$.
29. 0.5454 $\frac{1}{2}$.
30. 0.183 $\frac{1}{2}$.
31. 0.466 $\frac{1}{2}$.
32. 0.0016 $\frac{1}{2}$.
33. 0.388 $\frac{1}{2}$.
34. 0.2307 $\frac{1}{2}$.

35. 1.12.
36. 0.06 $\frac{1}{2}$; 6 $\frac{1}{2}$ %.
37. 78%.

Page 81

1. 10,400; 12,800;
8400; 8400.
2. \$1.15; \$1.38.
3. \$1.00.
4. 3756.22 $\frac{1}{2}$ ft.
5. 100 mi.
6. \$1,840.
7. \$728.89.
8. 90 sq. in.
9. \$115,520.98.
10. \$180.60.
11. 64 cd.
12. \$635.

Page 83

2. \$147.
3. \$87.
4. \$64.
5. \$1000.
6. \$67.87.
8. \$4.00.
9. \$1.88.
10. \$1.87.
11. \$3.72.
12. \$9.33.
13. \$33.26.
14. \$94.50.
15. \$11.63.

Page 86

24. 36 mi. 23,040 A.
25. 138 $\frac{1}{2}$ sq. ft.
26. 45 $\frac{1}{2}$ $\frac{1}{2}$ sq. rd.
27. 78,135 $\frac{1}{2}$ sq. ft.
28. 43,560 sq. ft.
29. 272 $\frac{1}{2}$ sq. ft.

30. 217,800 sq. ft.
 31. $6\frac{1}{11}$ sq. rd.
 32. $11\frac{1}{2}$ A.
 33. \$3411.50.
 34. $73\frac{59}{100}$ sq. rd.

Page 91

9. 500; $3\frac{1}{2}$.
 10. 1020.
 11. $8\frac{1}{2}$ sq. yd.
 12. 729.
 13. 4 A.
 14. \$3482.50.
 15. \$180.
 16. $5\frac{1}{3}$.
 17. 633,600.
 18. \$0.80.
 20. \$17.50.
 21. 75 ft.

Page 92

3. 6; 6; 36.
 4. 7; 7; 49;
 \$61.25.
 5. 5; $33\frac{1}{3}$.
 6. \$70.
 7. \$26.67.
 8. \$28.35.
 9. \$258.90.
 10. \$24.83.
 11. \$90.92.

Page 93

1. $21\frac{7}{11}$ sq. ft.
 2. 770.
 3. 14.
 4. $105\frac{1}{2}$; 2850.
 5. 26136.
 6. 70.
 7. $1\frac{1}{2}$ sq. ft. more.
 10. $71\frac{1}{2}$ sq. yd.

Page 95-96

1. \$4098.60.
 2. \$300.
 3. \$604.08.
 4. \$987.50.
 5. \$1350.
 6. \$2559.38.
 7. \$1728.
 8. \$2245.32.
 9. \$1975.59.
 10. \$1447.88.
 11. \$937.20.
 12. \$1929.60.
 13. \$272.68.
 14. \$350.35.
 15. \$57.00.
 16. \$391.
 17. \$67.50.
 18. \$124.
 19. \$320.
 20. \$1593.33.
 21. \$42.27.

Page 99

3. 360 sq. ft.
 4. 750 sq. ft.
 5. $1\frac{1}{2}$ sq. ft.
 6. 231 sq. ft.
 7. $73\frac{1}{2}$ sq. ft.
 8. $6975\frac{1}{2}$ sq. ft.
 9. 1248 sq. ft.
 10. $57\frac{1}{2}$ sq. yd.
 15. 396 sq. in.
 16. 7500 sq. ft.
 18. 75 sq. ft.
 19. 98 sq. in.

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8. $93\frac{1}{2}$ sq. ft.
 9. 10,560 sq. ft.
 10. $11\frac{1}{2}$ sq. ft.
 11. $3\frac{1}{2}$ sq. ft.

Page 101

2. 256 sq. in.
 3. $2\frac{1}{2}$ sq. ft.
 4. 90° .
 5. 465 sq. in.
 6. $74\frac{1}{2}$ sq. in.
 7. \$2756.25.
 8. $330\frac{7}{11}$ sq. rd.
 9. $\frac{1}{2}$ as large.
 10. 714 sq. in.
 11. 9. 82 sq. ft.
 12. 16,500 sq. ft.
 13. $17\frac{1}{2}$ sq. ft.
 14. 3,484,800 sq. ft.
 15. $153\frac{1}{2}$ sq. ft.
 16. 50 sq. yd.
 17. \$2.56.

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16. 62.832 ft.
 17. 65.2535 ft.
 18. 565.488 ft.
 19. 1.48 ft.
 20. 51.05 in.
 21. 1.19 ft.

Page 104

6. 37.6992 ft;
 113.0976 sq. ft.
 7. 37.6992 ft;
 113.0976.
 8. 31.8309;
 795.2269.
 9. 7.95; 198.925
 10. 706.86 sq. ft.
 11. 1385.45 sq. ft.
 12. 12,732.4 sq. in.
 13. 1590.435.
 14. 855.30 sq. rd.
 15. 5026.56 sq. yd.
 16. 928 + sq. ft.

17. 51 - acres.
 18. 1134.1176 sq.
 rd.
 19. 104,062.358 sq.
 ft.
 20. 7238.246 sq. in.
 21. 5944.69 sq. ft.

Page 108

6. $\frac{\pi}{3}$.
 7. 18.8496.
 8. 234.44 ft.
 9. 9685.84 sq. ft.
 10. 10.2744 sq. in.
 11. $63\frac{1}{2}\%$.
 12. 339.2928 sq. ft.
 13. $60\frac{1}{2}\%$.

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10. 216 cu. in.
 11. 512 cu. in.
 12. 125 cu. ft.
 13. 1728 cu. ft.
 14. 1000 cu. yd.
 15. 8000 cu. in.

Page 111

4. 180 cu. in.
 5. 400 cu. in.
 6. 7.5 cu. ft.
 7. 1280 cu. ft.
 8. 4.5 cu. ft.
 9. 4608 cu. ft.
 10. $11,333\frac{1}{3}$ cu. ft.
 11. 500 cu. ft.
 12. $177\frac{1}{2}$ cu. ft.
 14. 30 cu. in;
 300 cu. in.
 15. 96 cu. yd.
 16. $7\frac{1}{2}$ cu. ft.
 17. $8\frac{1}{2}$.

Page 112

10. \$18.
11. \$100.
12. \$112.50.
13. \$19.69.
14. \$29.30.
15. \$5.36.
16. 320 ft.

Page 113

4. 10 bd. ft.
5. 120 bd. ft.
6. 10 bd. ft.
7. 37.5 bd. ft.
8. 256 bd. ft.
9. 96 bd. ft.
10. 140 bd. ft.
11. 270 bd. ft.
12. 216 bd. ft.
13. 8 bd. ft.

Page 114

4. 5 in.
5. 486 cu. in.
6. 600 cu. in.
7. 1536 cu. in.
8. 8 in.
9. 10 in.
10. 7 in.
13. 114 sq. in.
16. 248 sq. in.
17. 468 sq. in.
18. 544 sq. in.
19. 848; 82.
20. 552; 88½.
21. 576; 22½.

Page 116

10. 502.656 cu. ft.
11. 752.0256 gal.
12. 628'32 cu. ft.
13. 7 in.

Page 117

1. 192.9376.
2. \$13.96.
3. 280 +.
4. 109.956 sq. ft.

Page 118

1. 24; 11.
2. 16.
3. 6.
6. 78 sq. ft.
7. 12 in.
8. 120 ft.
9. 3 yd.
10. 544½ ft.
11. 26 in.
12. 18 ft.
13. 9 in.
14. 96 rd.
15. 20 in.
16. 62,500 sq. ft.
17. 160 rd;
416 rd.
18. 6 ft. by 18 ft.

Page 119

1. \$10.
2. 4 days.
3. 6 in.
4. 12 ft.
5. 12 in.
6. 4 ft.
8. 30 in.
9. 1½ ft.
10. 2.8 ft.
11. 6 in.
12. 660 ft.

Page 120

1. 4½ ft.
2. 184,078½ gal.
3. \$48595.

4. \$133,650.
5. 7 ft.
6. \$2242.50.
7. 21,780 cu. ft.
8. \$192.
9. \$5.40.
10. 22050.
11. 21 sq. ft.;
252 cu. ft.

Page 121

1. \$50.63.
2. 10,939½ lb.
3. \$26.98.
4. 1565.44 T.
5. 37.6992 bbl.
6. 603.1872 sq. in.
7. \$276.67.
8. \$5760.
9. \$476.80.
11. 12½%.
12. \$92.80.

Page 122

1. 0.2146.
2. 909.
3. \$6106.88.
4. 142½.
5. 13¢.
6. 1 to 1000.
7. \$17.81.
8. 630.
9. 8+min.
10. 4950 ft.
11. 85 yd.
12. \$27.06.
13. 500 lb.

Page 124

1. \$10,000.
2. 433.
3. 45,375.

4. 40.2 mi.
5. 4 P.M. Mar. 21.
6. \$2822.40.
7. 1.
8. \$63.25.
9. \$51.84.
10. \$17.28.
11. 67½%.
12. 29,538.

Page 126

1. 152.210.
2. \$4.65.
3. \$8320.
4. \$287.10.
5. \$312.48.
6. 25% loss.
7. \$55.04.
8. \$345.
9. \$68.25;
\$123.50.
10. 5%.
11. \$1808; \$7.53.
12. 20½.

Page 128

1. \$133.35.
2. 55.13.
3. 22½ sq. yd.
4. 594 cu. yd.
5. 62½%.
6. \$18.
7. \$79.33.
8. ½ or 33⅓%.
9. 56¼%.
10. \$8320.
11. ⅓ or 10%.
12. \$4.60.
13. \$4095.24.
14. ⅔.
16. 42.

ANSWERS

THIRD BOOK—PART II

Page 136

5. 1419.4 bu.
6. \$2958.
7. \$337.50.
8. 4301 bu. corn.
3542 bu. oats.
4807 bu. wheat.
9. 935½ T.
10. 58.8 mi.
11. 955½ A.
12. \$2.44.
13. \$209.25.
14. \$51.19.
15. 100 to 1.
17. \$56,000.
18. \$560.
19. \$434.

Pages 137-8

2. 71 %.
3. 96 %.
4. 30,000 T.
5. \$17,634.37 ;
45½ %.
6. 32 %.
7. \$21.25.
8. 44½ %.
9. 60 %.
10. 17½ %.
11. 46,947 —.
2. \$963.
3. \$635.

4. \$467.
5. \$3125.
6. 66 gal.
7. 97 yd.
8. 250,000 ;
105,000.
9. 7900.
10. \$950,000.
11. 150,000,000 sq.
mi.
12. 656.
13. \$251,940.
14. 2032.
15. \$1175.

Page 139

1. Alike profit.
2. 100 to 1.
3. 25 %.
4. 58½ %.
5. 45 %.
6. 95 %.
7. 16½ %.
8. 4 ; 300 ; ¾ %.
9. 6 ; 1600 ; ½ %.
10. 112 ; 83½ % ;
16½ %.
11. 1½ ; 140 ; ½.
12. 33½ % ; 3½ % ;
25 %.
13. 625 ; 2000 ;
1000.

14. \$1241.67.
15. \$12,000.
16. 36½ %.
17. \$613.97.

Page 140

1. 12 %.
2. 960.
3. 27⅔.
4. 40 A.
5. 92 %.
6. \$1728.
7. 56½ %.
8. 225 cd.
9. ¾ %.
10. 7800 T.
11. 4 %.
12. 18 %.
13. 13½ %.
14. \$9600.
15. 5⅓.
16. 3456.
17. 3.8 + %.
18. \$4500.
19. \$72.90.
20. 1800.

Page 141

1. \$6500.
2. \$24,640.
3. \$60 ; \$72.
4. 33½ %.

5. 16½.
6. 25 %.
7. 33⅓ %.
8. 3½ % more is
gained by
buying for
\$4 and sell-
ing for
\$4.80.

9. \$9.
10. 33½ %.
11. Loss \$100.
12. Loss 20 %.
13. 145.4545 +.

Page 142

1. \$4.80.
2. \$72.
3. \$5568.
4. \$6000.
5. 71.
6. 300 A.
7. \$47,000.
8. \$2000.
9. 35 %.
10. \$1.60.
11. \$3.50.

Page 144

1. \$96.
2. \$48.
3. \$32.

4. \$30.
5. \$30.
6. \$35.
7. 25 %.
8. \$20 or 25 %.
9. 20 %
16 $\frac{2}{3}$ %.
10. 1 $\frac{1}{4}$ %.
11. \$870.
12. \$7.81.
13. \$5000.
14. \$50,000.

Page 145

1. 18 $\frac{3}{4}$ % or 14 $\frac{2}{7}$ %.
2. \$10.
3. 20 %.
4. \$48,000.
\$50,400.
5. \$140.
6. 15 $\frac{1}{2}$ %.
7. 4 $\frac{1}{2}$ %.
8. 8 $\frac{1}{2}$ %.
9. 50 %.
10. 14 $\frac{7}{12}$ %.
11. 8 %.
12. \$5200.
13. \$4.

Page 147

3. \$44.80.
4. \$137.60.
5. \$28.03.
6. \$5.88.
7. \$298.54.
8. \$375.
9. \$104.99.
10. \$125.16.

Page 148

9. \$8.71.
10. \$9.92.

11. \$1.56.
12. \$1.71.
13. \$1.36.
14. \$3.13.
15. \$20.83.
16. \$11.68.
17. \$38.65.
18. \$0.05.

Page 149

6. \$10.67.
7. \$15.53.
8. \$13.37.
9. \$14.21.
10. \$6.43.
11. \$1.90.
12. \$4.09.
13. \$9.06.
14. \$1.01.

Page 150

3. \$545.85.
4. \$7920.11.
5. \$1909.96.
6. \$95.06.
7. \$1135.24.
8. \$214.28.
9. \$92.52.

10. \$1187.25.
11. \$483.69.
12. \$18.48.
13. \$1266.66 $\frac{2}{3}$.

Page 151

7. 5 yr. 21 da.
8. 3 yr. 2 mo. 6 da.
9. 2 yr. 10 mo. 21 da.
11. 2 yr. 1 mo. 3 da.

Page 152

1. \$14.49.
2. \$95.
3. \$17.09.
4. \$7.99.
5. \$7.52.
6. \$37.50.
7. \$13.24.
8. \$4.61.
9. \$21.85.
10. \$166.45.
11. \$1.89.
12. \$1.67.
13. \$49.80.
19. \$88.

Page 153

4. \$144.
6. \$4.93.
7. \$9.70.
8. \$7.76.
9. \$11.05.
10. \$10.77.
11. \$29.59.
12. \$160.93.

Page 154

1. \$26.79.
2. \$759.54.
3. \$540.11.
4. \$13.93.
5. \$900.
6. \$540.08.
7. \$234.
8. \$60.67.
9. \$25.20.
10. \$3.15.
11. \$3.47.
12. \$97.86.
13. \$147.52.

14. \$4.41.
15. \$19.44.
16. \$10.
17. \$227.50.
18. \$746.67.
19. \$58.90.
20. \$105.22.
21. \$333.33.
22. \$1.29.
23. \$48.95.
24. \$1312.50.
25. \$4.25.

Page 155

1. \$555.21.
2. \$164.68.
3. \$71.35.
4. \$1316.10.
5. \$2607.85.
6. \$917.59.
7. \$5663.92.
8. \$96.63.
9. \$12,837.98.
10. \$265.45.
11. \$552.59.
12. \$26,614.07.
13. \$76.72.
14. \$33.77.
15. \$921.65.
16. \$10,086.95.
17. \$267,793.01.
18. \$56.52.
19. \$804.37.
20. \$1396.14.
21. \$15.24.
22. \$438.90 or \$438.85.
23. \$530.78.
24. \$443.06.
25. \$971.10.
26. \$189.24.

27. \$3014.35.
28. \$2853.33.
29. \$114.28.
30. \$449.50.

Page 156

1. \$475.31.
2. \$414.57.
3. \$773.44.
4. \$30.51.
5. \$11.20.
6. \$8.74.
7. \$121.33.
8. \$26.79.
9. \$444.01.
10. \$38.55.
11. \$854.91.
12. \$84.42.
13. \$20.30.
14. \$45.28.
15. \$49.23.
16. \$125.16.
17. \$41.36.
18. \$719.29.
19. \$33.66.
20. \$85.37.
21. \$511.25.
22. \$32.06.
23. \$1170.68.
24. \$38.35.
25. \$55.39.
26. \$27.99.
27. \$525.56.
28. \$36.83.
29. \$356.58.
30. \$54.17.
31. \$708.15.
32. \$49.47.
33. \$7.60.
34. \$22.14.
35. \$33.94.

36. \$19.74.
37. \$78.06.
38. \$16.04.
39. \$340.55.
40. \$12.10.
41. \$56.79.
42. \$38.56.
43. \$111.18.
44. \$4.22.
45. \$3.67.
46. \$98.02.
47. \$112.36.
48. \$201.56.
49. \$62.56.
50. \$38.49.
51. \$56.79.
52. \$39.32.
53. \$45.85.
54. \$314.53.
55. \$31.22.
56. \$48.01.
57. \$422.04.
58. \$22.27.
59. \$502.11.
60. \$40.83.
61. \$17.66.
62. \$6.84.
63. \$42.88.
64. \$64.22.
65. \$3.02.
66. \$126.34.
67. \$36.50.
68. \$10.75.
69. \$17.86.
70. \$12.25.
71. \$306.
72. \$17.25.
73. \$107.26.
74. \$10.55.
75. \$11.20.
76. \$14.58.

77. \$19.36.
78. \$37.12.
79. \$557.81.
80. \$41.52.
81. \$77.98.
82. \$4.33.
83. \$12.90.
84. \$36.32.
85. \$10.42.
86. \$29.51.
87. \$42.91.
88. \$1216.19.
89. \$28.89.
90. \$234.17.
91. \$8.51.
92. \$1412.15.
93. \$51.92.
94. \$121.48.
95. \$13.51.
96. \$540.00.
97. \$27.34.
98. \$54.44.
99. \$8.80.
100. \$15.34.
101. \$71.32.
102. \$516.98.
103. \$66.74.
104. \$1933.70.
105. \$48.12.
106. \$89.98.
107. \$67.24.
108. \$2.77.
109. \$1952.00.
110. \$327.36.
111. \$53.30.
112. \$17.66.
113. \$11.83.
114. \$64.80.
115. \$38.73.
116. \$16.90.
117. \$23.22.

118. \$20.27.
119. \$102.39.
120. \$186.91.
121. \$17.44.
122. \$13.25.
123. \$46.04.
124. \$4.48.
125. \$19.38.
126. \$16.20.
127. \$96.44.
128. \$15.10.
129. \$14.88.
130. \$228.99.
131. \$18.47.
132. \$570.65.
133. \$54.85.
134. \$52.08.
135. \$25.11.
136. \$62.50.
137. \$104.61.
138. \$363.48.
139. \$133.11.
140. \$551.54.
141. \$30.52.
142. \$117.79.
143. \$146.07.
144. \$53.32.
145. \$34.00.
146. \$580.32.
147.

Page 158

1. \$1900.
2. 10%.
3. \$500.
4. \$1128.
5. 3.46%.
6. 2000.
7. 16½%.
8. 4%.
9. \$1.50.

10. \$500.
11. 2%.
12. \$70.
13. 44.
14. 2%.

Page 159

2. \$114.
3. \$0.25.
4. First $\frac{1}{4}$ % or \$1.20 (better).
5. Third.
6. \$11.70.
7. \$24.26.
8. \$320.10.
9. \$5.24.
10. \$240.88.
11. \$2390.08.

Page 160

1. \$1400. \$1425.
2. \$10,590.
3. \$3920.
4. 73%.
5. 15%.
6. 20%.
7. \$313.63.
8. \$10,397.75.
9. \$598.50.
10. \$555.58.
11. \$675.88.
12. \$67.21.

Page 162

1. \$10,000.
\$10,000.
\$2000.
2. \$5000.
3. $1\frac{1}{4}$ %.
4. $1\frac{1}{2}$ %.
5. \$6857.14.

6. \$4000.
7. \$93.75.
8. \$31.60.
9. \$15,843.75.
10. \$23,333.33.
11. 2%.
12. \$2400.
13. \$1,500,000.
14. 18%.
15. \$50,000.
16. \$50,000.

Page 164

3. \$168.50.
4. \$10971.
5. $\frac{1}{2}$ %.
6. 3%.
7. 1.77-%
8. $2\frac{1}{2}$ %.
9. $1\frac{1}{2}$ %.
10. $14\frac{1}{2}$ %.
11. $3\frac{1}{12}$ %.
12. 3%.
13. 2%.
14. B's Com.
\$12.08; A's
profit \$40.42.
15. Com. 4 %.
Cost \$5000.
16. \$7783.16.

Page 166

1. \$19.50.
2. 200 bbl.
3. \$80.
4. \$4170.
5. 115 bbl.
6. \$39.
7. \$11,573.75.
8. \$170.50.
9. 5%.

10. $7\frac{2}{3}$ %.
11. 100 M.
12. $2\frac{1}{2}$ %.

Page 167

1. \$314.93.
2. \$12,000.
3. \$4123.38.
4. \$7600;
com. \$400.
5. \$16.87.
6. \$9,250,000.
7. \$4000.
8. \$30,000.
9. \$6297.63.
10. \$58.87.

Page 168

1. \$8.74;
\$428.26.
2. \$2750; \$2675.
3. \$25; 10%.
4. \$918.37;
\$18.37.
5. $1\frac{1}{2}$ %; \$1082.
6. \$1552.80;
\$67.20.
7. \$450.70; 3%.
8. \$421.05.
9. \$526.67.
10. \$200.00
9.63
\$214.00.
11. \$20,000.
12. \$589.20;
1.12 + %.
13. \$3.52;
2.37 + %.
14. \$777.65; 3%.
15. \$207.90; \$198.

16. \$1250;
\$43.75.
17. \$787.49;
\$15.39.

Page 171

1. \$261.52;
\$261.64
(grace).
2. \$931.60;
\$932.07
(grace).
3. \$737.08;
\$737.39
(grace).
4. \$1963.20;
\$1963.80
(grace).
5. \$801.95;
\$802.35
(grace).
6. \$297.57;
\$297.68
(grace).
7. \$76.50;
\$76.58(grace).

Page 176

7. \$780.
8. \$3180.
9. \$576.31.

Page 177

1. \$455.80.
2. \$700.62.
3. \$561.13.
4. \$1290.23.
5. \$664.80.
6. \$4431.71.
7. \$939.84.
8. \$110.30.
9. \$1755.

Page 179

1. \$300,000.
2. \$50,000;
\$10,000.
3. \$15 on \$1000;
1½¢ on \$1;
\$8000.
4. 1½%; \$0.012;
1½%.
5. 1½%; \$1.48.
6. \$33.33.
7. \$2.
8. \$66.67.
9. \$76.93--.
10. \$15.69--.
11. \$12.86--.
12. \$6.67.
13. \$12.56--.
14. \$17.
15. \$112.50.
16. \$3876.75.
17. \$12.30--.

Page 181

9. 8243.2.
10. \$123.65.
12. Specific duty
\$40 more.
13. \$60.75; 3¼%.
14. 122½%; \$150.
15. \$450.80.
16. 20%.
17. \$0.32 gain.

Page 182

6. \$1226.75.
7. \$111.40.

Page 183

1. \$78.81.
2. \$206.46.

3. \$449.95.
Int. Table.
1. \$551.91.
2. \$713.21.
3. \$367.58.
4. \$877.50.

Page 186

1. \$790.70
(grace);
\$791.
2. \$713.90
(grace);
\$714.
3. \$8.97 (grace);
\$8.75.

Page 187

1. \$2.89, \$522.11
(grace);
\$2.63,
\$522.37.
2. \$906.34
(grace);
\$906.68.
3. \$310.82
(grace);
\$311.04.
4. \$784.50
(grace);
\$785.
5. \$953.28
(grace);
\$953.60.
6. \$712.41
(grace);
\$712.74.

1. July 7/10; 27
da. or 30 da.

2. Feb. 15/18; 63
da. or 66 da.
3. Oct. 5/8; 87
da. or 40 da.
4. June 17/20; 47
da. or 50 da.
5. Aug. 14/17;
67 da. or
70 da.
6. Feb. 24/27; 26
da. or 29 da.
7. Jan. 19/22; 70
da. or 73 da.;
2 mo. 9 da.;
2 mo. 12 da.
8. May 11/14; 66
da. or 69 da.;
2 mo. 5 da.
or 2 mo. 8 da.

Page 188

1. 63 da. or 66 da.
2. 24 da.
3. \$438.
4. \$994.86.
5. \$795.33.
6. \$447.65
(grace);
\$447.80.
7. \$868.45
(grace);
\$868.81.
8. \$971.54
(grace);
\$972.03.
9. \$693.99
(grace);
\$694.45.
10. \$838.37
(grace);
\$838.76.

11. \$1248.12
(grace);
\$1248.59.
1. \$1080.
2. \$1063.80.
\$1063.26
(grace).

Page 189

1. \$1223.64.
2. \$500.09
(grace);
\$500.14.
3. \$726.92
(grace);
\$726.81.
5. \$396.44
(grace);
\$396.18.
6. \$981.14
(grace);
\$980.56.
7. \$1272.
8. \$1240.20
(grace);
\$1239.67.
9. \$1214.76
(grace);
\$1214.12.
10. \$1195.68
(grace);
\$1195.04.
11. 9%.
12. \$452.88
(grace);
\$452.96.
13. \$725.92
(grace);
\$725.98.

14. \$956.74
(grace);
\$956.64.
15. \$806.53
(grace);
\$806.47.
16. \$277.48
(grace);
\$277.53.
17. \$5039.03.
(grace);
\$5037.80.

Page 194

1. \$5022.50.
2. \$1977.50.
3. \$12,000.
4. 180 shares.
5. \$68,400.
6. $12\frac{1}{2}\%$
7. 31 shares;
\$92.
8. 316 shares.
9. 40 shares.
10. \$181,250.

Page 197

1. $6\frac{1}{4}\%$.
2. 4% .
4. 3% bond.
5. $4\frac{1}{2}\%$.
6. \$62.50 gain.
7. $7\frac{1}{2}\%$.
8. \$4875 loss.
9. Equal.
10. \$210.
11. 80 shares;
\$20.
12. 697 $\frac{2}{3}$.
13. \$40; latter.

Page 203

1. 2872.5.
2. \$21.56.
3. \$217.01 $\frac{1}{2}$.
4. \$2085.71.
5. \$312.65.
6. \$73.46.
7. \$6185.
8. \$1265.25.
9. \$12.09.
10. \$5.21.
11. \$40.41.
12. \$89.24
(grace);
\$87.70.
13. \$51.14
(grace);
\$49.61.
14. \$194.74
(grace);
\$193.95.

Page 204

1. \$250.
2. 0.605%.
3. \$150.
4. \$58.80.
5. \$256.10.
6. \$845.56.
7. \$941.11.
8. \$1886.70.
9. \$1213.21.
10. \$1590.
11. Gain \$14.25.
12. \$1960.78.
13. \$289.59.
14. \$511.31
(grace);
\$511.57.
15. \$75.
16. \$500.

Page 205

1. \$1125.
2. 5.13 qt.
3. \$5.47.
4. 3.278+%.
5. Loss \$407.69.
6. $28\frac{5}{13}\%$.
7. $45\frac{1}{3}\%$.
8. \$1890 net loss.
9. \$4,800,000.
10. Latter \$27.28.

Page 207

18. 300.
19. 33 $\frac{1}{2}$.
20. 80.
21. 9.
22. $\frac{1}{2}$.
23. 8 yd.
24. \$20.
25. 75.
26. 60.
27. 96.
28. 65.
29. 43.5.
3. \$15.
3. \$378.

Page 208

5. 9 da.; 3:4.
6. 7 mo.
7. \$11.50.
8. \$33.92.
9. 96 men.
10. $19\frac{5}{8}$.
11. $4\frac{1}{2}$.
12. 56 men.
13. \$4.80 per bbl.
Page 209
2. 60 $\frac{1}{2}$ da.
3. $2\frac{1}{11}$.

4. \$819.
5. 8100 times.
6. 112 ft.
7. 126 mi.: 231 mi.
(depends on
rate).
8. 960 bu.
9. \$3.40.
10. 11 bbl.

Page 210

1. \$62.50.
2. 2.7 mi.
3. \$2.50.
4. \$18.
5. 48 yd.
6. 14 rev.
7. 22 $\frac{1}{2}$ da.
8. 18 men.
9. \$126.
10. 5 $\frac{1}{4}$ hr.
11. 22 $\frac{1}{2}$ da.
12. 4285.71 mi.
13. \$133.35.
14. 24 A.

Page 211

5. 35.
8. 96.
9. 630.
10. 441.
11. 343.
12. 420.

Page 213

13. 23-.
14. 26.
15. 34.
16. 42.
17. 45.
18. 54.

19. 63.
20. 68.
21. 75.
22. 84.

Page 214

3. 28.
4. 58.
5. 92.
6. 56.
7. 83.
8. 52.
9. 73.
10. 67.
11. 99.
12. 87.
13. 53.
14. 97.

Page 215

1. 532.
2. 547.
3. 636.
4. 746.
5. 869.
6. 2453.
7. 728.
8. 696.
9. 799.
10. 852.
11. 877.
12. 987.
13. 1074.
14. 1217.
15. 1594.

Page 216

1. 0.75.
2. 0.96.
3. 6.5.

4. 0.8848+.
5. 0.9433+.
6. 4.4121+.
7. 28.7210+.
8. 4.1273+.
9. 0.8.

10. 0.2529.
11. 43.9590.
12. 27.9991+.
13. 15.03.
14. 1430.
15. 8.74.
16. 23.28+.
17. $\frac{7}{8}$.
18. 0.559+.
19. $2\frac{3}{4}$.
20. 2.380+.
21. 9.099+.
22. 0.5422+.
23. 12.8237+.
24. 1.4790+.

1. 307.
2. 0.8538+.
3. 3.7249+.
4. 1004.
5. 3136.
6. 7921.
7. 45.09.
8. 0.9682+.
9. $1\frac{1}{2}$.
10. 999.
11. $9\frac{1}{2}$.
12. 64.
13. 1.4142+.
14. 0.4472+.
15. 0.19364+.
16. 11.2743+.
17. 28.0178+.
18. 1856.
19. 30.2324+.
20. 0.860813+.

Page 217

1. 8.94+.
2. 3.21+.
3. 2.53+.
4. 3.13+.
5. 2.68+.
6. 4.17+.
7. 3.87+.
8. 2.92+.
9. 3.06+.
10. 2.64.
11. 1.82+.
12. 0.8862+.
13. 6.28+.
14. $\frac{8.18+}{3}$; 2.72+.
15. 0.6.
16. 2.39+.
17. 8.89+.
18. 5.87+.
19. 8.00+.
20. 6.068+.

Page 219

1. 60 ft.
2. 68 ft.
3. 57 ft.
4. 115 ft.
5. 42.42+ ft.
6. 25.61+ in.
7. 8.60+ ft.
8. 223.60+ rd.

Page 220

1. 8.
2. 8.66+.
3. 24.16+ ft.
4. 21.93+ ft.
5. 73.32+ sq. ft.

6. Altitude,
20.78+ ft.;
area,
249.36+ sq. ft.
7. \$141.40.
9. 259.8 sq. in.

Page 221

2. 29.67+ ft.
3. 29.6+ ft.
4. 13.4 ft.
5. 8.
6. 25.37+ rd.
7. 67.8+ mi.
8. 1017.8784 sq. in.
9. 5.56+.
10. 270.4+ ft.
11. 18.02+ ft.
12. 17.97+ ft.

Page 222

1. 6.9282+ rd.
2. 43.863 ft.
3. 5:4.
4. 21.089 rd.
5. 208.71+ ft.
6. 97.616+ ft.
7. 63.639 ft.
8. 101.98+ ft.
9. 108 ft. by 36 ft.
10. 20.78+ ft.
11. 27.12+ ft.
12. 137.08 sq. in.
13. 2368.074 ft.
14. 69.41 rd.

Page 225

1. $29\frac{1}{2}$ cu. ft., or
29.629+ cu. ft.

3. 471.4 cu. in.
4. 1500 cu. in.
5. 6600 lb.
10. 96 sq. in.
11. 384 sq. in.
12. 62.35+ sq. in.
13. 720 cu. in.
14. S. H. 16.15+
in.; c. sur-
face 387.6+
sq. in.

Page 227

1. 1 lb.
2. 14.1372 cu. in.
3. 24 cu. in.
4. 6 sq. in.
5. 37.6992 cu. ft.
6. 8 oz.
9. 40 sq. in.
10. 20.1219+ sq. ft.
11. 93.4626.
12. 219.912 sq. ft.
13. 35.51 sq. yd.

Page 230

1. 0.5236
2. 0.4764
3. 33.5194
4. 235.62
5. 1.1950
6. 4,188,600
7. 64.
8. 0.0226
9. 113.0976
10. 12.5684
11. 9.7054
12. 5.4810
13. 486.9971

Page 233

3. \$2700.
4. 4:9.
5. \$18.56.
6. 13 $\frac{1}{2}$.
7. 5.
8. \$78.75.
9. \$21.09.
10. 27 lb.
11. 1171.87 $\frac{1}{2}$ bu.
12. 96 hr.

Page 237

1. \$9.25.
2. 681.7925+ sq.
ft.
3. 16 rd.
5. 42 $\frac{1}{2}$ sq. ft.
6. 315 sq. ft.
7. \$32.26.
8. 124.686.
9. 875.
10. 1.27324 ft.
11. 3 ft.
12. 768 sq. ft.
13. 4.1888 cu. ft.

Page 238

1. 1080.
2. 6.2882
3. 12.65
4. 530.145
5. 24902.18112
6. 180 rd.
7. 169.38+ rd.
8. 150.40
9. 94.81+
10. 62.0100
11. 20.529+
12. 1607.8125
13. 14.9834

12. 28.2744.
13. 18,000 lb.
14. 706.86 sq. ft.

Page 244

1. 847,200 g.;
1867.75 lb.
2. 26.43 Hl.
3. 16.72 sq. m.
4. 1312.359 yd.
5. 395.38 sq. rd.
6. 1000 l.
7. \$10.63 gain.
8. 960 Kg.
9. 88.9056 Kg.
10. 2.845184 Mm.
11. \$3.3108.
12. 56 Ha.;
138.38 A.
13. 264.17 gal.
14. 800.
15. \$3.21.
16. 196.875 Kg.
17. 7500 m.
18. 13.928694 T.
19. 1.9 nearly.

Page 245

3. 12:55:30 p.m.
4. 7:15:45 a.m.
5. 5:53:20 p.m.
6. 6:00:00 a.m.
7. 6:00:00 a.m.
8. 6:00:00 a.m.
9. 11:59:50 a.m.
10. 5:21 p.m.
11. 12:00 p.m.
12. 3:45:18 a.m.
13. 1:00:00 a.m.
14. 1:00:00 a.m.

Page 251

1. \$60,000
2. \$100,000

3. 8 $\frac{1}{2}$ %.
4. \$21,000.
5. 6 in.
6. \$2.60.
7. \$1.57.
8. 67,200.
9. 8 $\frac{1}{2}$ %.
10. 940 bbl.
11. 33 $\frac{1}{2}$ %.

Page 252

1. \$33,280; \$80
brok.
2. Dec. 22, 1903.
3. \$148.29.
4. \$153.58.
5. 36 weeks.
6. \$10,844.
7. \$80.
8. 672 ft.
9. 112.77.
10. \$346.15.
11. 105,753.6 gal

Page 253

1. \$80
2. \$1800.
3. \$133.33
4. 0.9272
5. 1282.70
6. \$0.56
7. \$24 increase
8. \$77.94
9. \$135.00
10. 12.48
11. \$22,900
12. \$32.00
13. 489.54
14. 1.16

Page 254

1. \$379.88.
2. 100%.
3. \$500.
4. \$200.
5. $\frac{1}{2}$; $\frac{1}{2}$.
6. $\frac{1}{12}$; $1\frac{1}{2}$ da.
7. $5\frac{1}{12}$ hr.
8. $2\frac{1}{2}$ da.
9. 12 da.
10. 7.3 in.
11. Int. \$25.
12. \$647.90.

Page 255

1. \$11,848.80.
2. $578\frac{1}{2}$ cu. ft.
3. \$4649.80.
4. $6\frac{1}{2}$ %.
5. \$3854.17.
6. \$2172.50.
7. 43.30 ft.
8. \$318.06.
9. 25%.
10. \$55.36.
11. \$345.
12. \$905.42.

Page 256

1. Latter.
2. \$767.33.
3. \$2250.
4. 40¢.
5. 30 mi.
6. \$844.95.
7. $16\frac{1}{2}$ %.
8. \$59.63.
9. \$4704.
10. \$43.20.
11. \$42.86 per
\$100 share.

12. 3 mo.
13. $12\frac{1}{2}$ yr.
14. 6.336 in.

Page 257

1. \$1711.11.
2. $\frac{5}{11}$ gal.
3. 280 gal.;
200 gal.
4. $3\frac{1}{2}$ ft.
5. 1368 sq. ft.
6. \$5.60.
7. \$1050.
8. 4 times.
9. 114¢.
10. 55 mi. 2933 $\frac{1}{2}$ ft.
11. 36 mo.
12. $134\frac{1}{2}$ A.
13. \$310.
14. \$28.80.

Page 258

1. \$595.
3. 25%.
4. 49 : 98 = 98 :
196.
5. Latter;
30¢ per yd.
6. 56¢.
7. \$222.
8. 4 ft. 2 in.
9. 200,000 lb.
10. \$1279.94.
11. \$313.50.

Page 259

1. $14,912\frac{1}{2}$ %.
2. $18\frac{1}{2}$ %.
3. 7.854 sq. ft.
4. \$0.94.
5. \$17.63.

6. \$34.45.
7. \$41.48 more.
8. \$0.13¢.
9. 1257.1636 cu. in.
10. \$4936.60.

Page 260

1. $17\frac{1}{2}$ %.
2. First, $\frac{5}{12}$ %
better.
3. \$1412.
4. 35%.
5. \$156.08.
6. 63.8 : 36.2.
7. \$0.94.
8. \$0.01375+.
9. \$3.24.
10. 0.816.
11. \$9.21.
12. 11.55.
13. Increase \$15.
14. 608 ; 544.

Page 261

1. 3125.
2. 107.62 $\frac{1}{2}$.
3. 336.13+.
4. \$12,000.
5. Loss \$16.
6. \$425.
7. \$302.25.
8. First $\frac{1}{2}$ %.
9. 92¢.
10. \$402.
11. $9.36\frac{1}{2}$ %.
12. \$56.53.
13. 1963.5 sq. in.
14. 140.0616 sq. in.

Page 262

1. $\frac{7}{8}$.
2. 70.71 - mi.

3. \$206.45;
\$208.
4. \$495.08;
\$494.83
(grace).
5. \$2934.37.
6. @ 25% =
\$16.09 less.
7. 21 rd. \times 28 rd.;
588 sq. rd.
8. L. price
\$960;
cost \$400.
9. \$148 $\frac{1}{2}$.
10. \$26.16.
11. \$889.43.
12. \$0.86.
13. \$25,000.
14. $6\frac{1}{2}$ %.

Page 263

1. $\frac{1}{4}$.
2. 70+ gal.
4. 389.7+ ft.
5. $33\frac{1}{2}$ %.
6. 60%.
7. \$632.10.
8. $66\frac{1}{2}$ %.
9. \$94.50.
10. \$80.
11. \$640.25.
12. \$450.
13. \$34.31.

Page 264

1. 1.308+.
2. \$518.55.
3. 48 ft.
4. \$555.10.
5. 420.168+.
6. \$17.92.

7. \$352.837+.
8. \$3668.75.
9. \$35,000.
10. $87\frac{1}{2}\%$.
11. \$58,500.
12. \$2971.25.
13. 56.568+ rd.
14. \$344.53.

Page 265

1. $\frac{7}{8}$.
2. \$340.
3. \$40.32.
4. 160 bbl.
5. \$10.75.
6. \$56.77.
7. 16 rd.
8. \$260.40;
\$13.02 com.
9. 56.56+ rd.
10. \$5.50.
11. \$1930.43.
12. $7\frac{1}{2}\%$.
13. 5%.

Page 266

1. \$55,000.
2. 44.74- %
3. $54\frac{2}{3}\%$.
4. 413,013 $\frac{1}{2}$.
5. 9 A.
6. \$2.75.
7. $177\frac{1}{3}$.
8. $22\frac{1}{11}\%$.
9. $\frac{1}{3}$.
10. 166 $\frac{1}{2}$.
11. 3616.037+ bu.
12. $16\frac{2}{3}\%$.
13. 93.81 - rd.
14. \$22.68.
15. \$5190.

Page 267

1. $\frac{3}{4}$.
2. 12 ft.
3. 50%.
4. 80.
5. 154.284+ bu.
6. \$140.62 $\frac{1}{2}$.
7. \$5.60.
8. \$90.56.
9. 125%.
10. \$2.625.
11. \$12.
12. \$252.53.
13. 125 ft.
14. \$198.

Page 268

1. \$4.375.
2. $166\frac{2}{3}\%$.
3. 40¢.
4. \$8.27.
5. $4\frac{1}{11}\%$.
6. \$371.20.
7. \$40.
8. \$105.80.
9. \$391.27.
10. \$620.82.
11. \$187.20.
12. \$220.

Page 269

1. $32\frac{1}{2}$.
2. $8\frac{1}{2}\%$.
3. \$6150.
4. 165 ft.
5. \$3761.25.
6. $966\frac{2}{3}\%$.
7. \$421.80.
8. $5\frac{1}{2}$ ft.
9. \$5.47.

10. Six hund. six
thous. and
fifteen tril-
lionths.
11. Nothing.
12. $14\frac{2}{3}\%$.
13. \$625.

Page 270

1. $53\frac{1}{3}$.
2. \$67.20.
3. \$28.80.
4. \$17.36-.
5. \$51,568.
6. 9929.4.
7. 62.3376+ min.
8. 1765.17 gal.
9. 1:10.
10. \$20.
11. \$5157.43.
12. \$4499.25.

Page 271

1. 0.016 $\frac{2}{3}$.
2. \$21.
3. 44.74- %.
4. 2473.
5. $68\frac{1}{3}\%$.
6. \$93.75.
7. \$33,333 $\frac{1}{3}$.
8. 10¢.
9. 12 da.
10. \$283.50.
11. 32.725 lb.
12. $141\frac{1}{2}$ cu. yd.
13. 12 rd.

Page 272

1. $2\frac{7}{11}$.
2. \$5413 $\frac{1}{2}$.

3. \$94.52.
4. \$418.88.
5. \$56,250; $3\frac{1}{2}\%$.
6. \$25.92.
7. 64 men.
8. \$8000.
9. \$76.
10. \$377.32.
11. \$150.
12. \$421.55.

Page 273

1. $1\frac{3}{4}$.
2. 159,155 cu. ft.
3. 116 ft.
4. Loses \$10.
5. \$15.80.
6. $1\frac{1}{2}$.
7. \$96.21.
8. \$577.40.
9. \$386; cost
600 fr.
10. \$12.75.
11. 0.02078125.
12. 810.
13. \$5700.

Page 274

1. $21\frac{1}{2}$.
2. $21\frac{1}{4}$.
3. Seven thou-
sand four
hundred
eighty-five
and two
ninths ten-
thou-
sandths.
4. \$43.20.
5. \$82.08.

6. \$2007.50.
7. 125.
8. \$1337.
9. $24\frac{1}{2}$ bu.
10. \$2.13.
11. \$150.

Page 275

1. $2\frac{3}{4}$.
2. \$8625.
3. \$3978.72.
4. $66\frac{3}{4}$.
5. 125%.
6. \$9.45.
7. 8%.
8. \$21.84.
9. \$7.48.
10. \$1.80.
11. \$4824.
12. \$32.18; \$4.95
per thous.

Page 276

1. $12\frac{1}{2}$.
2. 720.288+.
3. $87\frac{3}{4}$ sq. yd.
4. \$7400, $4\frac{1}{2}\%$
5. \$39.65.
6. \$200 income;
 $3\frac{1}{3}\%$
7. \$12.
8. \$24.79.

9. 80.
10. \$26.40.
11. \$53.90.
12. \$48.
13. \$640.

Page 277

1. 4.
2. $6\frac{1}{2}\%$.
3. 17 ft.
4. \$9; 60%.
5. \$1115.
6. 15¢.
7. 71.55+rd.
8. 20%.
9. \$560.
10. 1728.
11. $5\frac{1}{2}\%$; \$0.75
per bu.; \$558.
12. \$495.58
(grace);
\$495.83.

Page 278

1. \$18.29.
2. \$36.75.
3. 20 in.
4. 5¢.
5. \$739.78.
6. 9 cd.
7. \$23.
8. \$26.18.

9. \$4.50.
10. \$88.36.
11. \$1562.50.
12. \$8.66.

Page 279

1. $6\frac{1}{4}$.
2. \$1546.79.
3. \$150.
4. 0.853+.
5. \$120.
6. 1.002.
7. 32 shares.
8. \$59.45.
9. \$10.33.
10. $\frac{2}{3}$.
11. 12.806+ft.
12. Variable.
13. July 2, noon,
or July 2, 12
p.m.
14. 75¢.
15. $12\frac{1}{4}\%$.

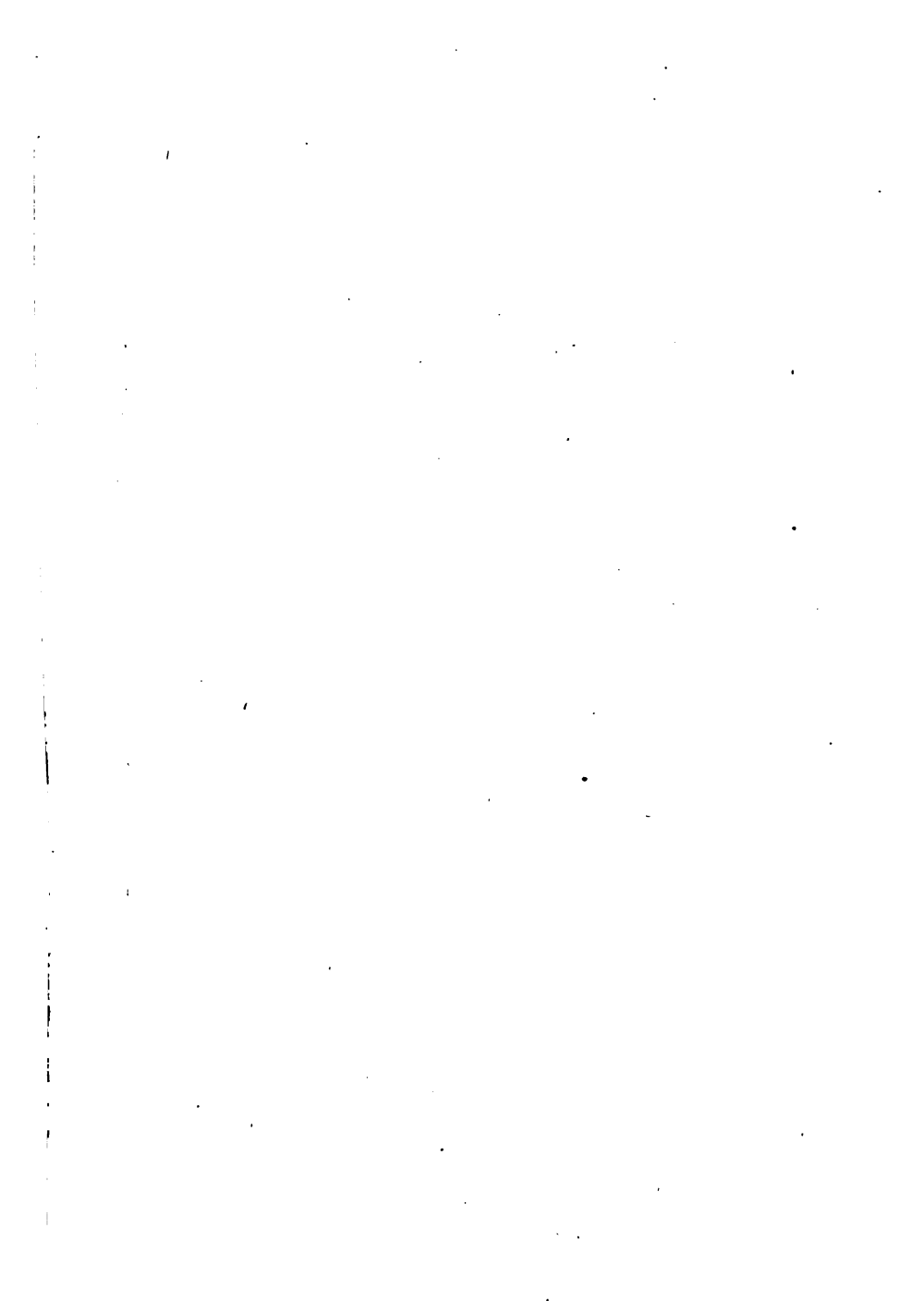
Page 283

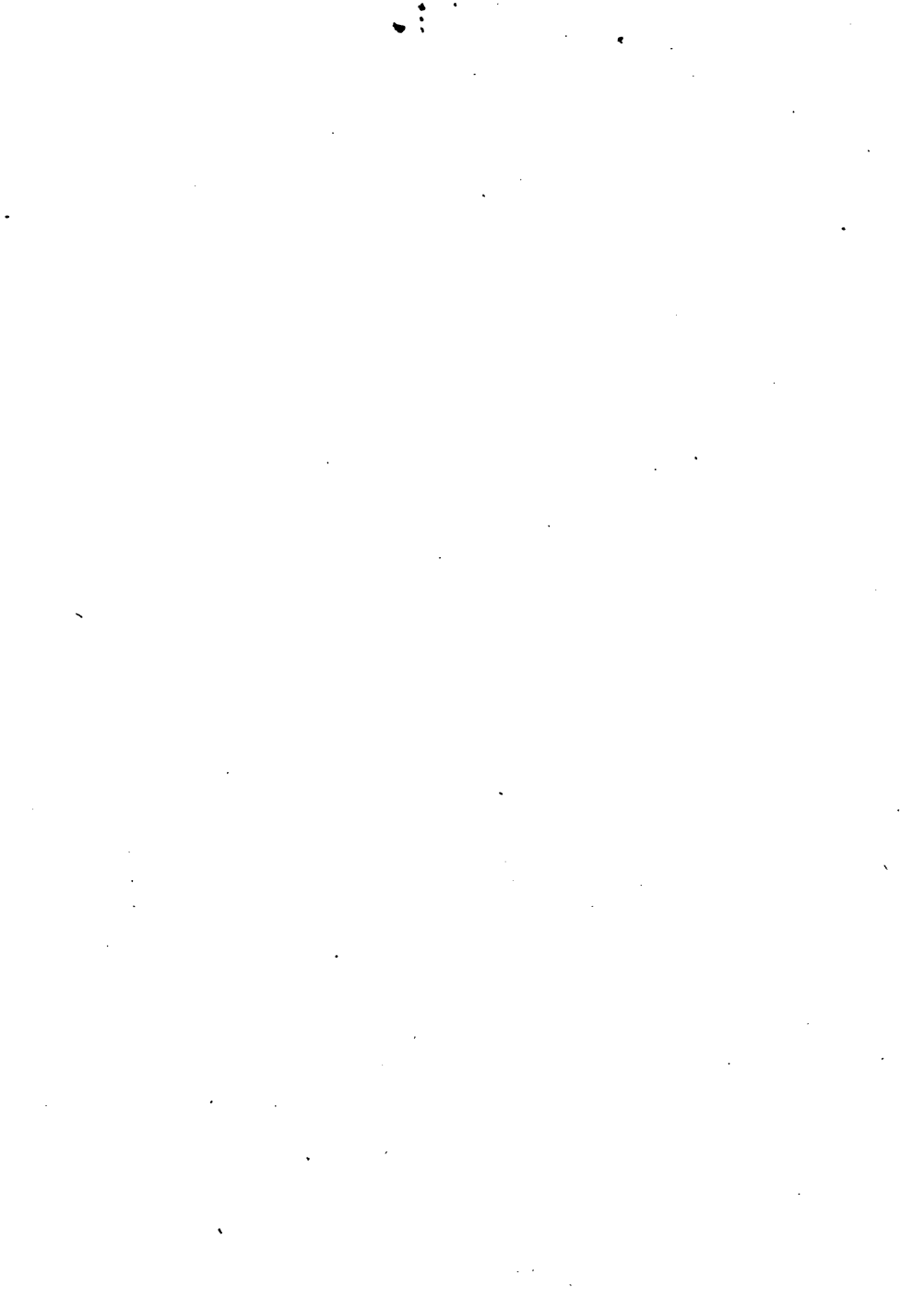
1. 23.
2. 32.
3. 36.
4. 42.
5. 47.
6. 54.
7. 56.

8. 62.
9. 72.
10. 78.
11. 85.
12. 96.
13. 75 in.
14. 156.
15. 235.
16. 3.34.
17. 0.4644+.
18. 0.444+.
19. 2.924+.
20. $1\frac{1}{2}$.
21. $\frac{2}{3}$.
22. $1\frac{1}{2}$.
23. $1\frac{1}{2}$.
24. $1\frac{1}{2}$.
25. 4.175+.
26. $7\frac{3}{4}$.

Page 284

1. 18 ft.
2. 44 in.
3. 32 ft.
4. 35.9 - in.
5. 3.96+ft.
6. \$43.20.
7. 12,150 sq. in.
8. 11.44+in.
9. 96 lb.
10. $1\frac{1}{2}$ oz.
11. 48 in.
12. 8.55 - ft.
13. $\frac{1}{2}$ lb.





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